

10/55

$$V_a(s) = R_2 + \frac{1}{sC_1} \quad V_2(s)$$

$$\Rightarrow V_a(s) = (sC_1 R_2 + 1) V_2(s)$$

KCL V_a :

$$V_a \left(\frac{1}{R_1} + \frac{1}{R_2} + sC_2 \right) = \frac{V_1(s)}{R_1} + V_2(s) \left(sC_2 + \frac{1}{R_2} \right)$$

$$V_2(s) (sC_1 R_2 + 1) \left(\frac{1}{R_1} + \frac{1}{R_2} + sC_2 \right) - V_2(s) \left(sC_2 + \frac{1}{R_2} \right) = \frac{V_1(s)}{R_1}$$

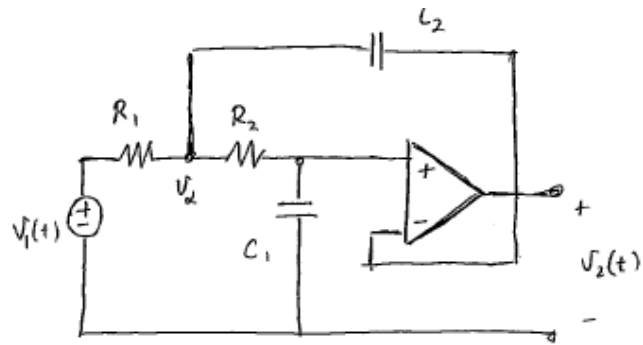
$$V_2(s) \left(\frac{sC_1 R_2}{R_1} + sC_1 + s^2 C_1 C_2 R_2 + \frac{1}{R_1} + \frac{1}{R_2} + sC_2 - sC_2 - \frac{1}{R_2} \right) = \frac{V_1(s)}{R_1}$$

$$\therefore V_1(s) = V_2(s) (sC_1 R_2 + sC_1 R_1 + s^2 C_1 C_2 R_1 R_2 + 1)$$

and

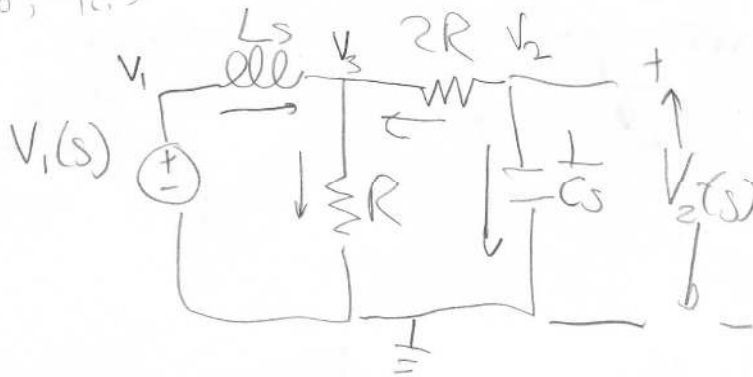
$$\frac{V_2(s)}{V_1(s)} = T(s) = \frac{(C_1 C_2 R_1 R_2)^{-1}}{s^2 + s[(C_2 R_1)^{-1} + (C_2 R_2)^{-1}] + (C_1 C_2 R_1 R_2)^{-1}}$$

$$\left(\text{equivalently } T(s) = \frac{1}{C_1 C_2 R_1 R_2 s^2 + (R_2 + R_1) C s + 1} \right)$$



11.5, 11.8, 11.9

11.5



$$\frac{V_1 - V_3}{Ls} - \frac{V_3}{R} + \frac{V_2 - V_3}{2R} = 0$$

$$(1) \quad \frac{V_2}{2R} - \left(\frac{1}{Ls} + \frac{1}{R} + \frac{1}{2R} \right) V_3 = -\frac{V_1}{Ls}$$

$$-\frac{V_2 - V_3}{2R} - \frac{V_2}{1/Cs} = 0$$

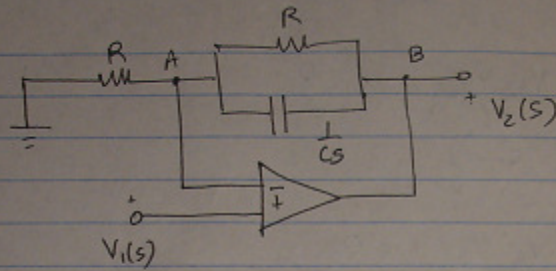
$$(2) \quad -\left(\frac{1}{2R} + Cs \right) V_2 + \frac{V_3}{2R} = 0$$

$$\begin{bmatrix} \frac{1}{2R} & -\left(\frac{1}{Ls} + \frac{1}{R} + \frac{1}{2R} \right) \\ -\left(\frac{1}{2R} + Cs \right) & \frac{1}{2R} \end{bmatrix} \begin{bmatrix} V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} -\frac{V_1}{Ls} \\ 0 \end{bmatrix}$$

using Matlab, $V_2 = \frac{R \cdot V_1}{3CLR^2s^2 + (2CR^2 + L)s + R}$

$$\boxed{\frac{V_2}{V_1} = \frac{R}{3CLR^2s^2 + (2CR^2 + L)s + R}}$$

11.7



$$V_B = V_2(s)$$

$$V_+ = V_-$$

$$V_A = V_1(s)$$

KCL At A
$$\frac{V_A}{R} + \frac{V_A - V_B}{R} + \frac{V_A - V_B}{\frac{1}{Cs}} = 0$$

$$\frac{V_A}{R} + \frac{V_A - V_B}{R\left(\frac{1}{Cs}\right)} = 0$$

$$\frac{V_A}{R} + \frac{V_A - V_B}{R} = 0$$

$$\frac{V_A}{R} + \frac{RCs+1}{R} V_A = \frac{RCs+1}{R} V_B$$

$$V_1(s) \left[\frac{1}{R} + \frac{RCs+1}{R} \right] = \frac{RCs+1}{R} V_B$$

$$V_1(s) \left[\frac{RCs+2}{R} \right] = \frac{RCs+1}{R} V_2(s)$$

$$\frac{V_2(s)}{V_1(s)} = \frac{(RCs+2)R}{R(RCs+1)} = \frac{RCs+2}{RCs+1}$$

pole at $-1k \text{ rad/s}$

$$RCs+1=0 \Rightarrow s = \frac{-1}{RC} = -1k$$

$$\boxed{R = 1k}$$

$$C = 1\mu F$$

11/8] $T_I(s) = \frac{I_2(s)}{I_1(s)} = \frac{R}{3R + sL}$ (immediate from source transformation)

$$T_I(s) = \frac{R/L}{s + 3R/L}$$

e.g. $R = \frac{500}{3} \Omega, L = 1 \text{ H}$

↗ This answer is non-unique

11-28



$$R_1 = 100 \Omega$$

$$R_2 = 400 \Omega$$

$$L = 100 \times 10^{-3} \text{ H}$$

repeat for $i_1(t) = 10 \cos(5000t) \cdot 10^{-3} \text{ A}$

$$\mathcal{L}\{\cos \beta t\} = \frac{s}{s^2 + \beta^2}$$

$$i_1(s) = \frac{10s}{s^2 + \beta^2} \cdot 10^{-3}$$

$$\beta = 5000$$

$$-R_2 i_2 - L s i_2 - R_1 (i_2 - i_1) = 0$$

$$R_1 i_1 - (R_1 + R_2 + Ls) i_2 = 0$$

$$\begin{bmatrix} 1 & 0 \\ R_1 & -(R_1 + R_2 + Ls) \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} \frac{10s}{s^2 + \beta^2} \\ 0 \end{bmatrix}$$

$$\text{for } \beta = 5000$$

$$i_2(s) = \frac{10^2 R_1 s}{L s^3 + (R_1 + R_2) s^2 + \beta^2 L s + \beta^2 (R_1 + R_2)}$$

$$i_1(s) = \frac{s}{s^2 + 5000^2 + 75000s + 1.25 \times 10^8}$$

$$i_1(s) = \frac{-0.002}{s + 5000} + \frac{.001 - .0001i}{s - 5000i} + \frac{.001 + .0001i}{s + 5000i}$$

$$i_2(t) = \underbrace{-0.002 e^{-5000t}}_{\text{decays to zero}} + (.001 - .0001i) e^{5000it} + (.001 + .0001i) e^{-5000it}$$

$$i_2(t) = -2 \left(\cos 5000t + \frac{\sin 5000t}{10} \right) \text{ mA} \approx -2 \cos(5000t - .0997)$$

$$\text{for } \beta = 5000, \quad i_2(s) = \frac{s}{L s^3 + 5000^2 s^2 + 2.5 \times 10^6 s + 1.25 \times 10^{10}} = \frac{-0.001}{s + 5000} + \frac{5 \times 10^{-4} + 5 \times 10^{-4}i}{s + 5000i} + \frac{5 \times 10^{-4} - 5 \times 10^{-4}i}{s + 5000i}$$

$$i_2(t) = \underbrace{-0.001 e^{-5000t}}_{\text{decays to zero}} + (5 \times 10^{-4} + 5 \times 10^{-4}i) e^{5000it} + (5 \times 10^{-4} - 5 \times 10^{-4}i) e^{-5000it}$$

$$i_{2ss}(t) = (\cos 5000t + \sin 5000t) \text{ mA} = \sqrt{2} \cos(5000t - \frac{\pi}{4})$$

$$11.71) T_V(s) = \pm \frac{2 \times 10^5}{(s+100)(s+10000)}$$

Case i)

$$T_V(s) = \frac{k_1}{(s+100)} + \frac{k_2}{s+10000}$$

$$\frac{k_1}{s+100} = \frac{k_1/s}{1+100/s} = \frac{Z_2(s)}{Z_1(s) + Z_2(s)}$$

$$Z_2(s) = \frac{k_1}{s} \quad Z_1(s) = 1 + \frac{100}{s} - \frac{k_1}{s}$$

$$\boxed{Z_2 = \frac{100}{s} \quad Z_1(s) = 1} \quad = 1 + \frac{(100 - k_1)}{s}$$

$$k_1 = 100 = 10^2$$

$$\frac{k_3}{s+10000} = \frac{Z_2(s)}{Z_1(s) + Z_2(s)} = \frac{k_3/s}{1 + 10000/s}$$

$$Z_2(s) = \frac{k_3}{s} \quad Z_1(s) = 1 + \frac{10000 - k_3}{s}$$

$$k_3 = 10000 = 10^4$$

$$\boxed{Z_2(s) = \frac{10^4}{s} \quad Z_1(s) = 1 + \frac{10000}{s}}$$

$$\text{Case (i): } k_1 k_2 k_3 = + 2 \times 10^5$$

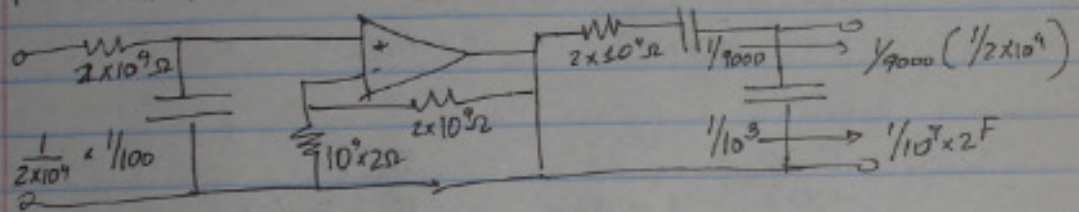
$$C < 1 \mu F = 10^{-6}$$

$$R > 10k = 10^4$$

$$10^5 k_2 = 2 \times 10^5$$

$$k_2 = \frac{Z_1}{Z_2} = \frac{Z_1 + Z_2}{Z_2} \quad \begin{matrix} Z_1 + Z_2 = 2Z_2 \\ Z_1 = Z_2 \end{matrix}$$

using scaling factor $k_m = 10^4 \times 2$

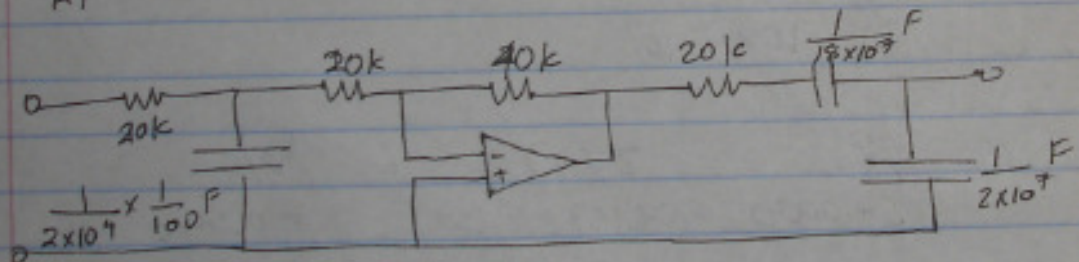


(Case (ii)) $k_1, k_2, k_3 = -2 \times 10^5$

$$10^5 k_2 = -2 \times 10^5$$

$$\Rightarrow k_2 = -2$$

$$\frac{-R_2}{R_1} = -2 \Rightarrow R_2 = 2R_1$$



11/58a | cct 1: $V_1(s) = V_p(s)$

$$V_p(s) \cdot \left(\frac{1}{10k} + \frac{1}{10k} + (100n)s \right) = V_2(s) \left(\frac{1}{10k} + (100n)s \right)$$

$$V_1(s) \cdot \left(2 \times 10^{-4} + 10^{-7}s \right) = V_2(s) \left(10^{-4} + 10^{-7}s \right)$$

$$\frac{V_2(s)}{V_1(s)} = T_v(s) = \left(\frac{s + 2000}{s + 1000} \right) \quad \blacksquare$$

cct 2: $V_2(s) = \frac{10k + (50n \cdot s)^{-1}}{20k + (50n \cdot s)^{-1}} V_1(s)$

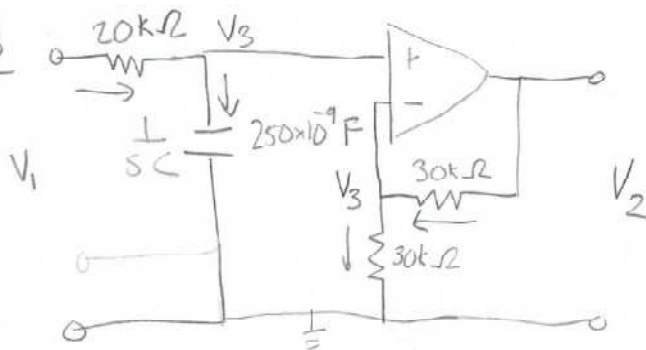
$$\begin{aligned} \frac{V_2(s)}{V_1(s)} = T_v(s) &= \frac{(10k)(50n)s + 1}{(20k)(50n)s + 1} \\ &= \frac{\frac{1}{2}s + 1000}{s + 1000} \times \frac{2}{2} \\ T_v(s) &= \frac{1}{2} \left(\frac{s + 2000}{s + 1000} \right) \quad \blacksquare \end{aligned}$$

11/58b | Since loading, especially for impedances of less than $10k\Omega$, changes the V_1 to V_2 characteristic of circuit 2, circuit 1 is clearly superior.

11/58c | An input impedance changes the V_1 to V_2 characteristic of circuit 2, so circuit 1 is superior; ~~however~~ however, one may argue that the 100Ω input impedance has negligible influence on the transfer function of circuit 2 (considering tolerances of practical components), so, because circuit 2 uses fewer & cheaper parts, it is superior.

11/58d | This assertion is true. Note that the transfer function of circuit 1 is unaffected by both input and output impedances. ~~The~~ The transfer function of circuit 2 may be affected by input or output impedances; however, circuit 1 introduces neither (this may be verified by a straightforward ~~Flawed~~ argument using Thevenin's Theorem... but only if you're feeling really anal). Hence the circuits may be connected in either order.

12.3

400
5+200

$$\frac{V_1 - V_3}{20k} - V_3 sC = 0$$

$$\frac{V_2 - V_3}{30k} - \frac{V_3}{30k} = 0$$

$$\frac{1}{20k} V_1 + 0V_2 - \left(\frac{1}{20k} + sC\right) V_3 = 0$$

$$0V_1 + \frac{1}{30k} V_2 - \left(\frac{1}{15k}\right) V_3 = 0$$

$$\frac{1}{20k} \cdot \frac{V_1}{\frac{1}{20k} + sC} = V_3$$

$$V_2 = 2V_3$$

$$V_2 = \frac{2}{20k} \cdot \frac{V_1}{\frac{1}{20k} + sC} \cdot \frac{20k}{20k} = \frac{2}{1 + 20ksC} = \frac{V_2}{V_1}$$

a)

$$\frac{V_2}{V_1} = \frac{400}{s + 200}$$

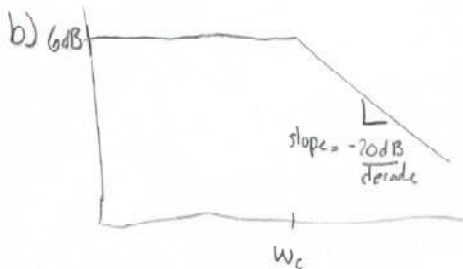
DC gain: 2

infinite frequency gain: 0

$$\frac{400}{\sqrt{\omega^2 + 200^2}} = \frac{2}{\sqrt{2}} \Rightarrow \frac{400^2}{2} = \omega^2 + 200^2$$

$$\omega_c = 200 \text{ rad/s}$$

low pass filter

c) gain at $0.5\omega_c$:

$$\frac{400}{\sqrt{100^2 + 200^2}} = 1.789$$

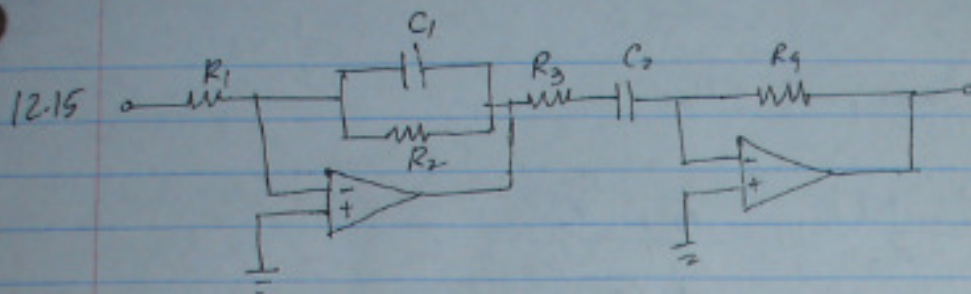
gain at ω_c :

$$1.414$$

gain at $2\omega_c$:

$$1.8944$$

e) increase the horizontal $30k\Omega$ resistor to a $270k\Omega$ resistor



Bandpass gain = 30

$$\omega_{c1} = 100 \text{ rad/s} \quad \omega_{c2} = 2500 \text{ rad/s}$$

$$R \geq 10 \text{ k} \quad C \leq 1 \text{ nF}$$

Stage 1: Inverter

$$K_1 = \frac{-Z_2}{Z_1} = \frac{-\left(\frac{1/C_1 \cdot R_2}{R_2 + \frac{1}{C_1 s}}\right)}{R_1} = \frac{-R_2}{(R_2 C_1 s + 1) R_1}$$

Stage 2:

$$K_2 = \frac{-Z_2}{Z_1} = \frac{-R_4}{R_3 + \frac{1}{C_2 s}} = \frac{-R_4 C_2 s}{R_3 C_2 s + 1}$$

$$T_V(s) = \frac{-R_2}{R_1 (R_2 C_1 s + 1)} \cdot \frac{-R_4 C_2 s}{(R_3 C_2 s + 1)}$$

$$T(s) = \left(\frac{R_2 / R_1 R_2 C_1}{s + \frac{R_1}{R_1 R_2 C_1}} \right) \cdot \left(\frac{\frac{R_4 C_2 s}{R_3 C_2}}{s + \frac{1}{R_3 C_2}} \right)$$

~~Eq 12.15~~ $\omega_{c1} = \frac{1}{R_2 C_1} \quad \omega_{c2} = \frac{1}{R_3 C_2} \quad K_1 = \frac{1}{R_1 C_1} \quad K_2 = \frac{R_4}{R_3}$

Band pass filter

$$\frac{|k_1 k_2|}{\omega_c} = \frac{\frac{1}{R_1 C_1} \cdot \frac{R_4}{R_3}}{\frac{1}{R_3 C_2}}$$

$$\Rightarrow \frac{R_4 C_2}{R_1 C_1} = 30$$

$$\frac{1}{R_3 C_2} = 100 \text{ rad/s} \Rightarrow$$

$$\frac{1}{R_3 C_1} = 2500$$

$$C_1 = \frac{1}{2500 R_3} \quad C_2 = \frac{1}{100 R_3}$$

$$R_3 = 10 \text{ k}\Omega$$

$$C_1 = 4 \times 10^{-9} \text{ F}$$

$$R_3 = 10 \text{ k}\Omega$$

$$C_2 = 10^{-7} \text{ F}$$

$$\frac{R_4}{R_1} \cdot \frac{10^{-7}}{4 \times 10^{-9}} = 30$$

$$\frac{R_4}{R_1} = \frac{6}{5} \Rightarrow R_4 = 60 \text{ k}\Omega$$

$$R_1 = 50 \text{ k}\Omega$$

12/57] 1st cct: filter characteristics

$$V_p(s) \approx \frac{1k}{100m + \frac{1}{s400n} + 1k} V_i(s) = \frac{10k}{100k} V_2(s)$$

$$\begin{aligned} \frac{V_2(s)}{V_1(s)} &= T_1(s) = \frac{10k}{s100m + \frac{1}{s400n} + 1k} \\ &= \frac{(10k)(400n)s}{s^2(100m)(400n) + (1k)(400n)s + 1} \\ &= \frac{10^5 s}{s^2 + 10^4 s + 25 \cdot 10^6} \\ &= \frac{10^5 s}{(s+5000)(s+5000)} \end{aligned}$$

The so-called pass-band gain is attained at roughly $\omega_0 = 5000 \text{ rad/s}$ (the center frequency)

$$|T(5000j)| = \left| \frac{10^5 (5000j)}{2(5000)(5000)} \right| = |10j| = \underline{10}$$

$$\Rightarrow |T(\omega_0 j)| = 10 \quad (\text{pass-band gain})$$

The cut-off frequencies are

$$\omega_{c1} = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

$$\omega_{c2} = \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}} =$$

$$B = \omega_{c2} - \omega_{c1} = \underline{10 \text{ krad/s} = B} \quad (\text{bandwidth})$$

Filter 1 meets specs almost exactly. ~~Then~~

Because it is also a superior design when comparing part count & use of standard components, there is no reason to investigate the filter specs of circuit 2 (though there is nothing wrong with doing so.)

Hence, filter 1 is the better design

(Should be worth half of points)