

1) find $F(s)$, poles & zeros

$$f(t) = A[e^{-\alpha t} - 2e^{-\gamma t}]u(t)$$

$$F(s) = A \left(\frac{1}{s+\alpha} - \frac{2}{s+\gamma} \right)$$

$$= -A \frac{(s+2\alpha-\gamma)}{(s+\alpha)(s+\gamma)}$$

poles: $-\alpha, -\gamma$ zeros: $-2\alpha + \gamma$ 4) find $F(s)$, poles & zeros

$$f(t) = A[\cos(\beta t) - \sin(\beta t)]u(t)$$

$$F(s) = A \left(\frac{s}{s^2 + \beta^2} - \frac{\beta}{s^2 + \beta^2} \right)$$

$$= \frac{A(s-\beta)}{s^2 + \beta^2}$$

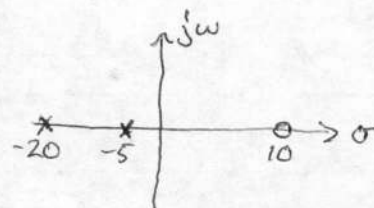
poles: $\pm j\beta$ zeros: β 7) find $F(s)$ & plot pole-zero diagrams

a) $f_1(t) = [5e^{-5t} - 10e^{-20t}]u(t)$

$$F_1(s) = \frac{5}{s+5} - \frac{10}{s+20}$$

$$= \frac{-5(s-10)}{(s+5)(s+20)}$$

(graph not to scale)



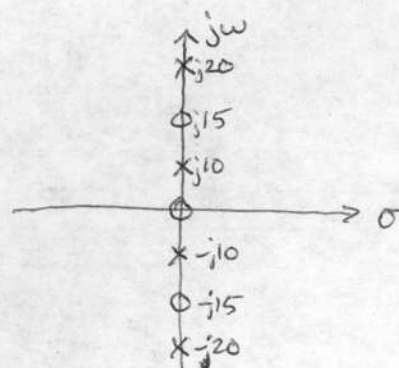
b) $f_2(t) = [5\cos(10t) + 7\cos(20t)]u(t)$

$$F_2(s) = \frac{5s}{s^2+100} + \frac{7s}{s^2+400}$$

$$= \frac{5s^3 + 2000s + 7s^3 + 700s}{(s^2+100)(s^2+400)}$$

$$= \frac{12s^3 + 2700s}{(s^2+100)(s^2+400)}$$

$$= \frac{12s(s^2+15^2)}{(s^2+100)(s^2+400)}$$



9) Find $F(s)$, poles & Zeros

a) $f_1(t) = \delta(t) - (625t e^{-50t})u(t)$

$$F_1(s) = 1 - \frac{625}{(s+50)^2}$$

$$= \frac{s^2 + 100s + 1875}{(s+50)^2}$$

$$= \frac{(s+25)(s+75)}{(s+50)^2}$$

poles: $-50, -50$

Zeros: $-25, -75$

b) $f_2(t) = [5 + e^{-20t} - 6\cos(10t) + 2\sin(10t)]u(t)$

$$F_2(s) = \frac{5}{s} + \frac{1}{s+20} - \frac{6s}{s^2+10^2} + \frac{20}{s^2+10^2}$$

$$= \frac{5(s+20)(s^2+100) + s(s^2+100) + (-6s+20)s(s+20)}{s(s+20)(s^2+100)}$$

$$= \frac{5s^3 + 500s + 100s^2 + 100^2 + s^3 + 100s - 6s^3 - 120s^2 + 20s^2 + 400s}{s(s+20)(s^2+100)}$$

$$= \frac{1000(s+10)}{s(s+20)(s^2+100)}$$

poles: $0, -20, \pm j10$

Zeros: -10

17) find $f(t)$

a) $F_1(s) = \frac{2(s+5)}{s(s+10)} = \frac{k_1}{s} + \frac{k_2}{s+10}$

$$k_1 = \lim_{s \rightarrow 0} s F_1(s) = 1$$

$$k_2 = \lim_{s \rightarrow -10} (s+10) F_1(s) = 1$$

$$F_1(s) = \frac{1}{s} + \frac{1}{s+10}$$

$$f_1(t) = [1 + e^{-10t}]u(t)$$

b) $F_2(s) = \frac{s^2}{(s+5)(s+10)}$ improper rational

$$\begin{array}{r} s^2 + 15s + 50 \quad \overline{) s^2} \\ \underline{-(s^2 + 15s + 50)} \\ -15s - 50 \end{array}$$

17) b) continued

now $F_2(s) = 1 - \frac{15s+50}{(s+5)(s+10)} = 1 - \frac{k_1}{s+5} - \frac{k_2}{s+10}$

$$k_1 = \lim_{s \rightarrow -5} (s+5) \frac{15s+50}{(s+5)(s+10)} = -5$$

$$k_2 = \lim_{s \rightarrow -10} (s+10) \frac{15s+50}{(s+5)(s+10)} = 20$$

$$F_2(s) = 1 + \frac{5}{s+5} - \frac{20}{s+10}$$

$$f_2(t) = \delta(t) + [5e^{-5t} - 20e^{-10t}] u(t)$$

18) find $f(t)$

a) $F_1(s) = \frac{20(s+20)}{(s+10)^2 + 400}$

poles: $-10 \pm j20$

$$= \frac{20(s+20)}{(s+10+j20)(s+10-j20)} = \frac{k_1}{s+10+j20} + \frac{k_1^*}{s+10-j20}$$

$$k_1 = \lim_{s \rightarrow -10-j20} (s+10+j20) F_1(s) = \frac{20(-10-j20+20)}{(-10-j20+10-j20)}$$

$$= \frac{20-j40}{-j4}$$

$$F_1(s) = \frac{10+j5}{s+10+j20} + \frac{10-j5}{s+10-j20} \quad = 10+j5$$

use table of transform pairs $f(t)$

simple complex poles $[2|k|e^{-\alpha t} \cos(\beta t + \angle k)] u(t)$

$$\frac{F(s)}{\frac{k}{s+\alpha-j\beta} + \frac{k^*}{s+\alpha+j\beta}}$$

for $F_1(s)$ $k = 10-j5$

$$|k| = 5\sqrt{5}$$

$$\angle k = \tan^{-1}(-5/10)$$

$$f_1(t) = [10\sqrt{5} e^{-10t} \cos(20t + \tan^{-1}(-1/2))] u(t)$$

$$18) b) F_2(s) = \frac{20(s-20)}{(s+10)^2 + 400} = \frac{k_1}{s+10-j20} + \frac{k_1^*}{s+10+j20}$$

$$k_1 = \lim_{s \rightarrow -10+j20} (s+10-j20) F_2(s) = \frac{-60+j40}{j4} = 10+j15$$

$$k_1^* = 10-j15$$

simple complex pole $k = 10+j15$ $|k| = 5\sqrt{13}$ $\angle k = \tan^{-1}(\frac{15}{10})$

$$f_2(t) = [10\sqrt{13} e^{-10t} \cos(20t + \tan^{-1}(3/2))] u(t)$$

24) find $f(t)$

$$a) F_1(s) = \frac{30(s+2)}{s(s^2+4s+5)} = \frac{k_1}{s} + \frac{k_2}{s+2-j} + \frac{k_2^*}{s+2+j}$$

$$k_1 = \lim_{s \rightarrow 0} s F_1(s) = 12$$

$$k_2 = \lim_{s \rightarrow -2+j} (s+2-j) F_1(s) = \frac{(j30)}{(-2+j)(j2)} = \frac{15}{-2+j} \frac{(-2-j)}{(-2-j)} = \frac{-30-j15}{5} = -6-j3$$

$$k_2^* = -6+j3$$

step function with simple complex pole $k = -6-j3$ $|k| = 3\sqrt{5}$ $\angle k = \pi + \tan^{-1}(3/6)$

$$f_1(t) = [12 + 6\sqrt{5} e^{-2t} \cos(t + \pi + \tan^{-1}(1/2))] u(t)$$

$$b) F_2(s) = \frac{2s}{(s+4)(s^2+4s+8)} = \frac{k_1}{s+4} + \frac{k_2}{s+2-j2} + \frac{k_2^*}{s+2+j2}$$

$$k_1 = \lim_{s \rightarrow -4} (s+4) F_2(s) = -1$$

$$k_2 = \lim_{s \rightarrow -2+j2} (s+2-j2) F_2(s) = \frac{2(-2+j2)}{(2+j2)(j4)} = \frac{(-1+j)}{(1+j)(j2)} = \frac{1}{2}$$

$$k_2^* = 1/2$$

$$|k| = 1/2 \quad \angle k = 0$$

$$f_2(t) = [e^{-4t} + e^{-2t} \cos(2t)] u(t)$$

32) Use Laplace transformation to find $y(t)$

$$\frac{dy}{dt} + 500y = [2500 e^{-250t}] u(t) \quad \text{with } y(0^-) = 0$$

$$\mathcal{L}\left\{\frac{dy}{dt} + 500y = [2500 e^{-250t}] u(t)\right\} \Rightarrow sY(s) - y(0^-) + 500Y(s) = \frac{2500}{s+250}$$

$$(s+500)Y(s) = \frac{2500}{s+250}$$

$$Y(s) = \frac{2500}{(s+500)(s+250)} = \frac{k_1}{s+500} + \frac{k_2}{s+250}$$

$$k_1 = \lim_{s \rightarrow -500} (s+500)Y(s) = -10$$

$$k_2 = \lim_{s \rightarrow -250} (s+250)Y(s) = 10$$

32) continued

$$Y(s) = -\frac{10}{s+500} + \frac{10}{s+250}$$

$$\mathcal{L}^{-1}\{Y(s)\} = y(t) = 10e^{-250t} - 10e^{-500t}$$

45) Use initial and final value properties to find the initial and final values of the waveform corresponding to the transforms

a) $F_1(s) = \frac{16}{(s+2)(s^2+12s+36)}$

$$\lim_{t \rightarrow 0^+} f_1(t) = \lim_{s \rightarrow \infty} s F_1(s) = 0$$

$$\lim_{t \rightarrow \infty} f_1(t) = \lim_{s \rightarrow 0} s F_1(s) = 0$$

b) $F_2(s) = \frac{2(s^2+2)}{s(s^2+4)}$

$$\lim_{t \rightarrow 0^+} f_2(t) = \lim_{s \rightarrow \infty} s F_2(s) = 2$$

$$\lim_{t \rightarrow \infty} f_2(t) = \lim_{s \rightarrow 0} s F_2(s) \quad \text{N/A because of } j\text{-axis poles}$$

47) a) $F_1(s) = \frac{2(s^2+5s+6)}{(s+2)(s+6)(s+12)}$

$$\lim_{t \rightarrow 0^+} f_1(t) = \lim_{s \rightarrow \infty} s F_1(s) = 2$$

$$\lim_{t \rightarrow \infty} f_1(t) = \lim_{s \rightarrow 0} s F_1(s) = 0$$

b) $F_2(s) = \frac{10(s^2+10s+40)}{s(s^2-100)}$

$$\lim_{t \rightarrow 0^+} f_2(t) = \lim_{s \rightarrow \infty} s F_2(s) = 10$$

$$\lim_{t \rightarrow \infty} f_2(t) = \lim_{s \rightarrow 0} s F_2(s) \quad \text{N/A because of pole in right side of plane}$$