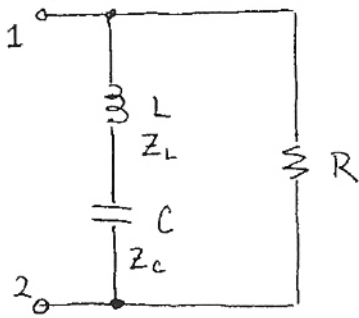


10/3

1st Transform circuit to s-domain

$$Z_L = sL$$

$$Z_C = \frac{1}{sC}$$

(No IC for an equivalent impedance problem)

2nd Apply methods for resistive networks

$$Z_{eq} = \left(\frac{1}{Z_L + Z_C} + \frac{1}{R} \right)^{-1}$$

$$= \left(\frac{1}{sL + \frac{1}{sC}} + \frac{1}{R} \right)^{-1}$$

$$= \left(\frac{sC}{s^2LC + 1} + \frac{1}{R} \right)^{-1}$$

$$= \left(\frac{sRC + s^2LC + 1}{s^2RLC + R} \right)^{-1}$$

$$= R \times \frac{s^2 + 1/(LC)}{s^2 + s(R/L) + (LC)^{-1}}$$

$$\text{where } \begin{cases} R = 4 \times 10^3 \Omega \\ L = 0.4 \text{ H} \\ C = 10^{-7} \text{ F} \end{cases}$$

Use the following MatLab command:

```
>> R = 4e3; L = 0.4; C = 1e-7; syms s
```

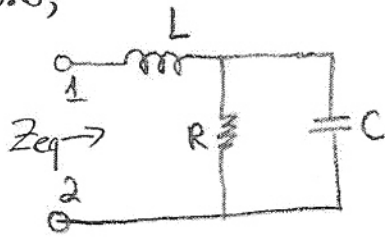
```
>> eval(solve('s^2 + 1/(L*C)', 's'))
```

```
>> eval(solve('s^2 + s*(R/L) + 1/(L*C)', 's'))
```

Zeroes: $\pm 5000j \text{ rad/s}$ pole: -5000 rad/s

$$Z_{eq} = 4 \times 10^3 \frac{(s + 5000j)(s - 5000j)}{(s + 5000)^2}$$

10.6,

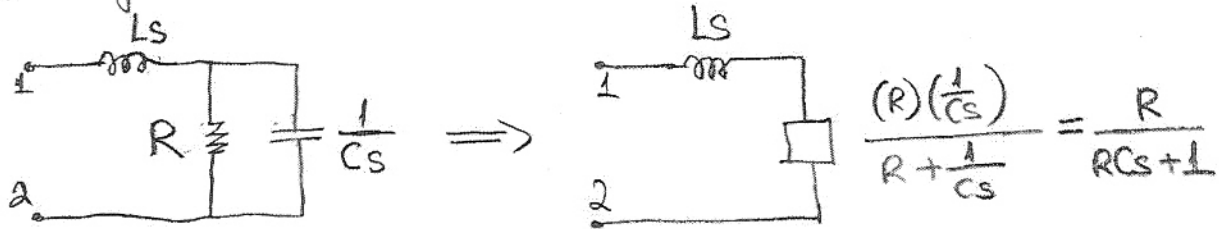


$$R = 1k\Omega$$

$$L = 1H$$

$$C = 500nF$$

→ transform ckt into s-domain:



$$\Rightarrow \text{Circuit diagram showing } Z_{eq} = Ls + \frac{R}{RCs + 1} = \frac{RLCs^2 + Ls + R}{RCs + 1}$$

→ sub in numbers $\Rightarrow Z_{eq} = \frac{s^2 + 2 \times 10^3 s + 2 \times 10^6}{s + 2 \times 10^3} = \frac{(s + 10^3)^2 + 10^6}{s + 2 \times 10^3}$

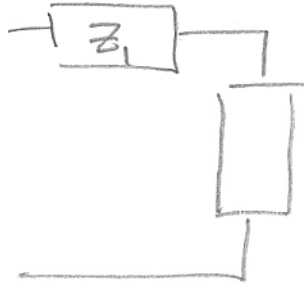
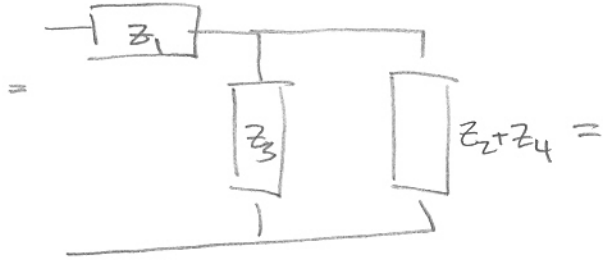
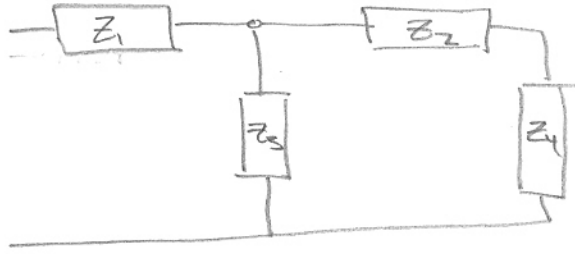
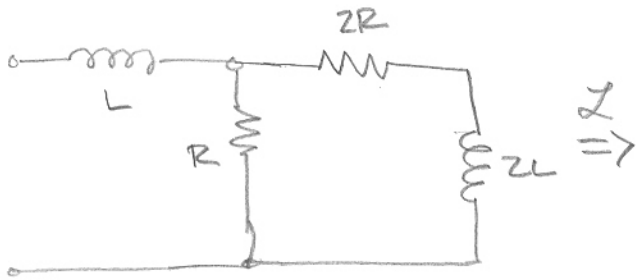
→ zeros: $\rightarrow (s + 10^3)^2 + 10^6 = 0 \Rightarrow (s + 10^3)^2 = -10^6$

$$\Rightarrow s + 10^3 = \pm 10^3 j$$

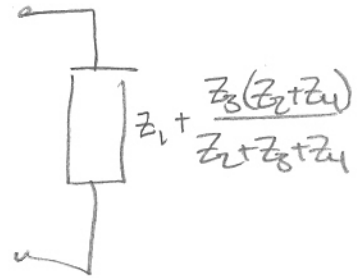
$$\Rightarrow \boxed{s = -10^3 \pm 10^3 j}$$

→ poles: $\rightarrow s + 2 \times 10^3 = 0 \Rightarrow \boxed{s = -2 \times 10^3}$

10.9



$$\frac{Z_3(Z_2 + Z_4)}{Z_2 + Z_3 + Z_4} =$$

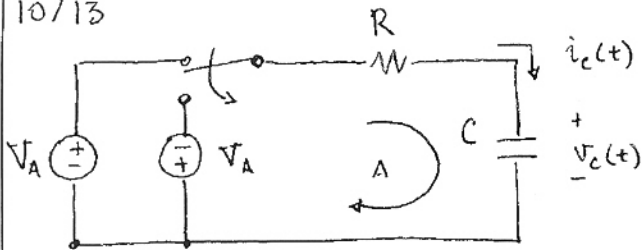


$$\Rightarrow Z_{eq} = Ls + \frac{R(ZR + ZLs)}{ZR + R + ZLs} = Ls + \frac{ZR^2 + ZRLs}{3R + ZLs} = \frac{3RLs + 2L^2s^2 + ZR^2 + ZRLs}{ZLs + 3R}$$

$$= \frac{2L^2s^2 + 5RLs + ZR^2}{ZLs + 3R} = \frac{s^2 + \frac{5R}{2L}s + (\frac{R}{L})^2}{\frac{1}{L}(s + \frac{3R}{2L})} = \frac{(s + \frac{R}{L})(s + \frac{3R}{L})}{\frac{1}{L}(s + \frac{3R}{2L})}$$

$$\Rightarrow \text{zeros @ } -\frac{R}{2L}, -\frac{3R}{L} \text{ and poles @ } -\frac{3R}{2L}$$

10/13

1st Transform circuit to s-domain

$$i_c(t) = C \frac{dv_c}{dt}$$

$$\Rightarrow i_c(s) = sC v_c(s) - C v_c(0)$$

$$\text{where } v_c(0) = V_A$$

$$\text{closing switch} \Rightarrow V_A u(t) \rightarrow V_A/s$$

2nd Apply KCL or KVL (KVL is slightly easier)

$$\text{note: } i_c(s) = sC v_c(s) - C v_c(0)$$

$$i_c(s) + C v_c(0) = sC v_c(s)$$

$$v_c(s) = (sC)^{-1} i_c(s) + s^{-1} v_c(0)$$

$$\text{KVL @ A: } s^{-1} V_A + i_c(s) R + (sC)^{-1} i_c(s) + s^{-1} v_c(0) = 0$$

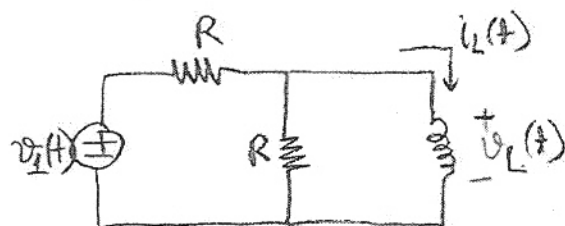
$$C V_A + i_c(s) sRC + i_c(s) + C V_A = 0$$

$$i_c(s) = \frac{-2C V_A}{sRC + 1}$$

$$i_c(s) = \frac{-2V_A}{R} \cdot \frac{1}{s + (RC)^{-1}}$$

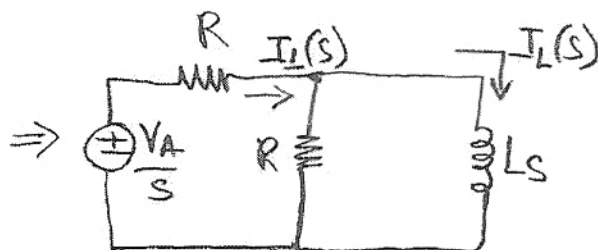
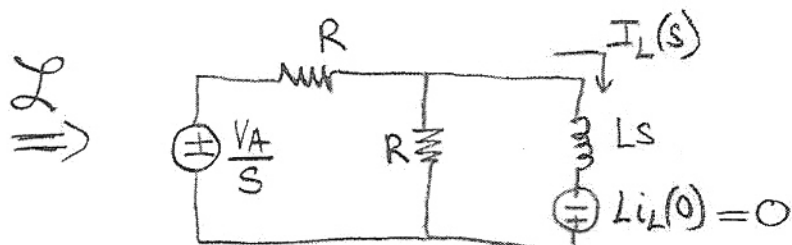
$$i_c(t) = \frac{-2V_A}{R} \cdot e^{-t/RC} u_t$$

10.15,



→ no initial energy

$$\rightarrow v_1(t) = V_A u(t)$$



$$\rightarrow I_1(s) = \frac{V_A/s}{Z_{eq}} = \frac{V_A/s}{R + \left(\frac{RLs}{R+Ls} \right)} = \frac{V_A(R+Ls)}{s(R^2 + 2RLs)}$$

→ use current divider to find $I_L(s)$

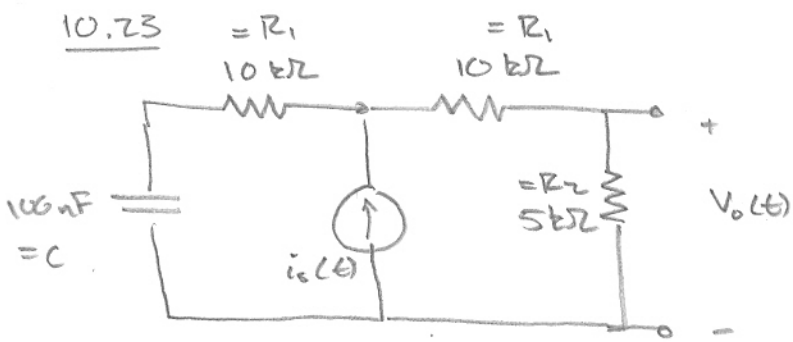
$$\Rightarrow I_L(s) = \frac{R}{R+Ls} I_1(s) = \frac{R}{R+Ls} \frac{V_A(R+Ls)}{s(R^2 + 2RLs)}$$

$$\Rightarrow \boxed{I_L(s) = \frac{V_A}{s(R+2Ls)}}$$

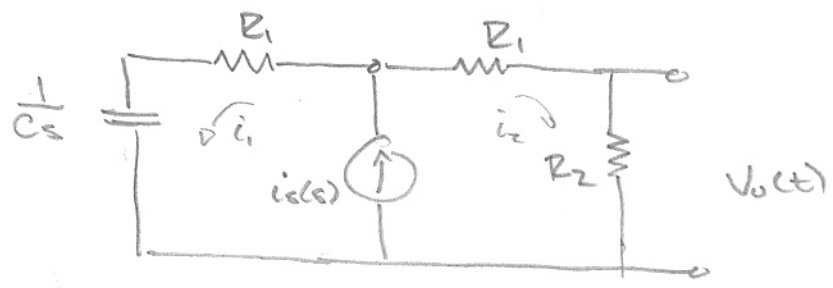
$$\Rightarrow I_L(s) = \frac{V_A/R}{s} + \frac{-V_A/R}{s + R/2L}$$

$$\Rightarrow i_L(t) = \frac{V_A}{R} u(t) - \frac{V_A}{R} e^{-\frac{R}{2L}t} u(t)$$

$$\Rightarrow \boxed{i_L(t) = \frac{V_A}{R} [1 - e^{-\frac{R}{2L}t}] u(t)}$$



⇓ Z



current divider $\Rightarrow i_2(s) = \frac{\frac{1}{R_1 + R_2}}{\frac{1}{R_1 + R_2} + \frac{1}{R_1 + 1/Cs}} (i_s(s))$

$V = iR$
 $V_o = i_2 R_2$

$$= \frac{1}{1 + \frac{R_1 + R_2}{R_1 + 1/Cs}} i_s(s) = \frac{R_1 + 1/Cs}{2R_1 + R_2 + 1/Cs} i_s(s)$$

$$\Rightarrow V_o = R_2 \left(\frac{R_1 Cs + 1}{(2R_1 + R_2)(s + 1)} \right) i_s(s)$$

$i_s(t) = 2.5e^{-100t} u(t) \text{ mA} \Rightarrow i_s(s) = \frac{(2.5 \times 10^{-3})}{s + 100}$

$$V_o = (5000) \left(\frac{(1 \times 10^{-3})s + 1}{0.0025s + 1} \right) \left(\frac{2.5 \times 10^{-3}}{s + 100} \right) = (5k) \left(\frac{(1 \times 10^{-3})s + 1}{s + 400} \right) \left(\frac{1}{s + 100} \right)$$

$$= \frac{A}{s + 400} + \frac{B}{s + 100}$$

$A: (5k) \left(\frac{.001s + 1}{s + 100} \right) \Big|_{s=-400} = -10$

$B: (5000) \left(\frac{.001s + 1}{s + 400} \right) \Big|_{s=-100} = 15$

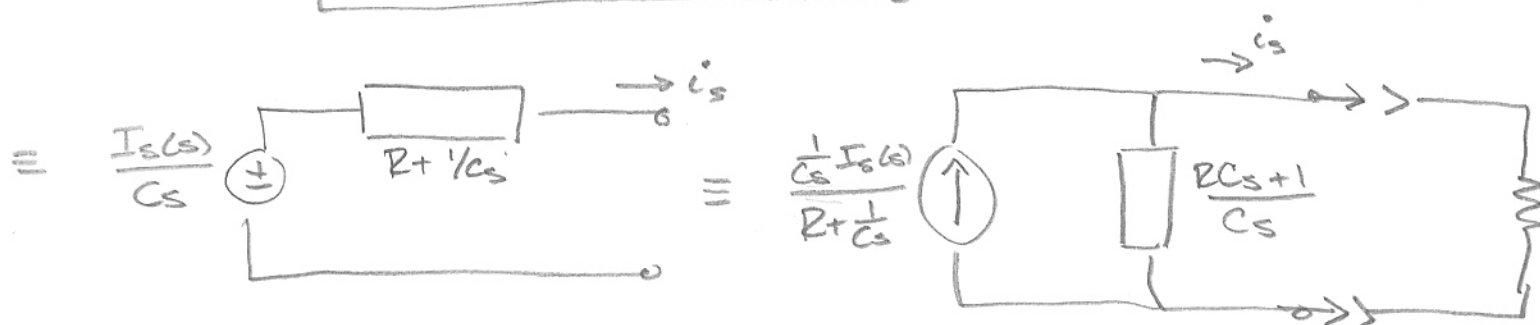
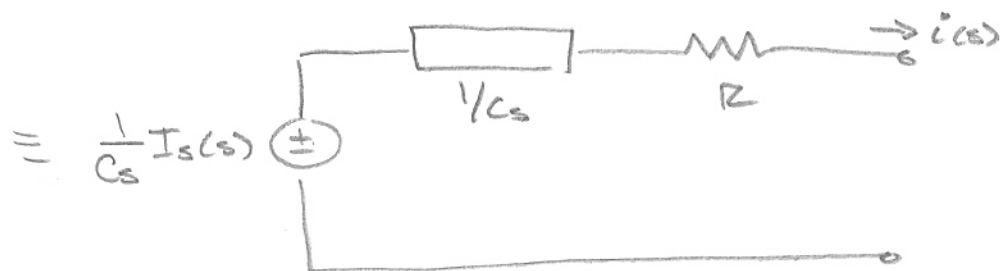
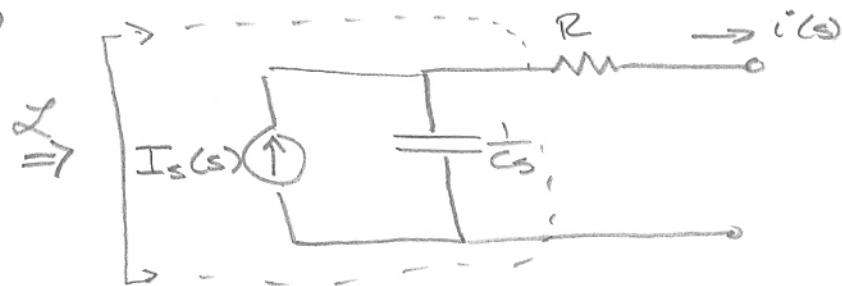
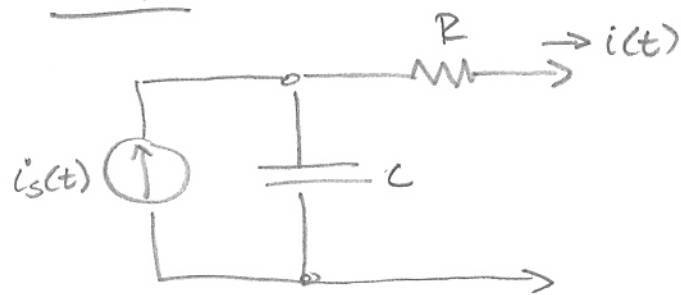
$$\Rightarrow V_o(s) = \frac{-10}{s+400} + \frac{15}{s+100}$$

$$\Rightarrow \boxed{V_o(t) = [-10e^{-400t} + 15e^{-100t}]u(t)}$$

where a forced pole is at -100 rad/s

and an unforced pole at -400 rad/s

10.26



using a current divider w/ the $\text{---}\mu\text{---}$ replaced $\uparrow$

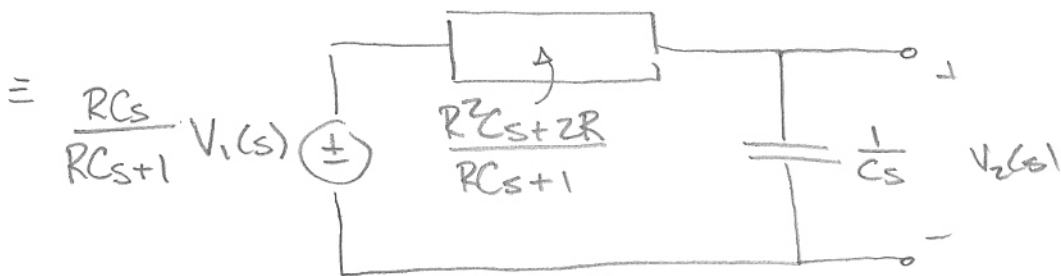
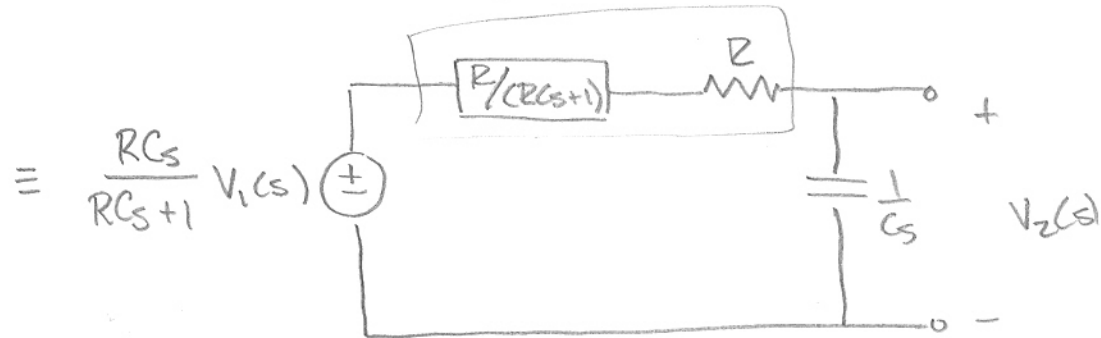
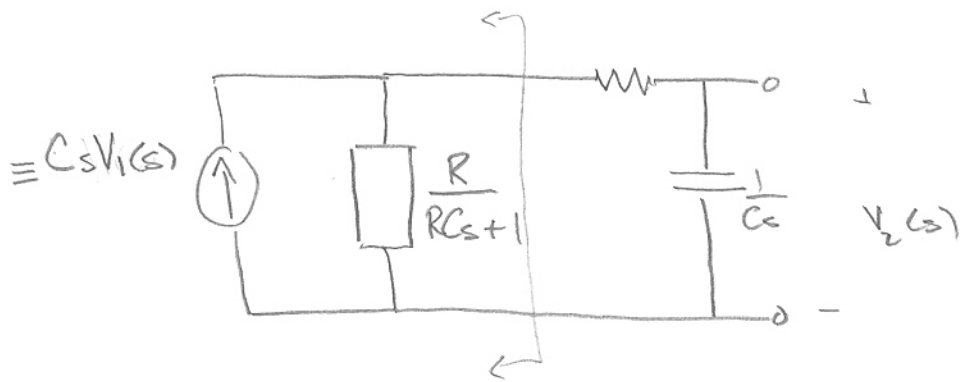
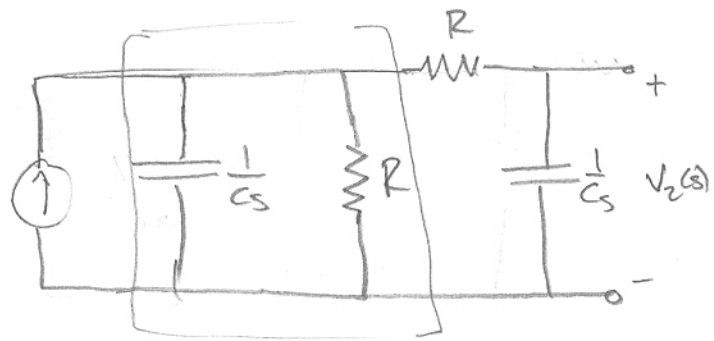
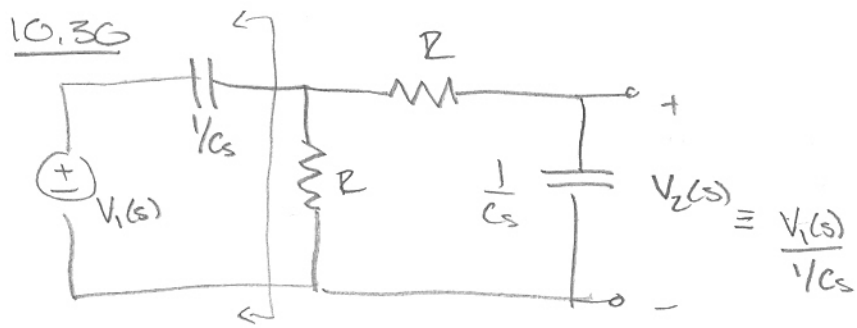
$$\Rightarrow i_s(s) = \left(\frac{\frac{1}{R}}{\frac{1}{R} + \frac{Cs}{RCs+1}} \right) \left(\frac{\frac{1}{Cs}}{R + \frac{1}{Cs}} \right) I_s(s)$$

$$= \left(\frac{1}{RCs+1} \right) \left(\frac{\frac{1}{R}}{\frac{1}{R} + \frac{Cs}{RCs+1}} \right) I_s(s)$$

$$= \left(\frac{1}{RCs+1} \right) \left(\frac{1}{1 + \frac{RCs}{RCs+1}} \right) I_s(s)$$

$$= \left(\frac{1}{RCs+1} \right) \left(\frac{RCs+1}{2RCs+1} \right) I_s(s)$$

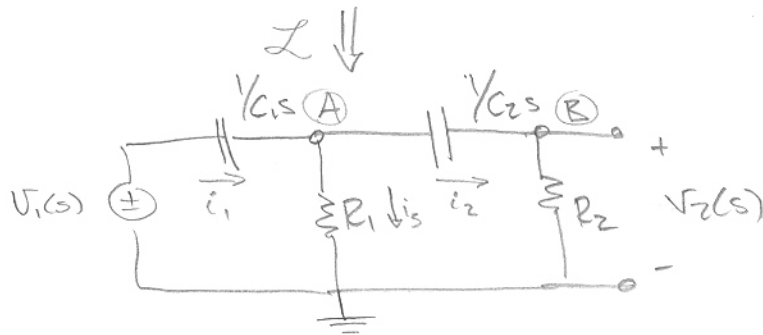
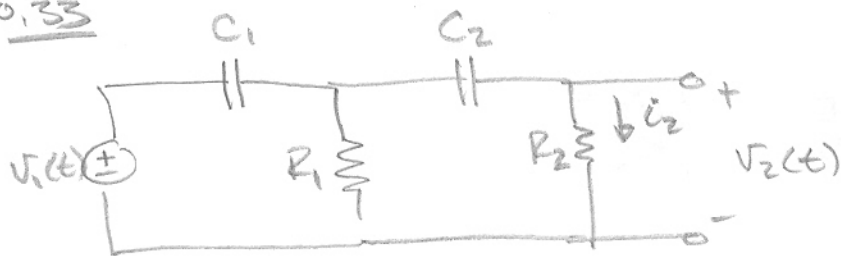
$$\Rightarrow i_s(s) = \left(\frac{1}{2RCs+1} \right) I_s(s)$$



Voltage divider: $V_2(s) = \frac{\frac{1}{C_s}}{\frac{1}{C_s} + \frac{R^2 C_s + 2R}{RC_s + 1}} \left(\frac{RC_s}{RC_s + 1} \right) V_1(s)$

$$\Rightarrow V_2(s) = \frac{RC_s}{R^2 C_s^2 + 3RC_s + 1} V_1(s)$$

10.33



(a)

@A: $i_1 = i_2 + i_3$

$$\frac{V_1 - V_A}{1/C_1 s} = \frac{V_A - V_B}{1/C_2 s} + \frac{V_A}{R_1} \quad \text{note } V_B = V_2!$$

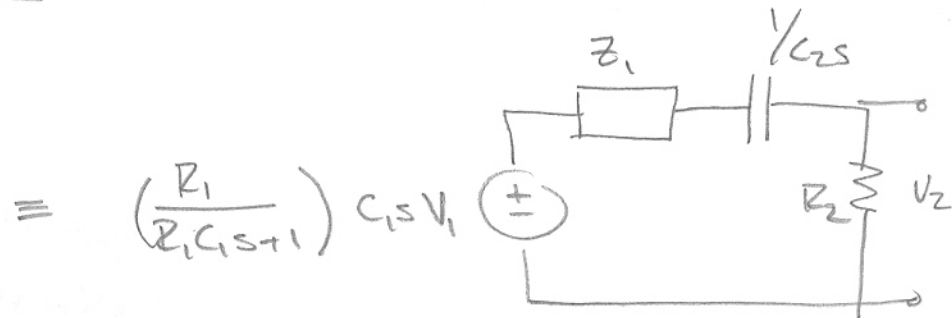
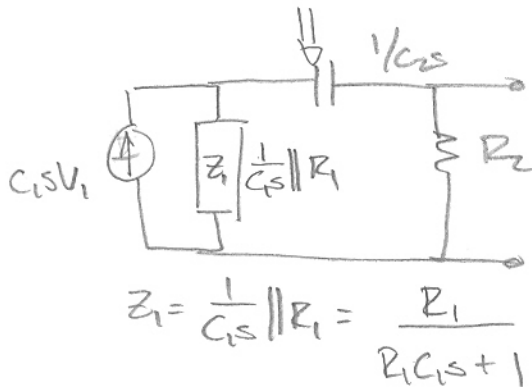
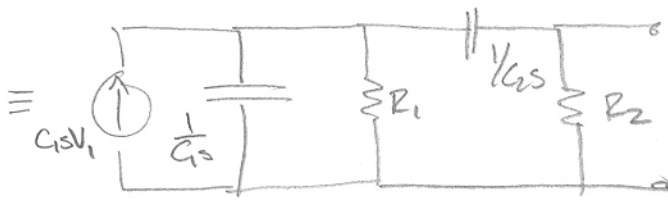
$$V_1 C_1 s - V_A C_1 s = V_A C_2 s - V_2 C_2 s + \frac{1}{R_1} V_A$$

$$\underline{V_1(s) C_1 s = (C_1 s + C_2 s + \frac{1}{R_1}) V_A - (C_2 s) V_2(s)}$$

@B: $\frac{V_A - V_B}{1/C_2 s} = \frac{V_B}{R_2}$

$$(C_2 s) V_A - (C_2 s) V_2(s) = \frac{1}{R_2} V_2(s) \Rightarrow \underline{(C_2 s + \frac{1}{R_2}) V_2(s) - (C_2 s) V_A(s) = 0}$$

$$\Rightarrow \begin{bmatrix} C_1 s + C_2 s + \frac{1}{R_1} & -C_2 s \\ -C_2 s & C_2 s + \frac{1}{R_2} \end{bmatrix} \begin{bmatrix} V_A(s) \\ V_2(s) \end{bmatrix} = \begin{bmatrix} V_1(s) C_1 s \\ 0 \end{bmatrix}$$



now we have a voltage divider

$$V_2(s) = \frac{R_2}{Z_1 + \frac{1}{C_2 s} + R_2} \left(\frac{R_1}{R_1 C_1 s + 1} \right) C_1 s V_1 = \frac{R_2}{\frac{R_1}{R_1 C_1 s + 1} + \frac{1}{C_2 s} + R_2} \left(\frac{R_1}{R_1 C_1 s + 1} \right) C_1 s V_1$$

$$= \frac{R_1 R_2 C_1 V_1(s) s}{R_1 + \frac{(R_1 C_1 s + 1)}{C_2 s} + R_2 (R_1 C_1 s + 1)} V_1(s)$$

$$= \frac{R_1 R_2 C_1 C_2 V_1(s) s^2}{R_1 C_2 s + R_1 C_1 s + 1 + R_2 (R_1 C_1 s + 1) C_2 s} V_1(s)$$

$$V_2(s) = \frac{R_1 R_2 C_1 C_2 s^2 V_1(s)}{R_1 R_2 C_1 C_2 s^2 + (R_1 C_1 + R_2 C_2 + R_1 C_2) s + 1}$$

$$(c) \quad V_1(t) = 30u(t) \Rightarrow V_1(s) = \left(\frac{30}{s}\right)$$

$$R_1 = 10 \text{ k}\Omega$$

$$R_2 = 20 \text{ k}\Omega$$

$$C_1 = 1 \mu\text{F}$$

$$C_2 = 0.5 \mu\text{F}$$

$$V_2(s) = \frac{(10 \text{ k}\Omega)(20 \text{ k}\Omega)(1 \mu\text{F})(0.5 \mu\text{F})s^2}{(10 \text{ k}\Omega)(20 \text{ k}\Omega)(1 \mu\text{F})(0.5 \mu\text{F})s^2 + [(10 \text{ k})(1 \mu) + (20 \text{ k})(0.5 \mu) + (10 \text{ k})(0.5 \mu)]s + 1} \left(\frac{30}{s}\right)$$

$$= \frac{(1 \times 10^{-4})s^2}{(1 \times 10^{-4})s^2 + (0.025)s + 1} \left(\frac{30}{s}\right)$$

$$= \frac{(0.003)s}{(0.0001)s^2 + (0.025)s + 1} = \frac{30s}{s^2 + 250s + 10000}$$

$$= \frac{30s}{(s+50)(s+200)}$$

$$= \frac{K_1}{s+50} + \frac{K_2}{s+200}$$

$$K_1 = \frac{30s}{(s+50)(s+200)} (s+50) \Big|_{s=-50} = \underline{-10}$$

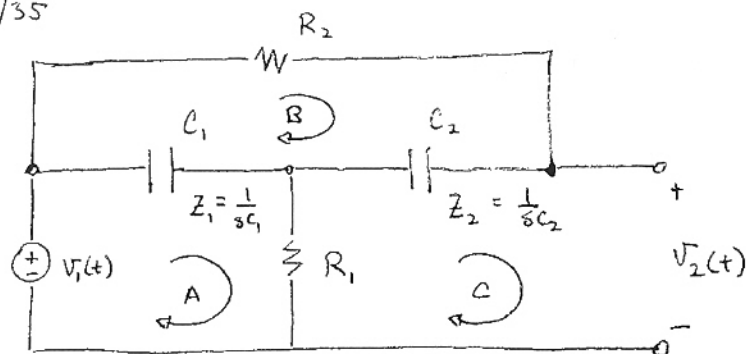
$$K_2 = \frac{30s}{(s+50)(s+200)} (s+200) \Big|_{s=-200} = 40$$

$$\Rightarrow V_2(s) = \frac{-10}{s+50} + \frac{40}{s+200}$$

$$\mathcal{L}^{-1} \Rightarrow -10e^{-50t} u(t) + 40e^{-200t} u(t)$$

$$\Rightarrow \boxed{V_2(t) = [40e^{-200t} - 10e^{-50t}] u(t)}$$

10/35



No initial energy $\Rightarrow v_{C1}(0) = v_{C2}(0) = 0$

(a) In the s-domain

$$Z_{C1} = \frac{1}{sC_1} \Leftrightarrow i_{C1}(s) = sC_1 v_{C1}(s)$$

$$Z_{C2} = \frac{1}{sC_2} \Leftrightarrow i_{C2}(s) = sC_2 v_{C2}(s)$$

Answer:

$$\frac{V_2(s)}{V_1(s)} =$$

$$\frac{s^2 R_1 R_2 C_1 C_2 + s(R_1 C_2 + R_2 C_1) + 1}{s^2 R_1 R_2 C_1 C_2 + s(R_1 C_2 + R_2 C_1 + R_1 C_1) + 1}$$

$$s^2 R_1 R_2 C_1 C_2 + s(R_1 C_2 + R_2 C_1 + R_1 C_1) + 1$$

Mesh equations:

$$\left. \begin{aligned} \text{A: } \frac{1}{sC_1} (i_A - i_B) + R_1 (i_A - i_C) &= V_1(s) \\ \text{B: } \frac{1}{sC_1} (i_B - i_A) + R_2 (i_B) + \frac{1}{sC_2} (i_B - i_C) &= 0 \\ \text{C: } R_1 (i_C - i_A) + \frac{1}{sC_2} (i_C - i_B) &= -V_2(s) \end{aligned} \right\} (1)$$

noting that $i_C(s) = 0$

$$(1) \Rightarrow \left(R_1 + \frac{1}{sC_1} \right) i_A + \left(-\frac{1}{sC_1} \right) i_B = V_1(s) \quad (2)$$

$$\left(-\frac{1}{sC_1} \right) i_A + \left(R_2 + \frac{1}{sC_1} + \frac{1}{sC_2} \right) i_B = 0 \quad (3)$$

$$\left(-R_1 \right) i_A + \left(-\frac{1}{sC_2} \right) i_B = -V_2(s) \quad (4)$$

$$(b) \quad 3 \Rightarrow i_A = i_B \left(sC_1 R_2 + 1 + \frac{C_1}{C_2} \right)$$

$$2 \Rightarrow \left[\left(R_1 + \frac{1}{sC_1} \right) \left(sC_1 R_2 + 1 + \frac{C_1}{C_2} \right) - \frac{1}{sC_1} \right] i_B = V_1(s)$$

$$1 \Rightarrow \left[-R_1 \left(sC_1 R_2 + 1 + \frac{C_1}{C_2} \right) - \frac{1}{sC_2} \right] i_B = -V_2(s)$$

$$V_2(s) \left[sR_1 R_2 C_1 + R_1 + \frac{R_2 C_1}{C_2} + \frac{1}{sC_2} \right]^{-1} = i_B(s)$$

$$\left[sR_1 R_2 C_1 + R_1 + \frac{C_1 R_1}{C_2} + R_2 + \cancel{\frac{1}{sC_1}} + \frac{1}{sC_2} - \cancel{\frac{1}{sC_1}} \right] \left(sR_1 R_2 C_1 + R_1 + \frac{R_2 C_1}{C_2} + \frac{1}{sC_2} \right)^{-1} = \frac{V_2}{V_1}$$

answer @ top