

$$9-1. \quad f(t) = 10[e^{-50t} - 2e^{-100t}]u(t)$$

$$F(s) = 10\left[\frac{1}{s+50} - \frac{2}{s+100}\right]$$

$$10\left[\frac{s+100 - 2s-100}{(s+50)(s+100)}\right] = \left[\frac{-10s}{(s+50)(s+100)}\right] = F(s)$$

$$\begin{aligned} \text{Zero: } s &= 0 \\ \text{Poles: } s &= -50, s = -100 \end{aligned}$$

$$8.8) a) f_1(t) = 4\delta(t) + [20e^{-20t} + 40e^{-40t}]u(t)$$

$$F_1(s) = 4 + \frac{20}{s+20} + \frac{40}{s+40}$$

$$\begin{aligned} b) f_2(t) &= [15 + 15 \cos 500t]u(t) \\ &= 15u(t) + 15 \cos 500t \\ &= \frac{15}{s} + \frac{15s}{s^2 + 500^2} \end{aligned}$$

9.10

(a) Since $f_1(t) = 3\delta(t-2) = 3\delta(t-2)u(t-2)$, the Laplace transform of $f_1(t)$ is $F_1(s) = 3e^{-2s}$ (to graders: explanation not necessary).

(b) The Laplace transform of $f_2(t) = 10e^{-50(t-1)}u(t-1)$ is $F_2(s) = \frac{10e^{-s}}{s+50}$.

(c) The Laplace transform of $f_3(t) = 10e^{-50(t-2)}u(t-2)$ is $F_3(s) = \frac{10e^{-2s}}{s+50}$.

$$\frac{9-16}{a)}$$

$$F_1(s) = \frac{s+160}{s(s+50)}$$

$$\frac{s+100}{(s+50)s} = \frac{A}{s} + \frac{B}{s+50}$$

$$s+100 = A(s+50) + B(s) \quad \text{let } s=0$$

$$100 = A(50) + 0 \quad A=2$$

$$50 = 0 + B(-50) \quad \text{let } s=-50$$

$$B=-1$$

$$F_1(s) = \frac{2}{s} + \frac{-1}{s+50}$$

$$f(t) = 2u(t) - e^{-50t}u(t) = (2 - e^{-50t})u(t)$$

$$b) F_2(s) = \frac{s+10}{s(s+50)(s+100)}$$

$$\text{let } s=0$$

$$s+10 = A(s+50)(s+100) + B(s)(s+100) + C(s)(s+50)$$

$$10 = A(50)(100) + 0 + 0 \quad A = \frac{1}{500}$$

$$\text{let } s=-50$$

$$-40 = 0 + B(-50)(50) + 0 \quad B = -\frac{2}{125}$$

$$\text{let } s=-100$$

$$-90 = 0 + 0 + C(-100)(-50) \quad C = \frac{-9}{500}$$

$$F_2(s) = \frac{\frac{1}{500}}{s} + \frac{-\frac{2}{125}}{s+50} + \frac{-\frac{9}{500}}{s+100}$$

$$F(t) = \left(\frac{1}{500} - \frac{2}{125} e^{-50t} - \frac{9}{500} e^{-100t} \right) u(t)$$

$$925.) a) F(s) = \frac{4(s^2 + 16)}{s(s^2 + 8s + 32)}$$

$$\frac{4(s^2 + 16)}{s(s^2 + 8s + 32)} = \frac{k_1}{s} + \frac{k_2}{s+4+j} + \frac{k_3}{s+4-j}$$

$$k_1 = sF(s)|_{s=0} = \frac{4(16)}{32} = 2$$

$$k_2 = (s+4+j)F(s)|_{s=-4-j} = \frac{4((-4-j)^2 + 16)}{(-4-j)(-4-j+4-j)}$$

$$= \frac{4(16 + 16j^2 + 32j + 16)}{(-4-j)(-8j)}$$

$$= \frac{2+4j}{(-1-j)(-j)} = \frac{2+4j(1+j)}{-1+j(1+j)} = \frac{2+4j+4j^2}{-1+j+1+j^2} = \frac{2-4+4j}{-j} = \frac{-2+4j}{-j} = \frac{2-4j}{j} = \frac{2-4j}{j} \cdot \frac{j}{j} = \frac{2j-4j^2}{-1} = \frac{2j+4}{-1} = -2-4j$$

$$K_3 = (s+4-j)F(s) \Big|_{s=-4+j}$$

$$= \frac{4 [(-4+j)^2 + 16]}{(-1+j)(-4+j+4+j)}$$

$$= \frac{4 [16 + 16j^2 - 32j + 16]}{(-4+j)(8j)} = \frac{[2 - 4j]}{(-1+j)(j)}$$

$$= \frac{2-4j}{-j+j^2} = \frac{2-4j(-1+j)}{-j-1(-1+j)}$$

$$= \frac{-2+2j+4j-4j^2}{6(-1)^2-j^2}$$

$$= \frac{-2+6j}{2} = 1+3j$$

$$= \sqrt{10} e^{j71.5^\circ}$$

$$= \sqrt{10} e^{-j128.5^\circ}$$

$$F_1(s) = \frac{2}{s} + \frac{1-3j}{s+4+j} + \frac{1+3j}{s+4-j}$$

$$f_1(t) = 2u(t) + 2\sqrt{10} e^{-4t} \cos\left(4t + \tan^{-1}(3)\right) u(t)$$

$$9.25b) F_2(s) = \frac{2(s^2 + 30s + 800)}{s(s^2 + 50s + 400)} = \frac{k_1}{s} + \frac{k_2}{s+10} + \frac{k_3}{s+40}$$

$$k_1 = sF(s)|_{s=0} = \frac{2(800)}{400} = 4$$

$$k_2 = (s+10)F(s)|_{s=-10} = \frac{2(100 - 300 + 800)}{(-10)(-100 - 400)} = \frac{2(600)}{-300} = -4$$

$$k_3 = (s+40)F(s)|_{s=-40} = \frac{2(1600 - 1200 + 800)}{(-40)(-40+10)} = \frac{2(1200)}{+1200} = +2$$

$$F_2(s) = \frac{4}{s} + \frac{-4}{s+10} + \frac{2}{s+40}$$

$$= [4 - 4e^{-10t} + 2e^{-40t}]u(t)$$

9.32

To solve the following ordinary differential equation,

$$\frac{di(t)}{dt} + 500i(t) = .100e^{-200t}u(t), \quad i(0^-) = 0,$$

begin by taking the Laplace transform;

$$sI(s) + 500I(s) = \frac{.100}{s + 200} \quad (\text{we have used the fact that } i(0^-) = 0).$$

We have

$$\begin{aligned} I(s) &= \frac{.100}{(s + 200)(s + 500)} \\ &= \frac{c_1}{s + 200} + \frac{c_2}{s + 500}, \end{aligned}$$

where the terms c_1 and c_2 are

$$\begin{aligned} c_1 &= \lim_{s \rightarrow -200} (s + 200)I(s) \approx \frac{1}{3000}, \\ c_2 &= \lim_{s \rightarrow -500} (s + 500)I(s) \approx -\frac{1}{3000} \end{aligned}$$

Now, taking the inverse Laplace transform of $I(s)$, we have that

$$i(t) \approx \frac{1}{3000} (e^{-200t} - e^{-500t}) u(t).$$

(This is an approximate solution due to the term .100 having only three significant digits.)

9-47

$$a) F_1(s) = \frac{50(s^2 + 3s + 6)}{(s+2)(s+6)(s+12)} = \frac{50(s+3)}{(s+6)(s+12)}$$

$$\text{initial value} \quad \lim_{s \rightarrow \infty} \frac{s \cdot 50(s+3)}{(s+6)(s+12)} = \boxed{50}$$

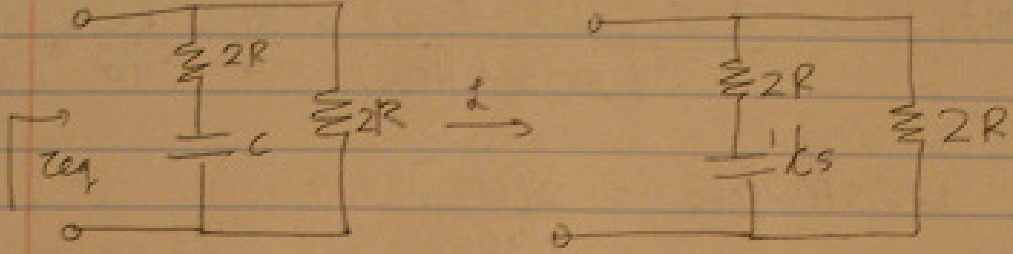
$$\text{final value} \quad \lim_{s \rightarrow 0} \frac{s \cdot 50(s+3)}{(s+6)(s+12)} = \boxed{0}$$

$$b) F_2(s) = \frac{10(s^2 + 10s + 40)}{s(s+25)(s-25)}$$

$$\text{initial value} \quad \lim_{s \rightarrow \infty} \frac{s \cdot 10(s^2 + 10s + 40)}{s(s+25)(s-25)} = \boxed{10}$$

final value doesn't exist. There is a pole in the Right-half plane so it's unstable.

10.1



$$Z_{eq} = \frac{(2R + 1/Cs)2R}{2R + \frac{1}{Cs} + 2R}$$

$$= \frac{(2RCs + 1)2R}{4RCs + 1} = \frac{4R^2Cs + 2R}{4RCs + 1}$$

$$= \frac{Rs + \frac{2R}{4RC}}{s + \frac{1}{4RC}} = \frac{Rs + \frac{1}{2C}}{s + \frac{1}{4RC}}$$

pole: $-\frac{1}{4RC}$

zero: $-\frac{1}{2RC}$

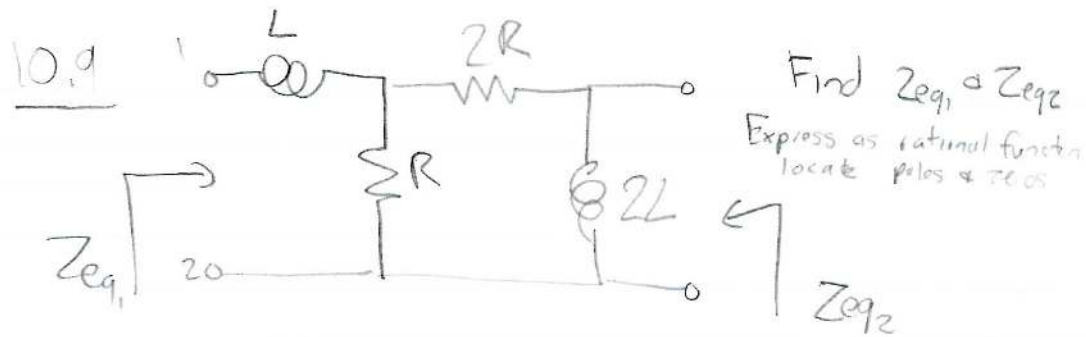
10.6

The equivalent impedance follows from transforming the circuit into the s -domain and assuming the initial conditions are zero (without loss of generality due to superposition). We have

$$\begin{aligned} Z_{eq} &= sL + \left(R \parallel \frac{1}{sC} \right), \\ &= sL + \frac{R \times \frac{1}{sC}}{R + \frac{1}{sC}}, \\ &= \frac{RLCs^2 + Ls + r}{RCs + 1}. \end{aligned}$$

Substituting the values for R , L , and C , we find a pole at $s = -1/(RC) = -2000$.

To find the zeros, we use MATLAB's root-finding function on the numerator polynomial (after substituting parameters). The two zeros are both at $s = -1000$.



$$Z_{eq1} = L + (R \parallel (2R + 2L))$$

$$2R + 2L = 2R + 2Ls$$

$$Z_{eq1} = \frac{R(2R + 2Ls)}{3R + 2Ls} + \frac{Ls(3R + 2Ls)}{3R + 2Ls}$$

$$Z_{eq1} = \frac{2R^2 + 2LsR + 3LsR + 2L^2s^2}{3R + 2Ls} = \frac{2L^2s^2 + 5LsR + 2R^2}{3R + 2Ls}$$

$$\text{pole @ } s = -\frac{3R}{2L}, \quad \frac{-5LR \pm \sqrt{25L^2R^2 - 4 \cdot 2L^2 \cdot 2R^2}}{4L^2} = \frac{-5LR \pm 3LR}{4L^2}$$

$$\text{zeros @ } s = -\frac{R}{2L}, \quad s = -\frac{2R}{L}$$

$$Z_{eq2} = 3R \parallel 2L = \frac{6LsR}{3R + 2Ls}$$

$$\text{pole @ } -\frac{3R}{2L}$$

$$\text{zero @ } 0$$