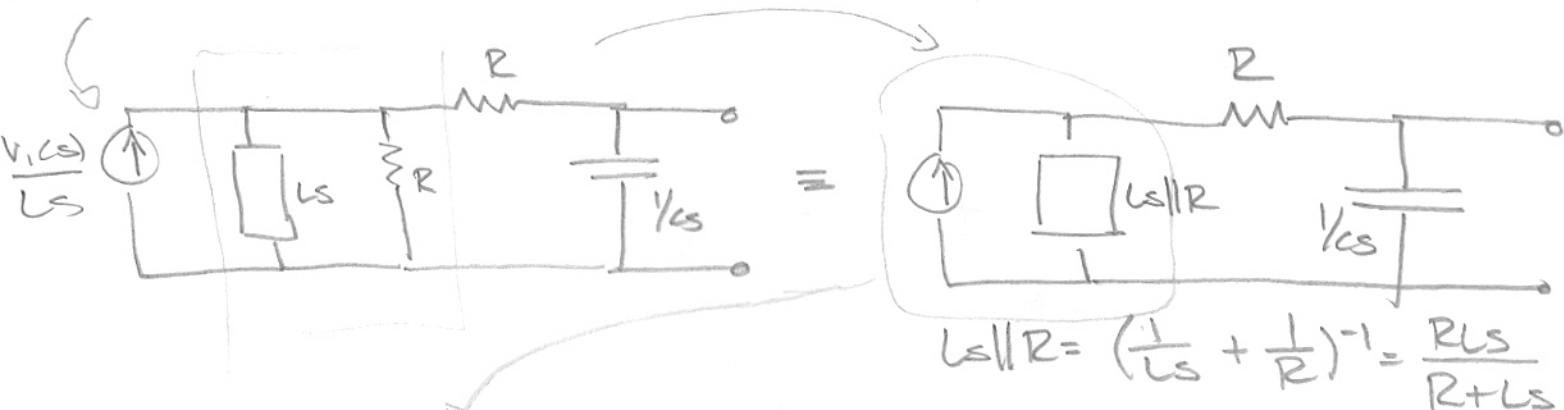
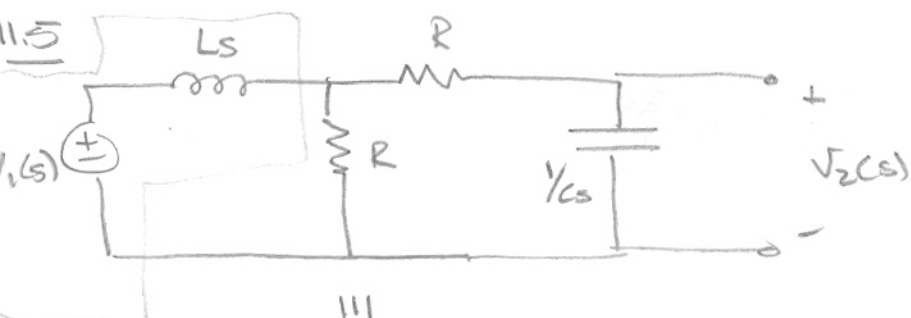


now we have a voltage divider!

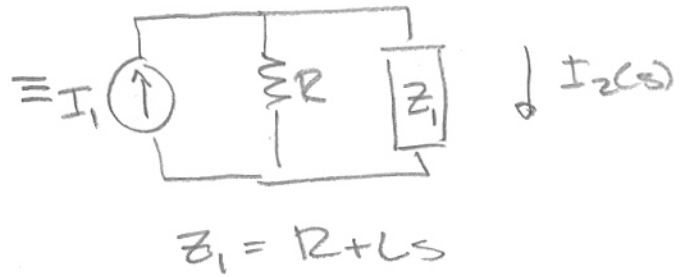
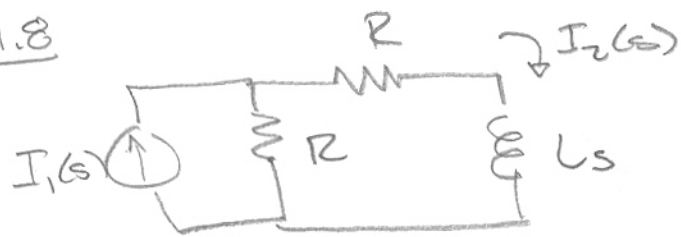
$$V_2(s) = \frac{\frac{1}{Cs}}{\frac{1}{Cs} + R + \frac{RLs}{R+Ls}} \left( \frac{V_1(s)R}{R+Ls} \right)$$

∴ (a few steps of algebra)

$$V_2(s) = \frac{R}{2RLCs^2 + (L+R^2C)s + R} V_1(s)$$

$$\Rightarrow \boxed{T_V(s) = \frac{V_2(s)}{V_1(s)} = \frac{R}{2RLCs^2 + (L+R^2C)s + R}}$$

11.8



current divider:

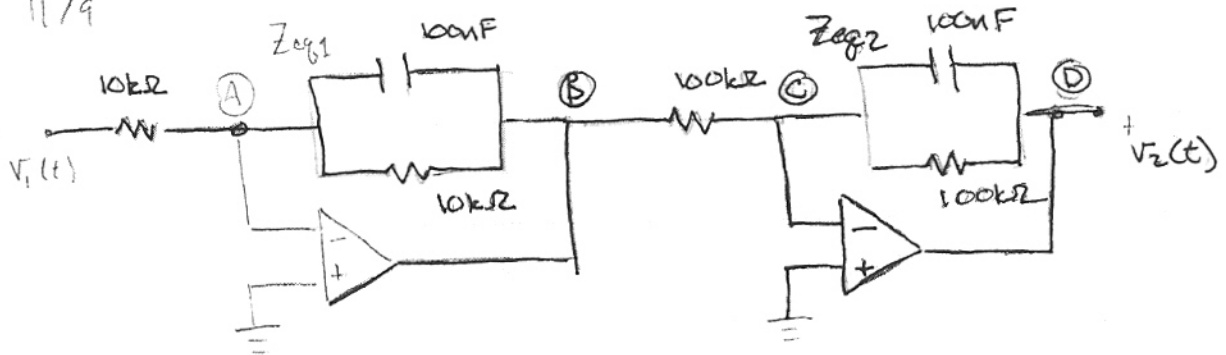
$$I_2(s) = \frac{\frac{1}{R+Ls}}{\frac{1}{R} + \frac{1}{R+Ls}} I_1(s) = \frac{1}{\frac{R+Ls}{R} + 1} I_1(s)$$

$$\Rightarrow I_2(s) = \frac{R}{2R+Ls} I_1(s) \Rightarrow \boxed{T_1(s) = \frac{I_2(s)}{I_1(s)} = \frac{R}{2R+Ls}}$$

the pole  $2R+Ls=0 \Rightarrow s = -\frac{2R}{L} = -500 \frac{\text{rad}}{\text{s}}$

if let  $L = 1 \text{ Henry} \Rightarrow R = 250 \Omega$

11/9



$$10 \text{ k}\Omega \parallel 100 \text{ nF} \\ \Rightarrow Z_{eq1} = \frac{10^4 \times \frac{1}{10^{-7}s}}{10^4 + \frac{1}{10^{-7}s}} = \frac{10^{11}}{10^4 s + 10^7} = \frac{10^7}{s + 1000}$$

$$100 \text{ k}\Omega \parallel 100 \text{ nF} \\ \Rightarrow Z_{eq2} = \frac{10^7}{s + 100}$$

$$V_A = V_C = 0$$

KCL @ A:

$$V_1(s) \times \frac{1}{10^4} + V_B(s) \times \left( \frac{10^7}{s + 1000} \right)^{-1} = 0$$

$$\therefore V_B(s) = -V_1(s) \times \frac{10^3}{s + 1000}$$

KCL @ C:

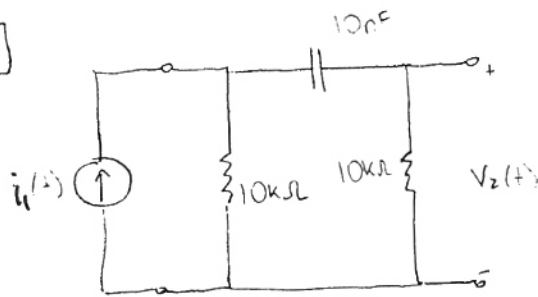
$$\frac{1}{10^5} V_B(s) + V_2(s) \left( \frac{10^7}{s + 100} \right)^{-1} = 0$$

$$\Rightarrow V_2(s) = -\frac{V_B(s)}{\frac{10^5}{s + 100}} \times \frac{10^7}{s + 100} \\ = V_1(s) \times \frac{10^5}{(s + 100)(s + 1000)} \\ T(s)$$

Zeros:  $\phi$

Poles:  $-100, -1000$

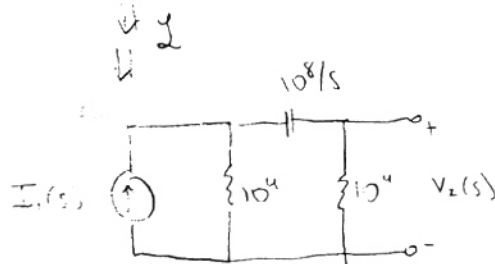
11.30



\* Steady state  $i_1(t) = 10 \cos 5000t$  mA

(a)

Find  $V_{2ss}(t)$



$$T(s) = \frac{V_2(s)}{I_1(s)} \Rightarrow V_2(s) = \frac{10^4}{10^{-8}/s + 2 \times 10^4} I_1(s) = \frac{10^8 s}{10^{-8}/s + 2 \times 10^4}$$

$$T(s) = \frac{5000 s}{s + 5000}$$

$$I_1(s) = \mathcal{L}\{i_1(t)\} = \frac{10 s}{s^2 + 5000^2} = \frac{10 s}{(s - j5000)(s + j5000)}$$

$$\Rightarrow V_2(s) = \underbrace{\left( \frac{5000 s}{s + 5000} \right)}_{T(s)} \underbrace{\left( \frac{10 s}{(s - j5000)(s + j5000)} \right)}_{F_{iss}(s)}$$

$$\Rightarrow V_2(s) = \frac{k}{s + j5000} + \frac{k^*}{s - j5000} + \frac{k_1}{s + 5000} \rightarrow \text{time domain response decays to zero at steady state}$$

$$k = \lim_{s \rightarrow -j5000} \frac{5000 s^2}{(s - j5000)(s + j5000)} = \frac{-5000(5000)^2}{j(j+1)(5000)(10000)} = \frac{-25000}{-1 + j}$$

$$= \frac{25000(1+j)}{2} = 12500(1+j) = 12500\sqrt{2} e^{j45^\circ}$$

$$\rightarrow V_{2ss}(t) = 25000\sqrt{2} \cos(5000t + 45^\circ) \text{ mV}$$

From 11.7.1

$$T(j\omega) = \frac{5000j}{(j+1)5000} = \frac{5000j}{j+1} = \frac{-5000 - 5000j}{-2}$$

$$= 2500 + 2500j = 2500\sqrt{2} e^{j45^\circ}$$

$$\Rightarrow V_{zss}(t) = 25000\sqrt{2} \cos(5000t + 45^\circ) \text{ mV}$$

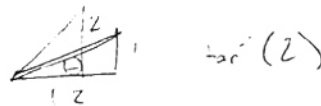
b) What about  $i_1(t) = 5 \cos 2500t \text{ mA}$

$$I(s) = \frac{5s}{(s+j2500)(s-j2500)}$$

$$T(j\omega) = \frac{5000(j\omega)}{j\omega + 5000} = \frac{5000(2500j)}{2500(j+2)} = \frac{5000 e^{j90^\circ}}{\sqrt{5} e^{j \tan^{-1}(1/2)}}$$

$$= \frac{-5000 - 10000j}{5} = 1000 + 2000j = 1000\sqrt{5} e^{j \tan^{-1}(2)}$$

$$\Rightarrow V_{zss}(t) = 5000\sqrt{5} \cos(2500t + \tan^{-1}(2)) \text{ mV}$$



$$11.51 \quad T_V(s) = \pm \frac{2 \times 10^4}{(s+250)(s+2500)}$$

$$\frac{\pm 2 \times 10^4}{(s+250)(s+2500)} = \frac{k_1}{s+250} \cdot \frac{k_2}{s+2500} \cdot k_2$$

$$\frac{k_1}{s+250} = \frac{k_1/s}{1+250/s} \cdot \frac{z_2(s)}{z_1(s)+z_2(s)}$$

$$z_2(s) = \frac{k_1}{s}$$

$$z_1(s) = 1 + \frac{250 - k_1}{s}$$

$$\text{if } k_1 = 250 \Rightarrow z_2(s) = \frac{250}{s} \quad z_1(s) = 1$$

stage 3.

$$\text{if } k_1 = 250, k_2 = 5 \Rightarrow k_3 = 16.$$

$$\frac{k_3}{s+2500} = \frac{k_3/s}{1+2500/s} = \frac{z_4(s)}{z_3(s)+z_4(s)}$$

$$z_4(s) = \frac{k_3}{s}$$

$$z_3(s) = 1 + \frac{2500 - k_3}{s}$$

$$\text{if } k_3 = 1600$$

$$z_4(s) = \frac{2500}{s}$$

$$z_3(s) = 1 + \frac{2484}{s}$$

Case (i)

Stage 2: find  $k_2$

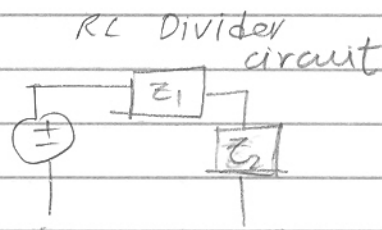
$$T_V(s) = \frac{+2 \times 10^4}{(s+250)(s+2500)}$$

$$\text{Using a non-inverting and } k_2 = 5 = \frac{R_1 + R_2}{R_1}$$

$$5R_1 = R_1 + R_2 \Rightarrow R_2 = 4R_1$$

$$R_1 = 10k$$

$$R_2 = 40k$$



We want  $R > 10k$  and  $C < 1\mu F$

stage 1:

$$Z_1(s) = 1$$

$$Z_2(s) = \frac{250}{s} \quad 1 \mu F s$$

$$= \frac{1}{4mF \cdot s}$$

stage 2:

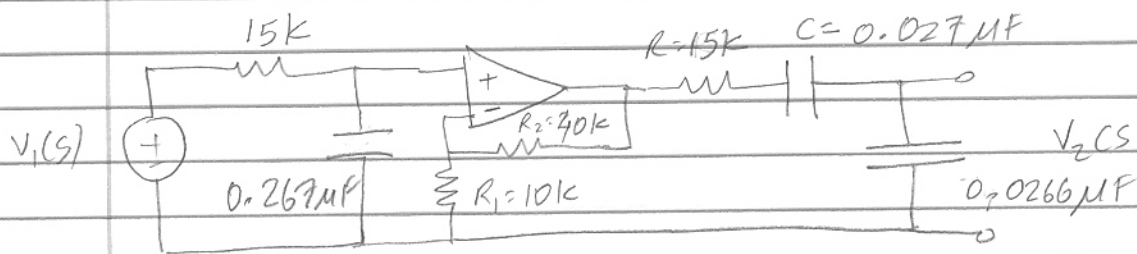
$$R_2 = 4R_1$$

stage 3:

$$Z_3(s) = 1 + \frac{2484}{s}$$

$$= 1 + \frac{1}{403 \mu F s}$$

$$Z_4(s) = \frac{1}{4 \times 10^{-4} F s}$$



stage 1

$$R_1 = 15k$$

$$Z_1(s) = 15k$$

$$C_2 = \frac{4mF}{15k} = 0.267 \mu F$$

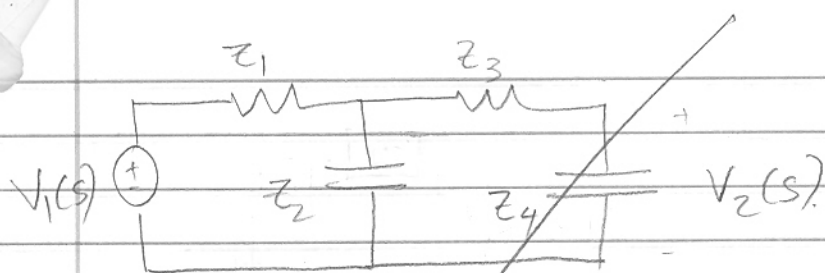
stage 3:

$$Z_3(s) = 1 + \frac{1}{403 \mu F s}$$

$$R = 15k$$

$$C = \frac{403 \mu F}{15k} = 0.027 \mu F$$

$$Z_4(s) = \frac{4 \times 10^{-4} F}{15k} = 0.0266 \mu F$$



$$Z_1 = 1$$

$$Z_2(s) = \frac{250}{s} = \frac{1}{0.004s}$$

$$Z_3 = 1$$

$$Z_4 = \frac{2500}{s}$$

Use scaling factor  $k_m = 10k\Omega$

$$Z_1 = R_1 = k_m(1) = 10k\Omega$$

$$Z_3 = R_3 = k_m(1) = 10k\Omega$$

$$C_2 = \frac{4mF}{1cm} = \frac{4mF}{10^{-4}} = 4 \times 10^{-7} F$$

$$C_4 = \frac{0.4mF}{1cm} = 4 \times 10^{-8} F$$

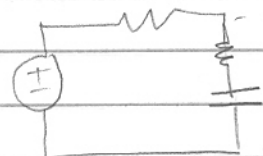
11.52 Design a circuit to realize the transfer function below using  $R$ ,  $C$ , and not more than one op-amp. Scale the circuit so that capacitors are exactly  $10nF$

$$T_V(s) = \frac{\pm 100(s+1000)}{(s+250)(s+2000)} = 1 \pm \frac{k_1}{s+250} - \frac{k_2(s+1000)}{s+2000}$$

$$\frac{k_1}{s+250} = \frac{k_1/s}{1 + 250/s} = \frac{Z_2(s)}{Z_1(s) + Z_2(s)}$$

$$Z_2(s) = \frac{k_1}{s} \quad Z_1(s) = 1 + \frac{250 - k_1}{s}$$

$$k_1 = 100 \Rightarrow Z_1 = 1 + \frac{150}{s} \quad Z_2 = \frac{100}{s}$$



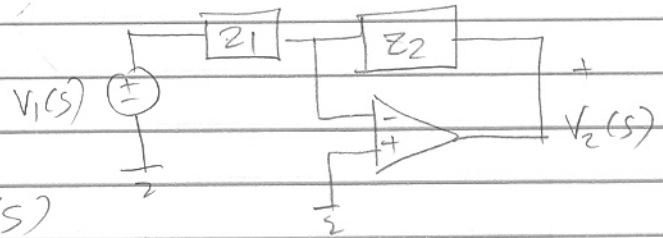
$$k_2 = 1$$

Design 1

$$T(s) = - \frac{(s+1000)}{s+2000}$$

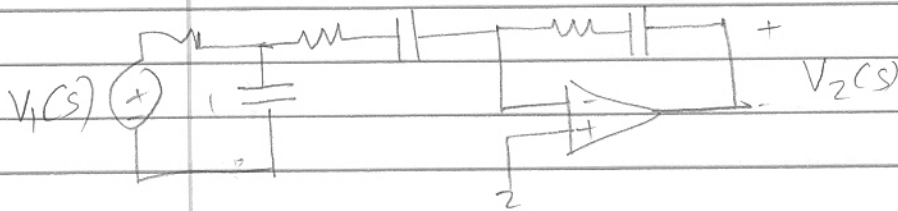
Using inverting op-Amp

$$-T(s) = \frac{-(s+1000)}{s+2000} = \frac{-Z_2(s)}{Z_1(s)}$$



$$Z_2(s) = 1 + \frac{1000}{s} \Rightarrow R = 1 \quad C = 10^{-3} F$$

$$Z_1(s) = 1 + \frac{2000}{s} \Rightarrow R = 1 \quad C = \frac{5 \times 10^{-4} F}{x}$$



Design 2

Using non-inverting op-Amp

$$T(s) = \frac{+(s+1000)}{s+2000} = \frac{1 + \frac{1000}{s}}{1 + \frac{2000}{s}} = \frac{Z_1(s) + Z_2(s)}{Z_1(s)}$$

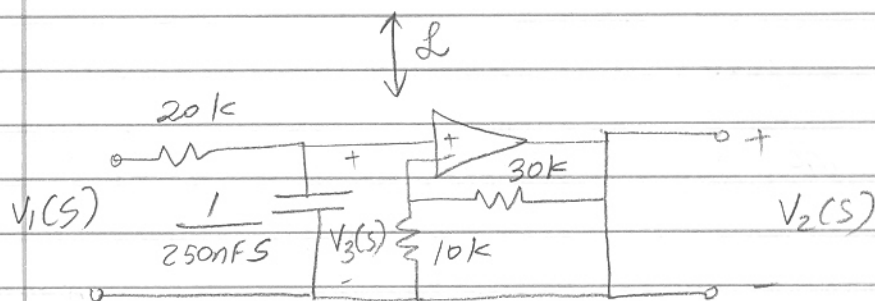
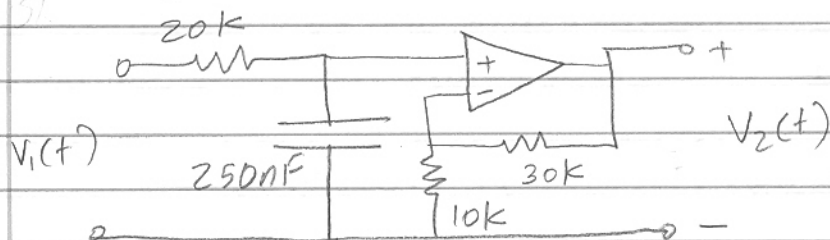
$$Z_1(s) = 1 + \frac{2000}{s}$$

$$Z_2(s) = 1 + \frac{1000}{s} - 1 - \frac{2000}{s} = \frac{-1000}{s}$$

12.3) Find the transfer function  $T_V(s) = V_2(s)/V_1(s)$

a) Find the dc gain, infinite frequency gain, and cutoff frequency. Identify the type of gain response.

b) 51



$$T_V(s) = T_{V1}(s) \cdot T_{V3}(s)$$

$$T_{V1}(s) = \frac{Z_2(s)}{Z_1(s) + Z_2(s)} = \frac{1}{20k + \frac{1}{250nF s}} = \frac{1}{(20k)(250nF s) + 1}$$

$$= \frac{1}{\frac{200}{200} s + 1} = \frac{200}{s + 200}$$

$$T_{V3}(s) = \frac{Z_1(s) + Z_2(s)}{Z_1(s)}$$

$$= \frac{10k + 30k}{10k} = \frac{40k}{10k} = 4$$

$$T_V(s) = \frac{4 \times 200}{s + 200} = \boxed{\frac{800}{s + 200}}$$

DC gain  $|T(0)| = \boxed{4}$

Infinite frequency gain  $|T(\infty)| = \frac{800}{\infty + 200} = \boxed{0}$

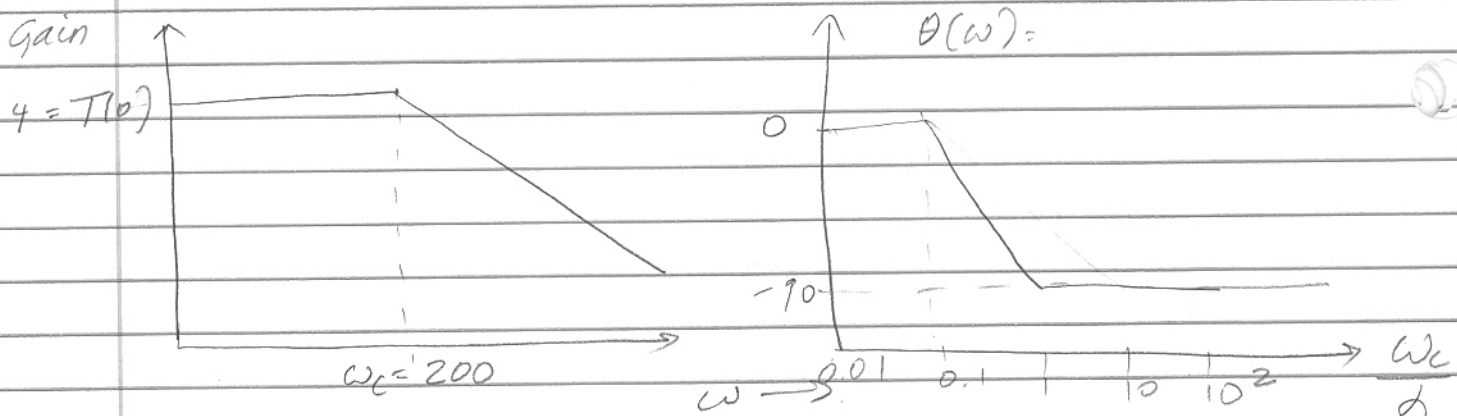
$$T(j\omega) = \frac{800}{j\omega + 200}$$

$$|T(j\omega)| = \frac{|800|}{\sqrt{\omega^2 + 200^2}} = \frac{|K|}{\sqrt{\omega^2 + \alpha^2}}$$

$$\boxed{\omega_c = \frac{1}{200} = 200 \text{ rad/s}} \rightarrow \text{cut-off frequency}$$

Low-Pass Gain Response — Type of gain response.

b) Sketch a straight-line approximation of the gain response.



c) What element values would you change to increase the passband gain to 10?

$$\frac{K}{\alpha} = 10$$

Change the value of  $z_2(s)$  of the op-Amp ( $R_2$ )

$$R_2 = 90k$$

$$\Rightarrow T_V(s) = \frac{10k + 90k}{10k} = 10$$

$$\Rightarrow T_V(s) = \frac{200(10)}{s + 200}$$

$$T_{max} = \frac{K}{\alpha} = \frac{200(10)}{200} = 10$$

12-9,

$$T(s) = \frac{2}{10^2 + 20/s} = \frac{200s}{s + 2000} \Rightarrow T(j\omega) = \frac{200j\omega}{j\omega + 2000}$$

→ high pass filter

$$\Rightarrow |T(j\omega)| = \frac{200\omega}{\sqrt{\omega^2 + 4 \times 10^6}}$$

→ passband gain = 200

→ cut off frequency  $\omega_c$  occurs when  $|T(j\omega_c)| = \frac{200}{\sqrt{2}}$

$$\Rightarrow \frac{\omega_c}{\sqrt{\omega_c^2 + 4 \times 10^6}} = \frac{1}{\sqrt{2}} \Rightarrow \omega_c = 2000 \text{ rad/s}$$

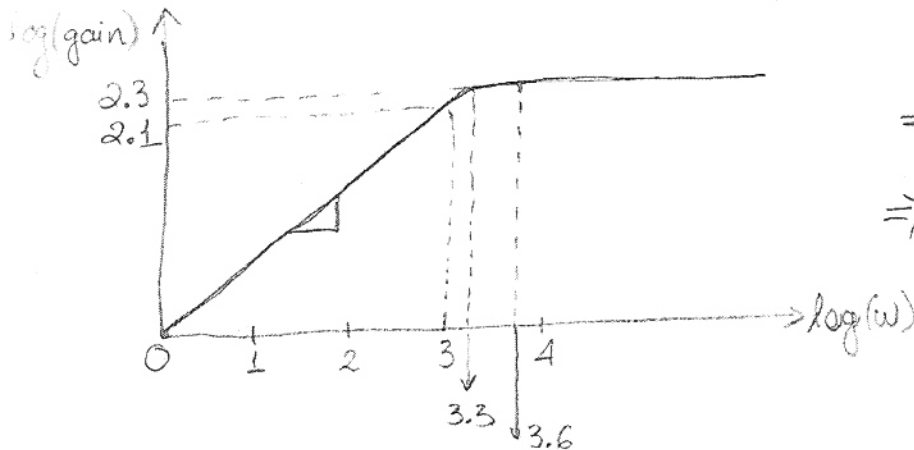
→ Bode-plot in log-log coordinate

$$\rightarrow \omega_c = 2000 \text{ rad/s} \Rightarrow \log_{10}(\omega_c) = 3.3$$

$$\rightarrow \text{gain} = 200 \Rightarrow \log_{10}(\text{gain}) = 2.3$$

$$\rightarrow \omega = 0.5\omega_c = 1000 \text{ rad/s} \Rightarrow \log_{10}(\omega) = 3$$

$$\rightarrow \omega = 2\omega_c = 4000 \text{ rad/s} \Rightarrow \log_{10}(\omega) = 3.6$$



$$\Rightarrow \text{slope} = 2.3/3.3 = 0.7$$

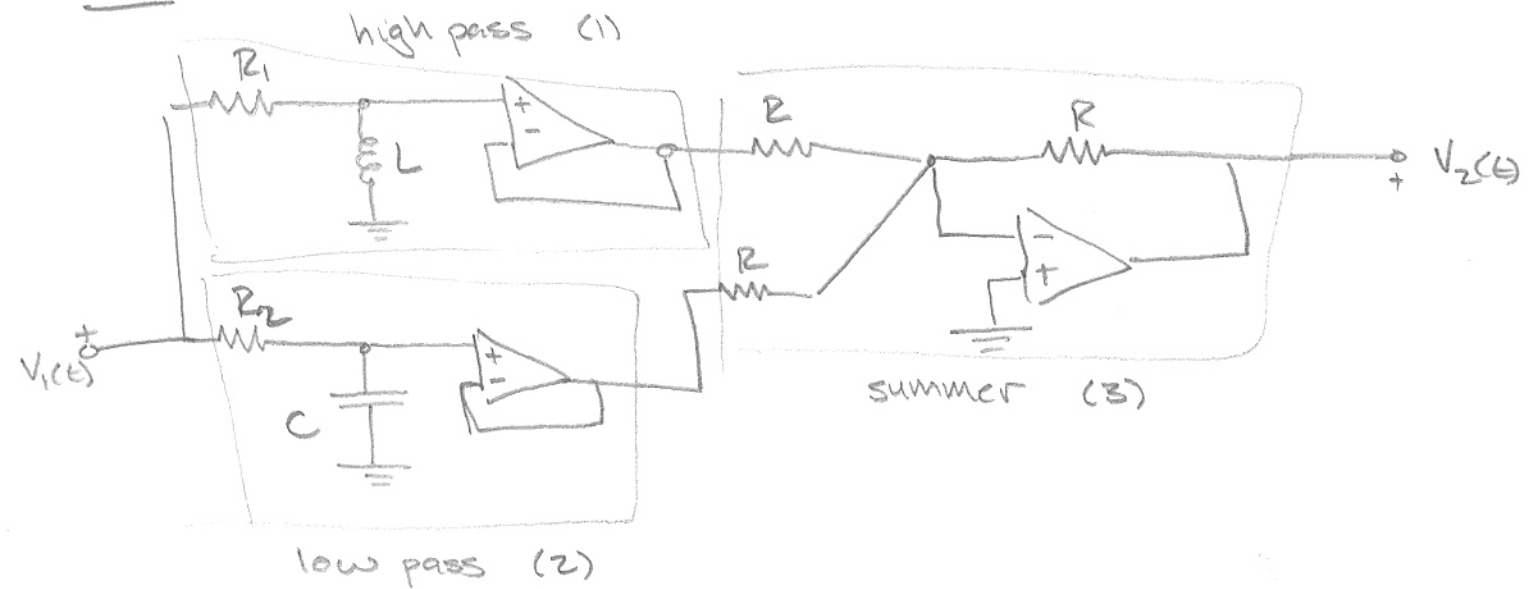
$$\Rightarrow \log(\text{gain}) = 0.7 \times \log(\omega)$$

$$\rightarrow \text{for } \omega = 0.5\omega_c, \log(\text{gain}) = 0.7 \times 3 = 2.1 \Rightarrow \text{gain} = 125.9$$

$$\rightarrow \text{for } \omega = \omega_c, \log(\text{gain}) = 2.3 \Rightarrow \text{gain} = 200$$

$$\rightarrow \text{for } \omega = 2\omega_c, \log(\text{gain}) = 2.3 \Rightarrow \text{gain} = 200$$

12.16



$$(1) T_1(s) \Rightarrow \text{high } \omega_c = \frac{-Ls}{R_1 + Ls} = \frac{-s}{s + (R_1/L)}$$

$$\omega_{c, \text{high}} = 500 \text{ rad/s} = R_1/L$$

$$\text{if } L = 100 \text{ mH} \Rightarrow R = 50 \Omega$$

$$(L = 5 \text{ mH} \Rightarrow R = 100 \text{ k}\Omega)$$

$$(2) T_2(s) = \frac{-1}{R_2Cs + 1} = \frac{-1/R_2C}{s + 1/R_2C}$$

$$\omega_{c, \text{low}} = 10 \frac{\text{rad}}{\text{s}} = \frac{1}{R_2C}$$

$$\Rightarrow \text{if } R_2 = 100 \text{ k}\Omega$$

$$\Rightarrow C = 1 \mu\text{F}$$