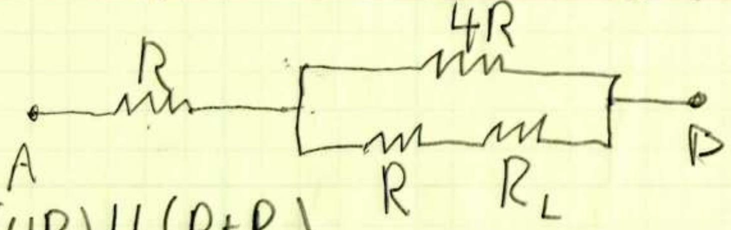
MAE 140 | HW Solution | P.S. #2

2.38 Redraw



$$\Rightarrow R_{AB} = R + (4R) \parallel (R+R_L)$$

$$= R + \frac{4R^2 + 4RR_L}{5R + R_L}$$

$$= \frac{5R^2 + RR_L + 4R^2 + 4RR_L}{5R + R_L}$$

$$R_{AB} = \frac{9R^2 + 5RR_L}{\cancel{5R+R_L}} = R_{AB} \cdot \frac{5R+R_L}{\cancel{5R+R_L}}$$

Now we have

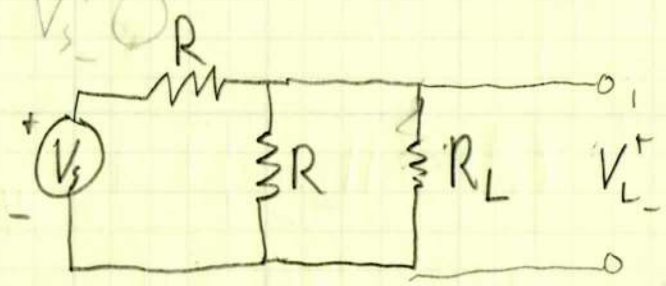
$$5R_{AB}R + R_{AB}R_L = 9R^2 + 5RR_L$$

Let $R_{AB} = R_L$ and subsitute in: $R_L^2 = 9R^2$

$$\Rightarrow \boxed{R = R_L/3}$$

This R Value will make $R_L = R_{AB}$

2.41.



To use Voltage divider first find Req across V_L: $R_{eq} = \frac{R \cdot R_L}{R + R_L}$

Now apply Voltage divider rule:

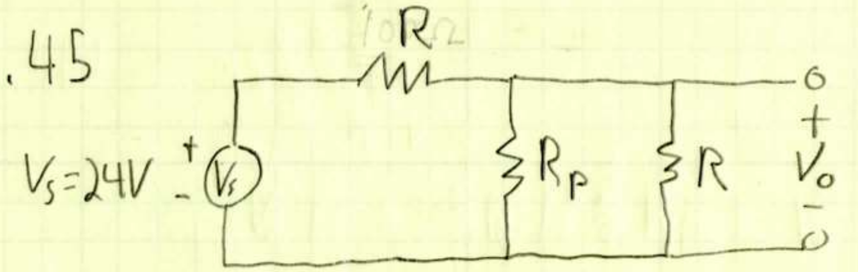
$$V_L = \left(\frac{R_{eq}}{R + R_{eq}}\right) V_s$$

Substitute $R_{eq} = \frac{RR_L}{R+R_L}$ and do some algebra* to get:

$$\boxed{V_L = \left(\frac{R_L}{R + 2R_L}\right) V_s}$$

* you may want to investigate Matlabs Symbolic toolbox.

2.45



$R = 100\Omega$, R_p is a potentiometer
$0 < R_p < 100\Omega$

Trivial Solution: $R_p = 0$

$R_p = 0$, all current flows through R_p which now acts as a wire
$\Rightarrow$ $V_o = 0V$

Now let $R_p = 100\Omega$ Use Voltage division to get

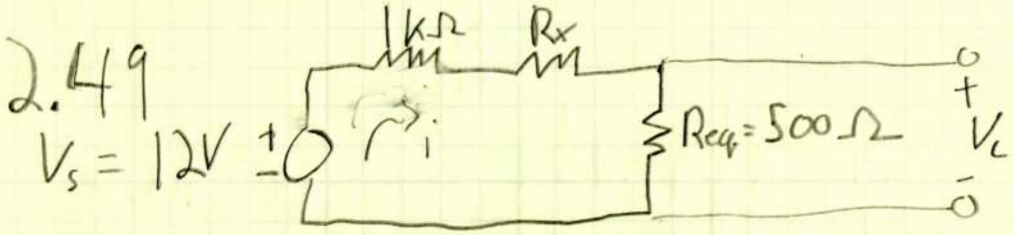
$$V_o = \left(\frac{R_p}{R + 2R_p}\right) V_s \quad \text{(see 2.41 solution)}$$

Sub in values to get $V_o = 8V$

also notice if $R_p = 0$ then $V_o = 0V$

$$\Rightarrow \boxed{0V < V_o < 8V}$$

2.49



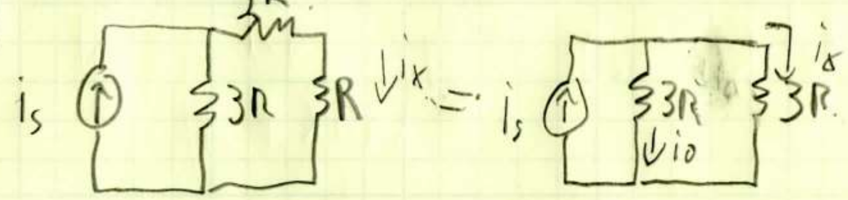
Simple Voltage Divider.

$$V_L = i\,R_{eq} = \left(\frac{R_{eq}}{1k\Omega + R_x + R_{eq}}\right) \cdot V_s$$

Solve for $R_x = R_{eq} \cdot \frac{V_s}{V_L} - 1k\Omega - R_{eq}$

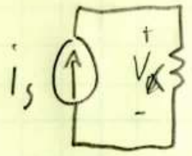
$$\boxed{R_x = 500\Omega}$$

2.51



KCL $i_s = i_o + i_x$ but $i_o = i_x$ So $\boxed{i_x = i_s/2}$

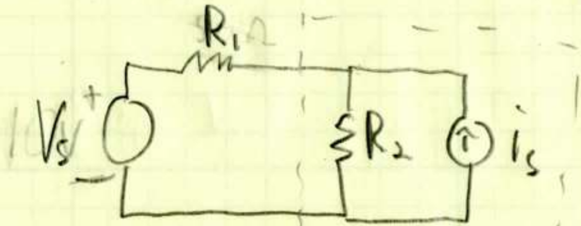
reduce circuit further to see



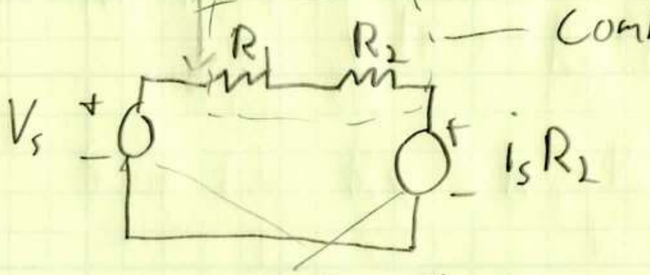
$$\frac{3R \cdot 3R}{3R + 3R} = \frac{3}{2}R \qquad \boxed{V_x = iR = \frac{3}{2} R\, i_s}$$

Alternatively $V_x = 3R \cdot i_x = 3R \cdot \frac{i_s}{2} = \underline{\frac{3}{2} R\, i_s}$

2.55 Note; more than one way to go about this.



replace this



Combine these

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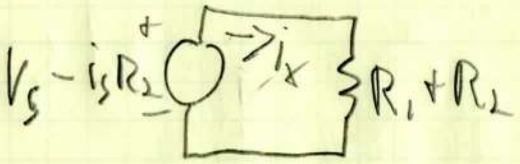
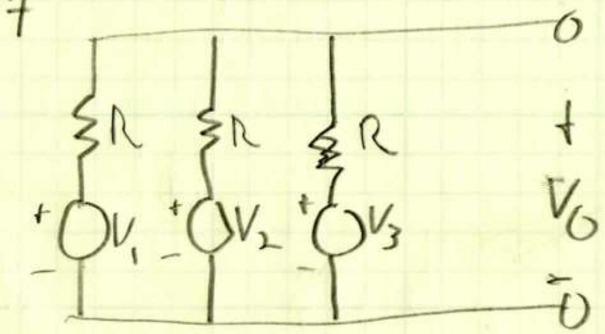


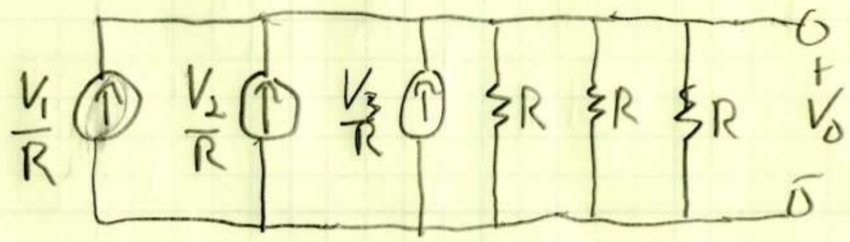
$V = iR \Rightarrow i = \frac{V}{R}$

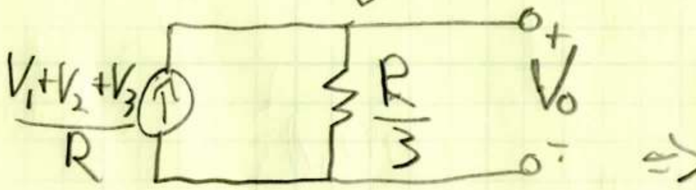
$$\Rightarrow \boxed{i_x = \frac{V_s - i_s R_2}{R_1 + R_2} = 50mA}$$

2.57



Replace with current source

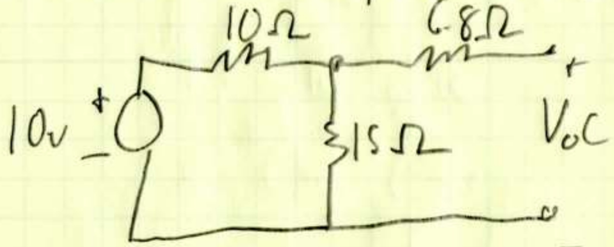




$V = iR$

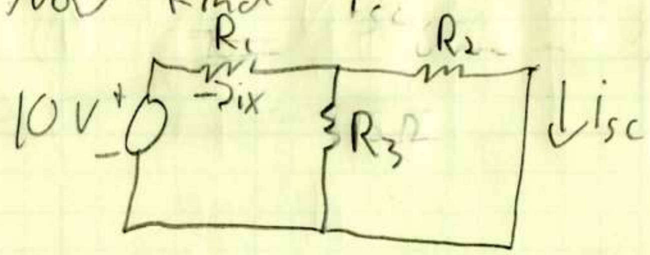
$$\Rightarrow \boxed{V_o = \frac{V_1 + V_2 + V_3}{3}}$$

3.35 a. Open Circuit Voltage Voc:



Use Voltage divider $V_{oc} = \left(\frac{15\Omega}{10\Omega + 15\Omega}\right) \cdot 10V = \underline{6V}$

Now Find i_{sc}

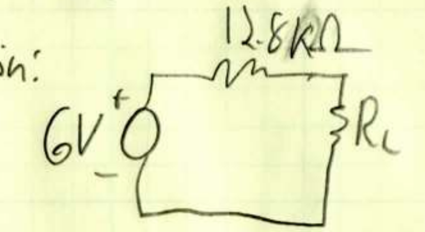


$i_x = 10V / \left(R_1 + \frac{R_2 R_3}{R_2 + R_3}\right) = .681\ mA$

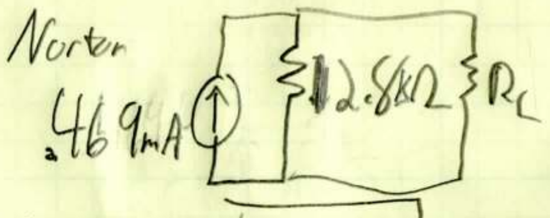
$i_{sc} = \frac{R_3}{R_2 + R_3} \cdot i_x = \underline{.469\ mA}$

$R_{TH} = \frac{V_{oc}}{i_{sc}} = \underline{12.8k\Omega}$

Thevenin:

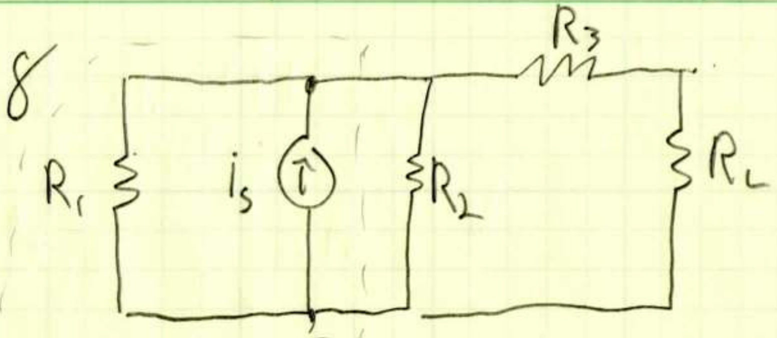


Norton



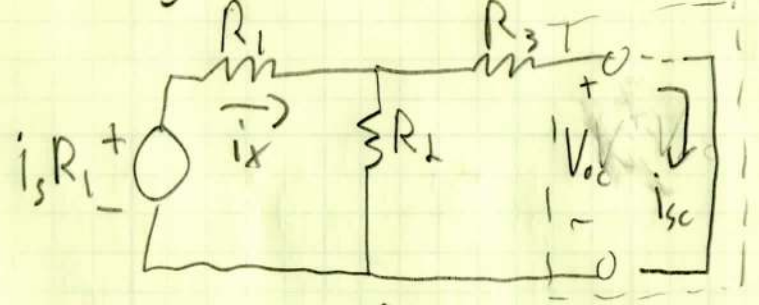
b. Find i_L; $i_L = \frac{V}{R} = \frac{6V}{12.8k\Omega + R_L} = \boxed{.1mA}$

3.38



$$\begin{bmatrix} R_1 \\ R_2 \\ R_3 \\ i_s \end{bmatrix} = \begin{bmatrix} 15k\Omega \\ 5.6k\Omega \\ 8.1k\Omega \\ 2.5mA \end{bmatrix}$$

Using Circuit Reduction from Chpt. 2:



Doesn't this look a lot like the last problem. Maybe you could use the same Matlab Script!

$V_{oc} = \left(\frac{R_2}{R_1 + R_2}\right) i_s R_1 = \underline{10.19\ V}$

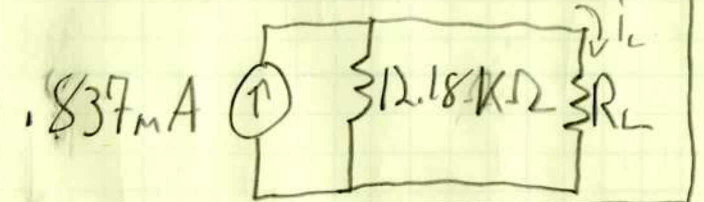
Now Short Circuit

$i_x = i_s R_1 / \left(R_1 + \frac{R_2 R_3}{R_2 + R_3}\right) = \underline{2.05\ mA}$

$i_{sc} = \left(\frac{R_2}{R_2 + R_3}\right) \cdot i_x = \underline{.837\ mA}$

$R_N = \frac{V_{oc}}{i_{sc}} = \underline{12.18\ k\Omega}$

Norton Eq:

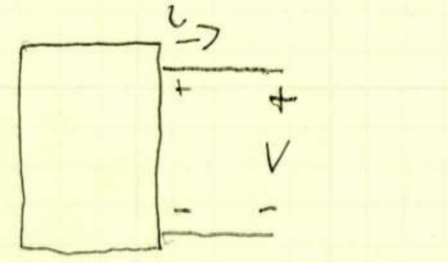


b. Current Division

$$i_L = \left[\frac{1/R_L}{\frac{R_L + R_N}{R_L \cdot R_N}}\right] i_{sc} = \left[\frac{R_N}{R_N + R_L}\right] i_{sc}$$

For $R_L = \begin{bmatrix} 4.7 \\ 15 \\ 68 \end{bmatrix} k\Omega$ $\boxed{i_L = \begin{bmatrix} .64 \\ .38 \\ .13 \end{bmatrix} mA}$

3.42

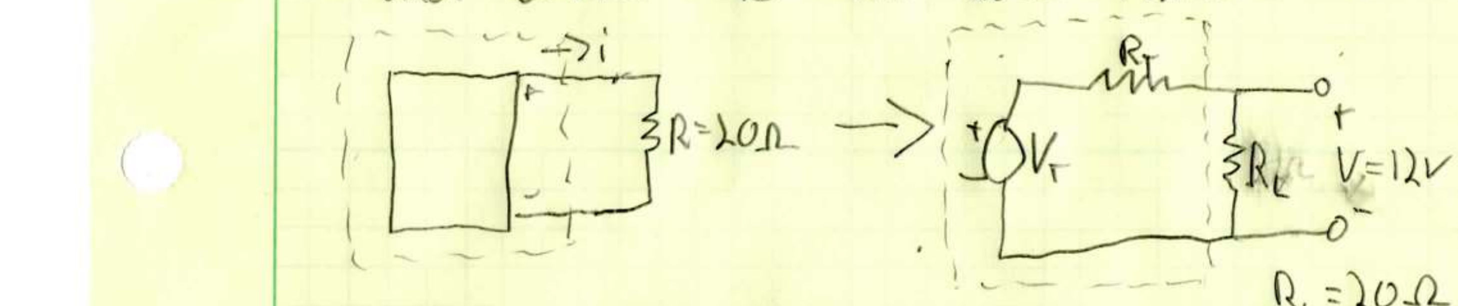


Solve by finding Thévenin equivalent! (Norton Could be used too)

First, $V=0V$ gives us i_{sc} => $\underline{i_{sc} = 1\,A}$

Now, we need V_T and R_T.

Replace Car port with Thévenin equivalent and connect to the 20Ω resistor!



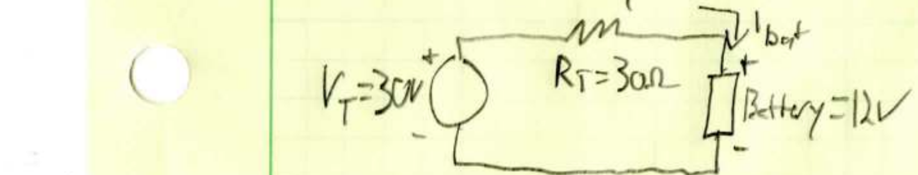
We know $\dfrac{V_T}{R_T} = i_{sc} = 1\,A$ (1)

And we can write $V_T - iR_T - \underbrace{iR_L}_{=V=12V} = 0$, where $i = \dfrac{V}{R_L} = \dfrac{12V}{20\Omega}$

$\Rightarrow \dfrac{V_T - 12V}{R_T} = \dfrac{12}{20}$ (2)

Solve (1) and (2) to get $\underline{R_T = 30\Omega,\ V_T = 30V}$

Connect Battery



$$\boxed{i_{bat} = \frac{V_T - V_{Bat}}{R_T} = \frac{30V - 12V}{30\Omega} = 600mA}$$