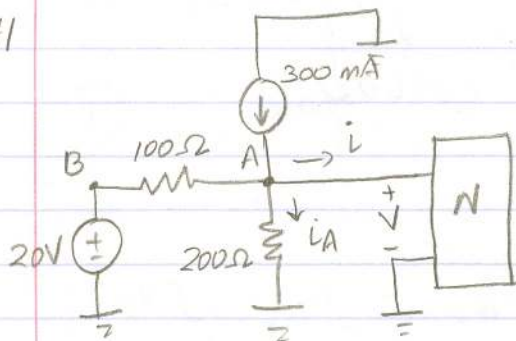


MAE 140
HW #3 SOLUTIONS

3.71



$$i = (5 \times 10^{-3} V - 0.5)$$

$$V = V_A$$

At A

$$0.3 = i + \frac{V_A - 20}{100} + \frac{V_A}{200}$$

$$0.3 = (0.005 V_A - 0.5) + 0.01 V_A - 0.2 + V_A (0.005)$$

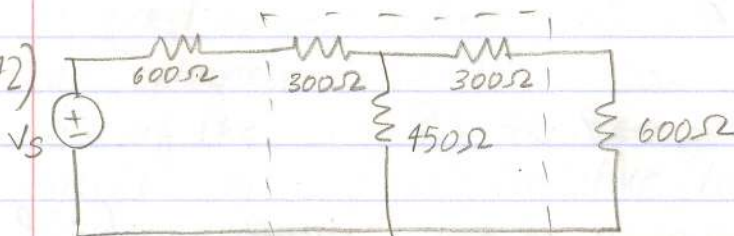
$$0.3 = 0.01 V_A + 0.01 V_A - 0.7$$

$$1 = 0.02 V_A$$

$$V_A = \frac{100}{2} = 50 V$$

$$V = V_A = 50 V$$

3.72



$$P = \frac{V^2}{R}$$

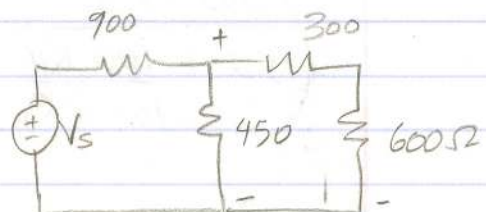


$$\frac{1}{R_{eq}} = \frac{1}{450} + \frac{1}{900}$$

$$R_{eq} = \frac{900(450)}{1350}$$

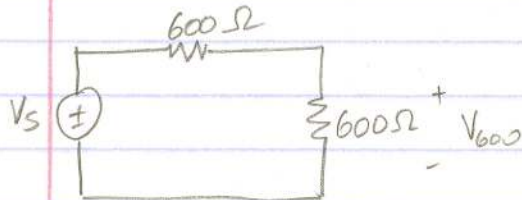
$$V_{300} = \frac{300}{1200} V_s = \frac{V_s}{4}$$

$$V_{600} = \frac{\frac{600}{3}}{\frac{900}{3}} \left(\frac{V_s}{4} \right) = \frac{V_s}{6}$$



$$P = \frac{V_{600}^2}{R} = \frac{V_s^2}{36 \times 600} = \frac{V_s^2}{21600}$$

Without Attenuator



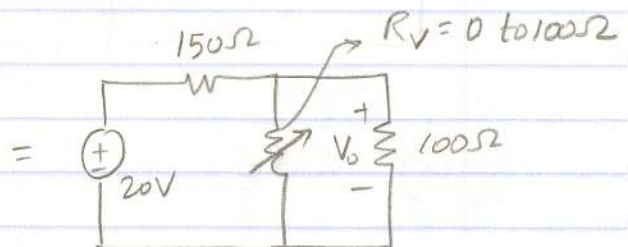
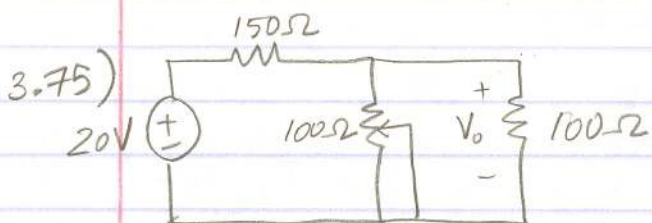
$$V_{600} = \frac{600}{600 + 600} (V_s) = \frac{600 (V_s)}{1200} = \frac{V_s}{2}$$

$$P = \frac{V_{600}^2}{R} = \frac{V_s^2}{4(600)} = \frac{V_s^2}{2400}$$

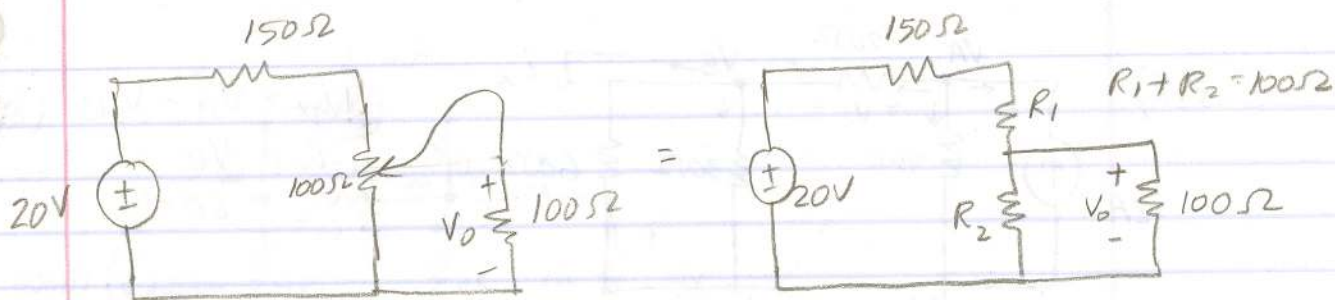
$$\frac{P_{w/o \text{ attenuator}}}{P_{w/ \text{ attenuator}}} = \frac{V_s^2 \times 21600}{2400 \times V_s^2} = 9$$

$$\frac{P_{w/ \text{ attenuator}}}{P_{w/o \text{ attenuator}}} = \frac{1}{9}$$

$$\Rightarrow 10 \log\left(\frac{1}{9}\right) = -9.54 \text{ dB}$$



$$V_o = \left(\frac{100 R_v}{100 + R_v} \right) (20V) = 20 \left(\frac{100 R_v}{15000 + 250 R_v} \right)$$



$$V_0 = 20 \left(\frac{\frac{100 R_2}{100 + R_2}}{150 + R_1 + \frac{100 R_2}{100 + R_2}} \right)$$

$$= 20 \left(\frac{100 R_2}{(150 + R_1)(100 + R_2) + 100 R_2} \right)$$

$$R_1 + R_2 = 100$$

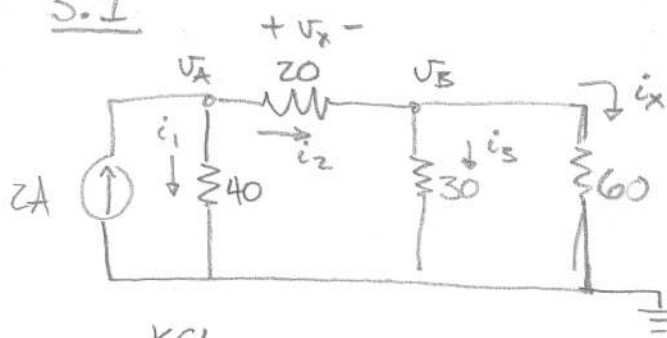
$$V_0 = 20 \left(\frac{100 R_2}{25000 + 250 R_2 - R_2^2} \right)$$

We need to find a circuit in which the output voltage can ^{be} obtained close to 0. Looking at V_0 equations for each of the circuit, we can see that in the first circuit V_0 is more sensitive to the changes in R_1 .

Thus, getting V_0 close to 0 could be difficult.

In the second circuit, V_0 is less sensitive to changes in R_2 . Therefore, it is more desirable.

3.1



I) Select a reference node

KCL

@ A: $2 = i_1 + i_2$

@ B: $i_2 = i_3 + i_4$

Ohm's Law: $v = iR \Rightarrow i = \frac{v}{R}$

@ A: $2 = (\frac{1}{40})v_A + (\frac{1}{20})(v_A - v_B)$

@ B: $(\frac{1}{20})(v_A - v_B) = (\frac{1}{30})v_B + (\frac{1}{60})v_B$

$$\Rightarrow \begin{cases} (\frac{1}{40} + \frac{1}{20})v_A - (\frac{1}{20})v_B = 2 \\ (-\frac{1}{20})v_A + (\frac{1}{20} + \frac{1}{30} + \frac{1}{60})v_B = 0 \end{cases}$$

or

$$\begin{bmatrix} (\frac{1}{40} + \frac{1}{20}) & -(\frac{1}{20}) \\ -(\frac{1}{20}) & (\frac{1}{20} + \frac{1}{30} + \frac{1}{60}) \end{bmatrix} \begin{bmatrix} v_A \\ v_B \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

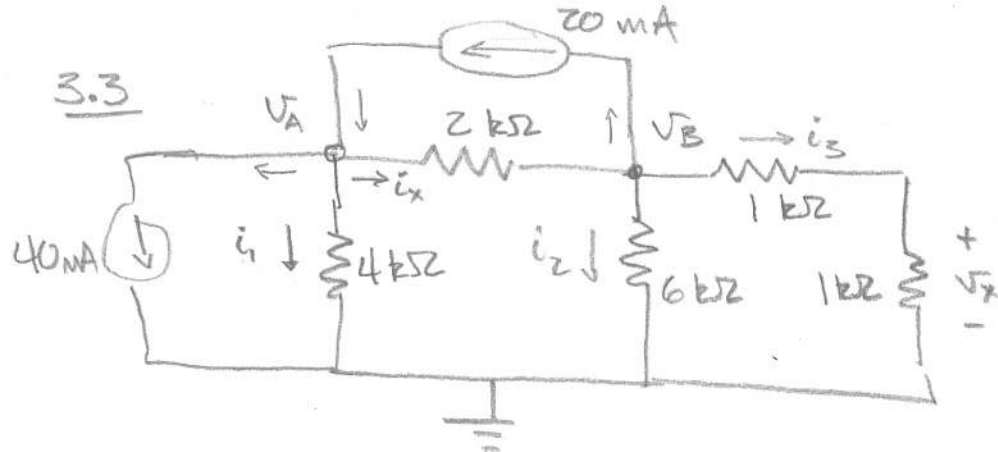
(b) Find v_x and i_x :

$$\begin{bmatrix} v_A \\ v_B \end{bmatrix} = \begin{bmatrix} (\frac{1}{40} + \frac{1}{20}) & -\frac{1}{20} \\ -\frac{1}{20} & (\frac{1}{20} + \frac{1}{30} + \frac{1}{60}) \end{bmatrix}^{-1} \begin{bmatrix} 2 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} v_A \\ v_B \end{bmatrix} = \begin{bmatrix} 40 \\ 20 \end{bmatrix}$$

from our layout: $v_A - v_B = v_x \Rightarrow (40 - 20) = \boxed{v_x = 20 \text{ V}}$

and $v_B = i_x(60) \Rightarrow 20 = i_x(60) \Rightarrow i_x = \frac{20}{60} = \boxed{\frac{1}{3} \text{ A} = i_x}$

3.3

KCL @ A/B

$$A: 20 \text{ mA} - 40 \text{ mA} - i_1 - i_x = 0$$

$$B: i_x - 20 \text{ mA} - i_2 - i_3 = 0$$

$$\text{w/ Ohm's Law: } V = iR \Rightarrow i = V/R$$

$$i_1 = \left(\frac{1}{4000}\right)V_A \quad i_x = \left(\frac{1}{2000}\right)(V_A - V_B) \quad i_2 = \left(\frac{1}{6000}\right)V_B$$

$$i_3 = \left(\frac{1}{1000 + 1000}\right)V_B$$

$$A: -20 \text{ mA} - \left(\frac{1}{4000}\right)V_A - \left(\frac{1}{2000}\right)(V_A - V_B) = 0$$

$$B: \left(\frac{1}{2000}\right)(V_A - V_B) - \left(\frac{1}{6000}\right)V_B - \left(\frac{1}{2000}\right)V_B - 20 \text{ mA} = 0$$

$$\Downarrow$$

$$A: \left(\frac{1}{4000} + \frac{1}{2000}\right)V_A - \left(\frac{1}{2000}\right)V_B = -20 \text{ mA}$$

$$B: \left(\frac{1}{2000} + \frac{1}{6000} + \frac{1}{2000}\right)V_B - \left(\frac{1}{2000}\right)V_A = -20 \text{ mA}$$

or

$$\begin{bmatrix} \left(\frac{1}{4000} + \frac{1}{2000}\right) & -\left(\frac{1}{2000}\right) \\ -\left(\frac{1}{2000}\right) & \left(\frac{1}{2000} + \frac{1}{6000} + \frac{1}{2000}\right) \end{bmatrix} \begin{bmatrix} V_A \\ V_B \end{bmatrix} = \begin{bmatrix} -20 \times 10^{-3} \\ -20 \times 10^{-3} \end{bmatrix}$$

3.3 cont

which, when solved

$$\Rightarrow \begin{bmatrix} V_A \\ V_B \end{bmatrix} = \begin{bmatrix} -160/3 \\ -40 \end{bmatrix}$$

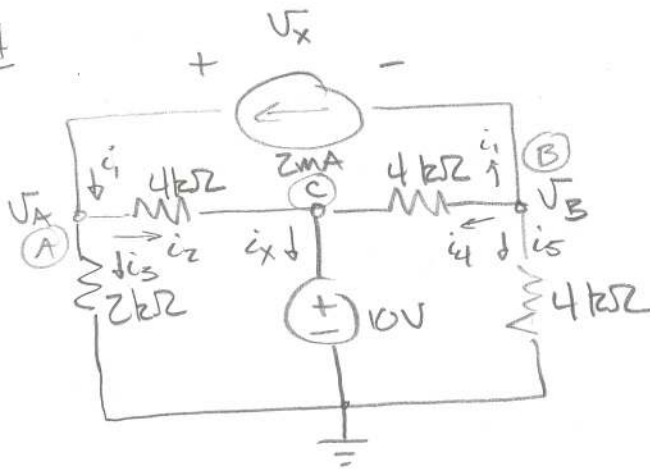
voltage divider:

$$V_x = \left(\frac{1000}{2000} \right) V_B = \left(\frac{1}{2} \right) (-40) = \boxed{-20V}$$

$$i_x = \frac{(V_A - V_B)}{2000} = \frac{-160/3 + 40}{2000} = \frac{-40/3}{2000} = \frac{-40}{6000} = \frac{-20}{3000} A$$

or $i_x = -\frac{20}{3} \text{ mA}$

3.4



things we know: $i_1 = 2 \text{ mA}$, $V_C = 10 \text{ V}$

Ohm's law for each element:

$$i_2 = \frac{V_A - V_C}{4000} \Rightarrow \frac{V_A - 10}{4000}$$

$$i_3 = \frac{V_A}{2000}$$

$$i_4 = \frac{V_B - V_C}{4000} \Rightarrow \frac{V_B - 10}{4000}$$

$$i_5 = \frac{V_B}{4000}$$

KCL @ A: $i_1 - i_2 - i_3 = 0$

@ B: $-i_1 - i_4 - i_5 = 0$

KCL + elements:

$$\Rightarrow \text{@ A: } \frac{2}{1000} - \frac{V_A - 10}{4000} - \frac{V_A}{2000} = 0 \Rightarrow \left(\frac{1}{4000} + \frac{1}{2000} \right) V_A = \frac{9}{2000}$$

$$\text{@ B: } -\frac{2}{1000} - \frac{V_B - 10}{4000} - \frac{V_B}{4000} = 0 \Rightarrow \left(\frac{1}{4000} + \frac{1}{4000} \right) V_B = \frac{1}{2000}$$

$$\underline{\text{or}}$$

$$\begin{bmatrix} \left(\frac{1}{4000} + \frac{1}{2000} \right) & 0 \\ 0 & \left(\frac{1}{4000} + \frac{1}{4000} \right) \end{bmatrix} \begin{bmatrix} V_A \\ V_B \end{bmatrix} = \begin{bmatrix} \frac{9}{2000} \\ \frac{1}{2000} \end{bmatrix}$$

3.4 cont.

solving this system $X = A^{-1}B$

$$\Rightarrow \begin{bmatrix} V_A \\ V_B \end{bmatrix} = \begin{bmatrix} 6 \\ 1 \end{bmatrix}$$

and we know that

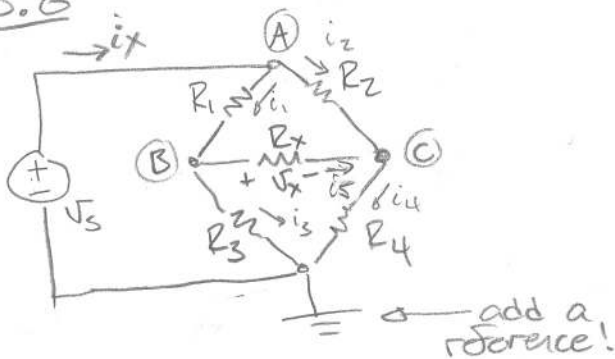
$$\textcircled{C}: i_x = i_2 + i_4 = \frac{V_A - 10}{4000} + \frac{V_B - 10}{4000} = \frac{-4}{4000} + \frac{-9}{4000} = \frac{-13}{4000} \text{ A}$$

or $= i_x$

$$\frac{-13}{4} \text{ mA}$$

and $V_x = V_A - V_B = 6 - 1 = \boxed{5 \text{ V} = V_x}$

3.8



element laws:

$$i_1 = \frac{V_A - V_B}{R_1}, \quad i_2 = \frac{V_A - V_C}{R_2}$$

$$i_3 = \frac{V_B}{R_3}, \quad i_4 = \frac{V_C}{R_4}, \quad i_5 = \frac{V_B - V_C}{R_x}$$

things we know:

$$\underline{V_A = V_S!}$$

KCL equations

A: no need: $V_S = V_A!$

B: $i_1 - i_5 - i_3 = 0$

C: $i_2 + i_x - i_4 = 0$

(plug element laws)

$$\Rightarrow \text{B: } \frac{V_A - V_B}{R_1} - \frac{V_B - V_C}{R_x} - \frac{V_B}{R_3} = 0$$

$$\text{C: } \frac{V_A - V_C}{R_2} + \frac{V_B - V_C}{R_x} - \frac{V_C}{R_4} = 0$$

3.8 cont

simplifying the previous equations

note $V_B = V_A = \text{constant}$

$$B: \frac{V_A}{R_1} - \frac{V_B}{R_1} - \frac{V_B}{R_x} + \frac{V_C}{R_x} - \frac{V_B}{R_3} = 0 \Rightarrow \left(\frac{1}{R_1} + \frac{1}{R_x} + \frac{1}{R_3}\right)V_B - \left(\frac{1}{R_x}\right)V_C = \left(\frac{1}{R_1}\right)V_S$$

$$C: \frac{V_A}{R_2} - \frac{V_B}{R_2} + \frac{V_B}{R_x} - \frac{V_C}{R_x} - \frac{V_C}{R_4} = 0 \Rightarrow \left(\frac{1}{R_2} + \frac{1}{R_x} + \frac{1}{R_4}\right)V_C - \left(\frac{1}{R_x}\right)V_B = \left(\frac{1}{R_2}\right)V_S$$

$$\begin{bmatrix} \left(\frac{1}{R_1} + \frac{1}{R_x} + \frac{1}{R_3}\right) & -\left(\frac{1}{R_x}\right) \\ -\left(\frac{1}{R_x}\right) & \left(\frac{1}{R_2} + \frac{1}{R_x} + \frac{1}{R_4}\right) \end{bmatrix} \begin{bmatrix} V_B \\ V_C \end{bmatrix} = \begin{bmatrix} \left(\frac{1}{R_1}\right)V_S \\ \left(\frac{1}{R_2}\right)V_S \end{bmatrix}$$

(b) given $R_1 = R_4 = 1000 \Omega$

$R_2 = R_3 = 250 \Omega$

$R_x = 500 \Omega$

$V_S = 15 \text{ V}$

find V_x and i_x

$$\Rightarrow \begin{bmatrix} \left(\frac{1}{1000} + \frac{1}{500} + \frac{1}{250}\right) & -\frac{1}{500} \\ -\frac{1}{500} & \left(\frac{1}{250} + \frac{1}{500} + \frac{1}{1000}\right) \end{bmatrix} \begin{bmatrix} V_B \\ V_C \end{bmatrix} = \begin{bmatrix} \left(\frac{1}{1000}\right)(15) \\ \left(\frac{1}{250}\right)(15) \end{bmatrix}$$

||

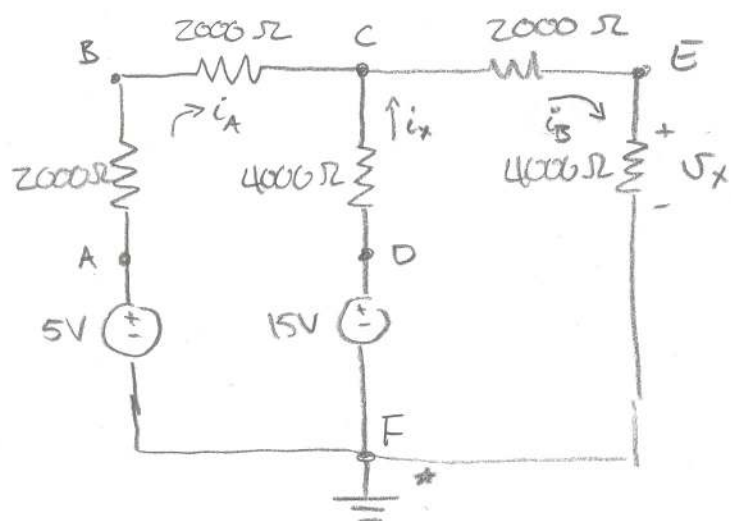
$$\begin{bmatrix} \frac{7}{1000} & -\frac{1}{500} \\ -\frac{1}{500} & \frac{7}{1000} \end{bmatrix} \begin{bmatrix} V_B \\ V_C \end{bmatrix} = \begin{bmatrix} \frac{3}{200} \\ \frac{3}{50} \end{bmatrix} \Rightarrow \begin{bmatrix} V_B \\ V_C \end{bmatrix} = \begin{bmatrix} 5 \\ 10 \end{bmatrix} \text{ V}$$

and note $V_x = V_B - V_C = 10 - 5 = \boxed{5 \text{ V}}$

$$@A: i_x = i_1 + i_2 = \frac{V_A - V_B}{R_1} + \frac{V_A - V_C}{R_2} = \frac{V_S - V_B}{R_1} + \frac{V_S - V_C}{R_2}$$

$$= \frac{15 - 5}{1000} + \frac{15 - 10}{250} = \frac{10}{1000} + \frac{5}{250} = \boxed{\frac{30}{1000} \text{ A}} \\ \text{or } 30 \text{ mA}$$

3.16



We need to deal w/
 $\oplus$'s, choose option 2
 and add a ground ($\frac{1}{\infty}$)
 @ F

$$\Rightarrow \underline{V_A = 5V} \text{ and } \underline{V_D = 15V}$$

By analysis inspection:

$$@ B: \left(\frac{1}{2000} + \frac{1}{2000} \right) V_B - \left(\frac{1}{2000} \right) V_A - \left(\frac{1}{2000} \right) V_C = 0$$

$$@ C: \left(\frac{1}{2000} + \frac{1}{4000} + \frac{1}{2000} \right) V_C - \left(\frac{1}{2000} \right) V_B - \left(\frac{1}{4000} \right) V_D - \left(\frac{1}{2000} \right) V_E = 0$$

$$@ E: \left(\frac{1}{2000} + \frac{1}{4000} \right) V_E - \left(\frac{1}{2000} \right) V_C - \left(\frac{1}{4000} \right) V_F = 0$$

$$\rightarrow V_F = 0 \text{ b/c } F \text{ is now } \frac{1}{\infty}$$

remember V_A and V_D are constants

so we move them to the "B" matrix

$$\begin{bmatrix} \left(\frac{1}{2000} + \frac{1}{2000} \right) & -\left(\frac{1}{2000} \right) & 0 \\ -\left(\frac{1}{2000} \right) & \left(\frac{1}{2000} + \frac{1}{4000} + \frac{1}{2000} \right) & -\left(\frac{1}{2000} \right) \\ 0 & -\left(\frac{1}{2000} \right) & \left(\frac{1}{2000} + \frac{1}{4000} \right) \end{bmatrix} \begin{bmatrix} V_B \\ V_C \\ V_E \end{bmatrix} = \begin{bmatrix} \left(\frac{1}{2000} \right) V_A \\ \left(\frac{1}{4000} \right) V_D \\ 0 \end{bmatrix}$$

|||

$$\begin{bmatrix} \frac{1}{1000} & -\frac{1}{2000} & 0 \\ -\frac{1}{2000} & \frac{1}{800} & -\frac{1}{2000} \\ 0 & -\frac{1}{2000} & \frac{3}{4000} \end{bmatrix} \begin{bmatrix} V_B \\ V_C \\ V_E \end{bmatrix} = \begin{bmatrix} \frac{5}{2000} \\ \frac{15}{4000} \\ 0 \end{bmatrix}$$

solving this system:

$$\begin{bmatrix} V_B \\ V_C \\ V_E \end{bmatrix} = \begin{bmatrix} 6.25 \\ 7.5 \\ 5 \end{bmatrix}$$

B/c we used a gnd. at F $\Rightarrow V_x = V_E - V_F = V_E$

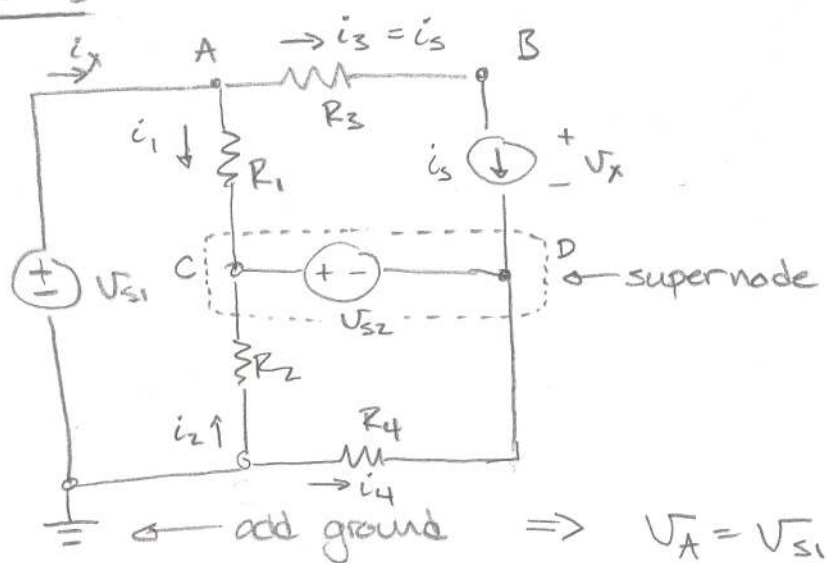
$$\Rightarrow \boxed{V_x = 5 \text{ V}}$$

Consider Ohm's Law for the middle 4 k Ω Resistor:

$$V = iR \Rightarrow (V_D - V_E) = i_x (4000)$$

$$i_x = \frac{15 - 7.5}{4000} = \frac{7.5}{4000} = \boxed{\frac{15}{8} \text{ mA}}$$

3.13



super-node equation: $i_1 + i_2 + i_s + i_4 = 0$

$$= \frac{1}{R_1}(V_A - V_C) + \frac{1}{R_2}(0 - V_C) + i_s + \frac{1}{R_4}(0 - V_D) = 0$$

and $V_C - V_D = V_{s2}$

KCL @ B: $\frac{1}{R_3}(V_A - V_B) = i_s$

which gives us 3 equations:

$$\frac{1}{R_1}V_A - \left(\frac{1}{R_1} + \frac{1}{R_2}\right)V_C - \frac{1}{R_4}V_D = -i_s$$

$$V_C - V_D = V_{s2}$$

$$\frac{1}{R_3}V_A - \frac{1}{R_3}V_B = i_s$$

and, b/c of the $\frac{1}{\infty}$, $V_A = V_{s1}$

$$-\left(\frac{1}{R_1} + \frac{1}{R_2}\right)V_C - \frac{1}{R_4}V_D = -i_s - \frac{1}{R_1}V_{s1}$$

$$V_C - V_D = V_{s2}$$

$$\frac{1}{R_3}V_B = \frac{1}{R_3}V_{s1} - i_s$$

which gives us 3 eq for 3 unknowns (V_B, V_C, V_D) and can be written as

$$\begin{bmatrix} 0 & -(\frac{1}{R_1} + \frac{1}{R_2}) & -\frac{1}{R_4} \\ 0 & 1 & -1 \\ \frac{1}{R_3} & 0 & 0 \end{bmatrix} \begin{bmatrix} V_B \\ V_C \\ V_D \end{bmatrix} = \begin{bmatrix} -i_s - \frac{1}{R_1} V_{s1} \\ V_{s2} \\ \frac{1}{R_3} V_{s1} - i_s \end{bmatrix}$$

(b) when $R_1 = R_2 = 10,000 \Omega$
 $R_3 = 2,000 \Omega$
 $R_4 = 1,000 \Omega$
 $i_s = 2.5 \times 10^{-3} A$
 $V_{s1} = 12 V$
 $V_{s2} = 0.5 V$

$$\Rightarrow V_B = V_{s1} - R_3 i_s = 12 - (2.5)(2) = 12 - 5 = \boxed{7 V = V_B}$$

and

$$\begin{bmatrix} -\frac{2}{10} & -1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} V_C \\ V_D \end{bmatrix} = \begin{bmatrix} -3.7 \\ 0.5 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} V_C \\ V_D \end{bmatrix} = \begin{bmatrix} 3.5 \\ 3 \end{bmatrix}$$

(b)

$$\Rightarrow V_x = V_B - V_D = 7 - 3 \Rightarrow \boxed{V_x = 4 V}$$

KCL @ A: $i_x = i_1 + i_s = \frac{1}{R_1} (V_A - V_C) + i_s = \frac{1}{10000} (12 - 3.5) + 2.5 \times 10^{-3}$

$$\Rightarrow \boxed{i_x = 3.35 \text{ mA}}$$

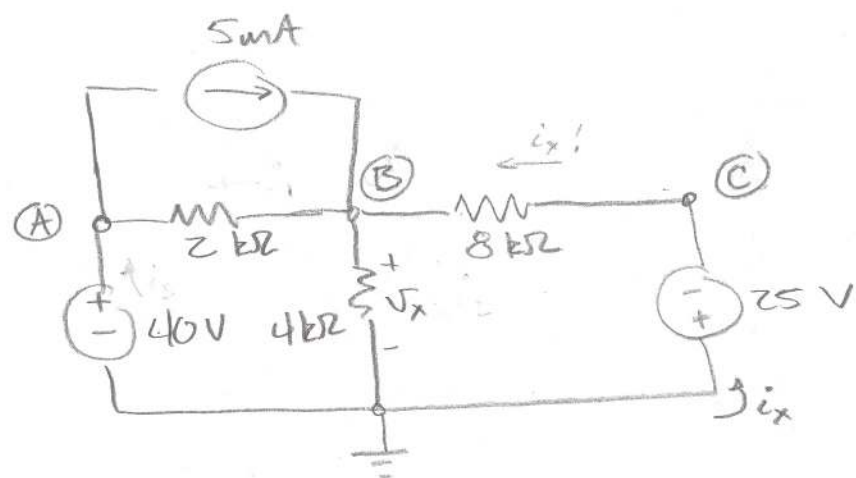
3.13 cont.

(c) the power supplied by V_{s1}

$$P = V i \Rightarrow P = (V_{s1})(i_x) = (12V)(3.35 \times 10^{-3})$$

$$\begin{aligned} &= 0.0402W \\ &P = 40.2mW \end{aligned}$$

3.15



w/ $\frac{1}{\square}$:

$$\Rightarrow V_A = 40 \text{ V}$$

$$V_C = -25 \text{ V}$$

(Method 2)

Need only write 1 nodal equation

$$\left(\frac{1}{2000} + \frac{1}{4000} + \frac{1}{8000} \right) V_B - \left(\frac{1}{2000} \right) V_A - \left(\frac{1}{8000} \right) V_C = (5 \times 10^{-3})$$

$$\left(\frac{7}{8000} \right) V_B = \left(\frac{1}{2000} \right) (40) + \left(\frac{1}{8000} \right) (-25) + (5 \times 10^{-3}) \quad \frac{5}{1000}$$

$$V_B \left(\frac{7}{8000} \right) = \frac{160}{8000} - \frac{25}{8000} + \frac{40}{8000}$$

$$\Rightarrow V_B = \frac{175}{7} \Rightarrow \boxed{V_B = 25 \text{ V}}$$

(b)

$$V_x = V_B = 0 \Rightarrow \boxed{V_x = V_B = 25 \text{ V}}$$

$$i_x = \frac{V_C - V_B}{8000} = \frac{-25 - 25}{8000} = \frac{-50}{8000} = \boxed{-\frac{25}{4} \text{ mA}}$$

$$(c) P = \frac{V^2}{R}$$

$$P_T = \sum_{n=1}^3 P_n$$

$$P_T = \frac{(V_B - V_A)^2}{2000} + \frac{(V_B - V_C)^2}{8000} + \frac{(V_B - 0)^2}{4000} = \frac{225}{2000} + \frac{2500}{8000} + \frac{225}{4000}$$

$$\boxed{= 581.25 \text{ mW or } 0.581 \text{ W}}$$