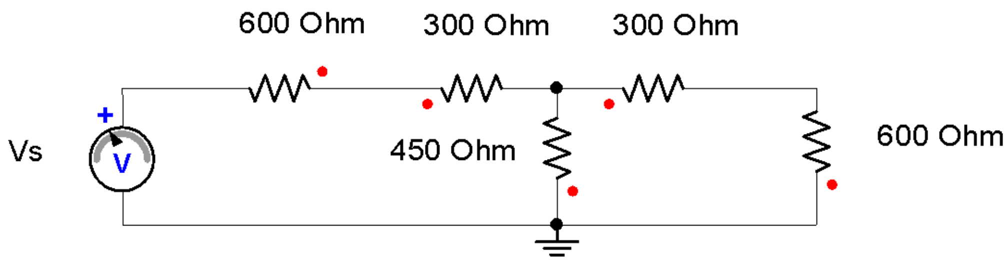
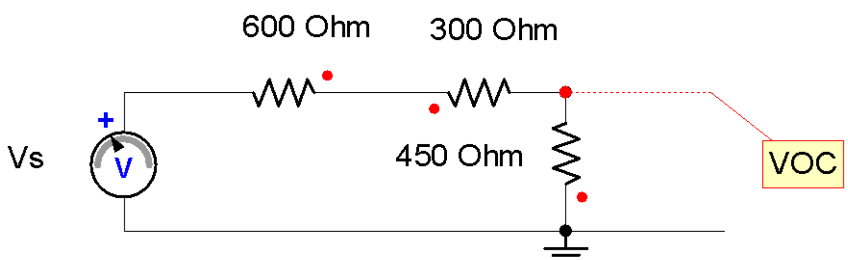
MAE140 HW3 Solutions

3.72)

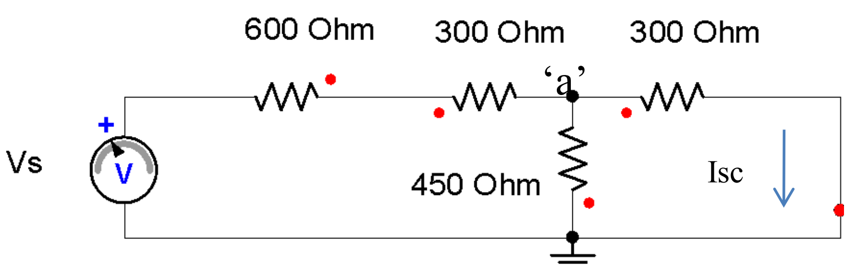


Method: Remove load resistor and find Thevenin equivalent circuit



$$VOC = Vs\frac{450}{600+300+450} = \frac{Vs}{3}$$

To find Isc, use a voltage divider at 'a'



$$Va = Vs\frac{300\,||\,450}{300\,||\,450+300+600} = Vs\frac{180}{180+900} = \frac{Vs}{6}$$

$$Isc = \frac{Va}{300} = \frac{Vs}{6*300} = \frac{Vs}{1800}\,Amps$$

$$Rt = Voc\,/\,Isc = \frac{Vs/3}{Vs/1800} = 600\Omega$$

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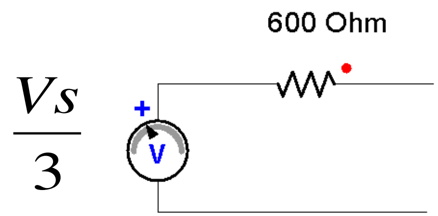
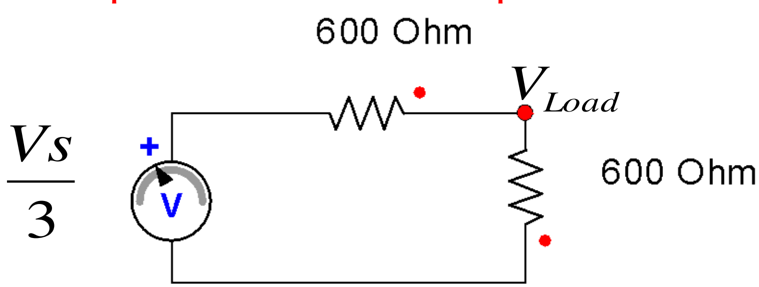
3.72)Continued



Thevenin Equivalent

Now put RL back to find power delivered



$$V_{Load} = \frac{1}{6}Vs$$ Another voltage divider

$$Power = \frac{V_{Load}^2}{R_{Load}} = \frac{Vs^2}{36*600} = \frac{Vs^2}{21600}Watts$$

Without Attenuator: $V_{Load} = \frac{1}{2}Vs$

$$Power = \frac{V_{Load}^2}{R_{Load}} = \frac{Vs^2}{4*600} = \frac{Vs^2}{2400}Watts$$

21600/2400 = 9: Power decreases by 1/9

In dB $$10Log_{10}\left(\frac{Vs^2/21600}{Vs^2/2400}\right) = 10Log_{10}\left(\frac{2400}{21600}\right) = -9.54dB$$

For dB calculation, use 10Log(Pa/Pb), not 20Log(Va/Vb) (Because power is proportional to V^2, we use 20 Log(A/B) for Voltage or Amplitude comparisons, and 10Log (A/B) for Power comparisons

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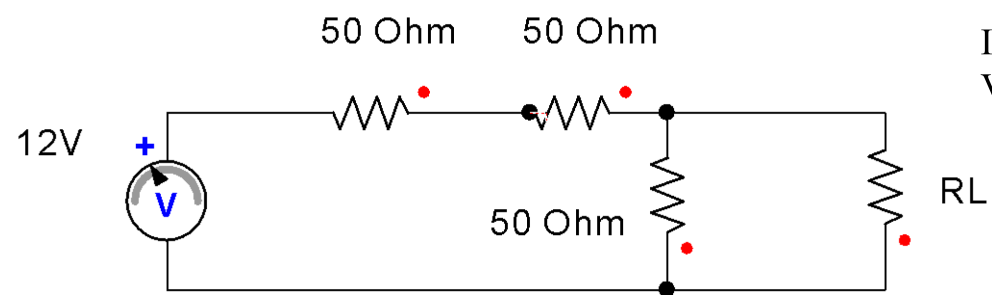
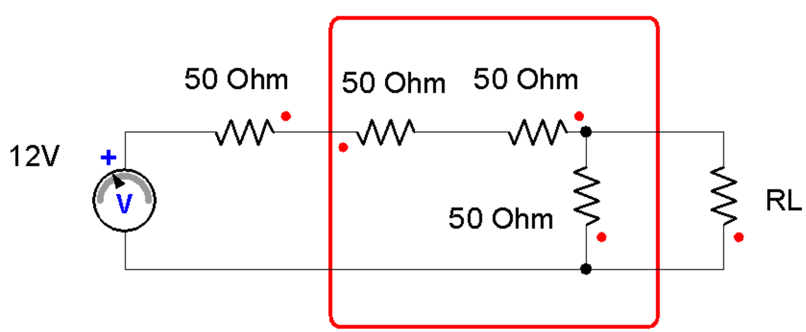
3.73)



Initial guess:50 Ohm
Voltage Divider

Check boundaries, Let RL = infinity.
Then I = 0 which satisfies I<= 100 mA
VL = 12*(50/150) = 4V which also satisfies VL<= 4V.
Note any decrease in RL would only lower VL so the voltage requirement will remain satisfied.
Let RL = 0
I = 12/100 = 120 mA>100mA. We need to reduce this current
So add another series 50 Ohm to do this. This will also reduce our voltage across the load, maintaining VL<=4V



Check: Vmax (when RL = infinity) = 12*(50/200) = 3V
Imax (When RL = 0) = 12/150 = 80 mA

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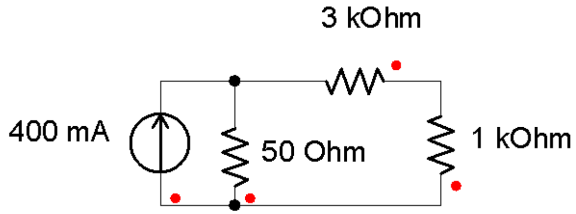
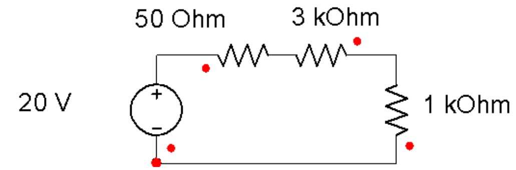
3.76)

Team A





It is sometimes convenient to convert to the Thevenin equivalent

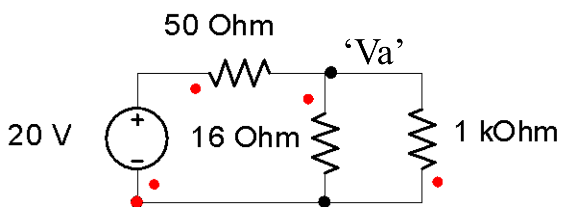
$$I = \frac{20V}{4050} = 12.345mA$$

$$P_{Load} = I^2 R_{Load} = 24.38mW$$

$$P_{Loss} = I^2 R_{3k\Omega} = 73mW$$

$$Error = \frac{25 - 24.38}{25} * 100 = 2.48\%$$

Team B



Find Va: $Va = 20 * \frac{16 \| 1000}{50 + 16 \| 1000} = 20 * \frac{15.74}{50 + 15.74} = 4.788V$

$$P_{Load} = \frac{Va^2}{R_{Load}} = 22.9mW$$

$$P_{Loss} = \frac{Va^2}{R_{16\Omega}} = 1.432W$$

$$Error = \frac{25 - 22.9}{25} * 100 = 8.4\%$$

Team A clearly wins, they have
1) A closer tolerance
2) They use standard resistor values (16 Ohm isn't standard)
3) They waste less power (or require less power from the source)

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3.2) (Refer to circuit drawing from HW, pg 132)

Note this circuit is linear with just resistors and independent current sources, allowing us to directly write the equations by inspection (see T,R &T pg 75)

A) Node A: (1/20 + 1/10 + 1/8)Va – 1/8 Vb = 3A

Node B: -1/8 Va + (1/4+1/8) Vb = 1A-3A

Simplifying and putting into matrix form we get

$$\begin{bmatrix} \frac{11}{40} & \frac{-1}{8} \\ \frac{-1}{8} & \frac{3}{8} \end{bmatrix} \begin{bmatrix} Va \\ Vb \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \end{bmatrix} \Rightarrow \begin{bmatrix} Va \\ Vb \end{bmatrix} = \begin{bmatrix} \frac{11}{40} & \frac{-1}{8} \\ \frac{-1}{8} & \frac{3}{8} \end{bmatrix}^{-1} \begin{bmatrix} 3 \\ -2 \end{bmatrix}$$

Computing the determinant we get $\Delta = \left[\frac{11}{40} * \frac{3}{8}\right] - \left[\frac{-1}{8} * \frac{-1}{8}\right] = .0875$

$$\begin{bmatrix} Va \\ Vb \end{bmatrix} = \frac{1}{\Delta} \begin{bmatrix} \frac{3}{8} & \frac{1}{8} \\ \frac{1}{8} & \frac{11}{40} \end{bmatrix} \begin{bmatrix} 3 \\ -2 \end{bmatrix} = \begin{bmatrix} 10 \\ -2 \end{bmatrix}$$

B)

Vx = Vb = -2V, Ix = Va/20 = 10/20 = 0.5 Amps

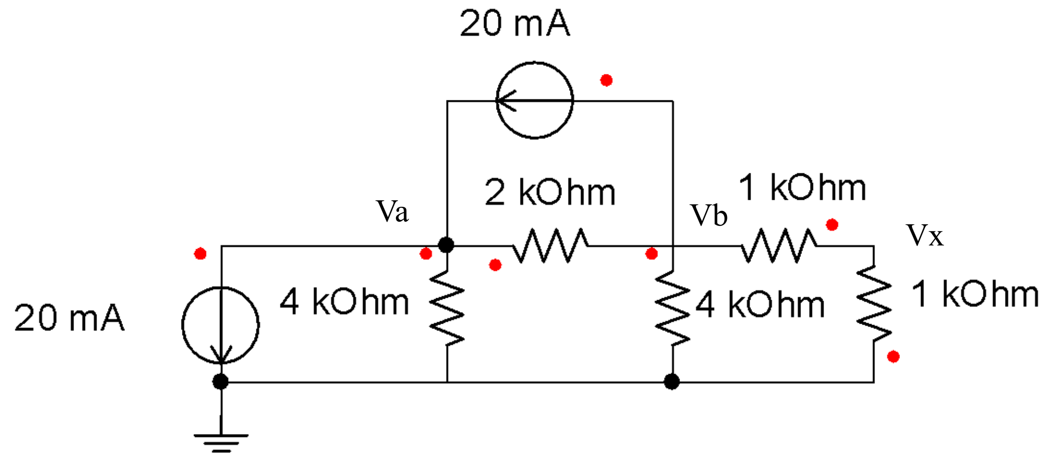
MAE140 HW3 Solutions

3.3)

A) $\frac{Va}{4k}+\frac{Va-Vb}{2k}=20mA-20mA=0$

$\frac{Va-Vb}{2k}-\frac{Vb}{4k}+\frac{Vx-Vb}{1k}=20mA$

$\frac{Vb-Vx}{1k}-\frac{Vx}{1k}=0$



B) To find Va and Vb, we can combine the two 1k resistors in series to remove a node equation, then once armed with Vb, use a voltage divider equation to get Vx.

Node A $\frac{Va}{4k}+\frac{Va-Vb}{2k}=0 \longrightarrow \frac{3}{2}Va=Vb$

Node B $\frac{Vb}{4k}+\frac{Vb}{2k}+\frac{Vb-Va}{2k}=-20mA$

$\frac{5}{4k}Vb-\frac{1}{2k}Va=-20mA$

$\frac{5}{4k}Vb-\frac{1}{2k}\frac{2}{3}Vb=-20mA \longrightarrow Vb=-21.8V$

$Va=\frac{2}{3}Vb=-14.55V$

C)

$Vx=Vb*\frac{1k}{1k+1k}=-21.8*\frac{1k}{2k}=-10.9V$

$Ix=\frac{Va-Vb}{2k}=\frac{-14.55+21.8}{2000}=3.6mA$

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3.4) See Figure 3-4 in HW for reference on page 132

A) By inspection, (Note Vb is known)

Node A $Va/2k + (Va-Vb)/4k = 1mA$

Node C $Vc/4k + (Vc-Vb)/4k = -1mA$

B) Use given values to solve for Va and Vc

$Va(1/2k + 1/4k) = 1mA + 5V/4k \rightarrow Va = 3V$

$Vc(2/4K) = -1mA + 5V/4k \rightarrow Vc = 0.5V$

C) Solve for Vx and Ix (Vb = 5V)

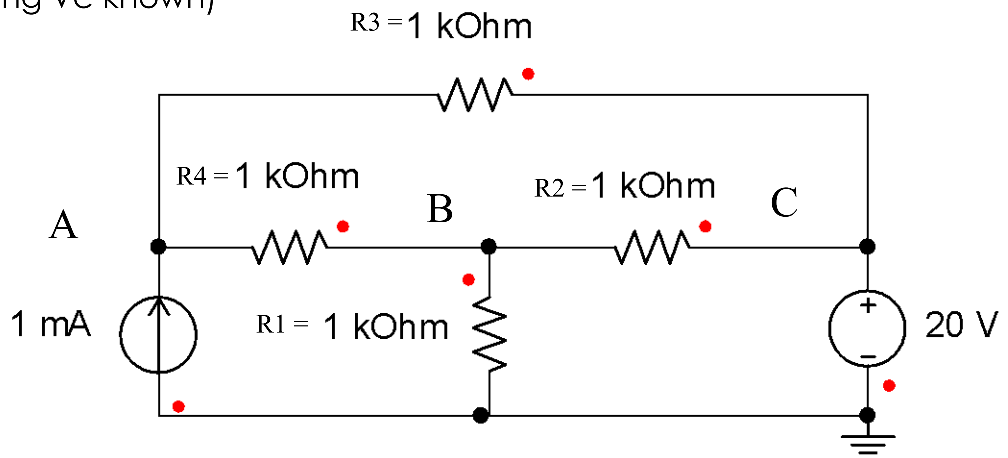
$Vx = Va-Vc = 2.5V$

$Ix = (Va-5V)/4k + (Vc-5V)/4k = -1.625mA$

MAE140 HW3 Solutions

3.5) Choose ground so one side of the voltage source is connected to it

(in this case, let ground be connected to –Vs, yielding Vc known)



A)

Node A (Va-Vc)/R3 + (Va-Vb)/R2 = Is

Node B (Va-Vb)/R4 + (Vc-Vb)/R2 = Vb/R1

Since Vc is known, move it to the RHS and put node A and node B equations into matrix form

Va(1/R3 + 1/R4) – Vb/R4 = Is + Vc/R3
Va/R4 – Vb(1/R4 + 1/R2 + 1/R1) = -Vc/R2

B)

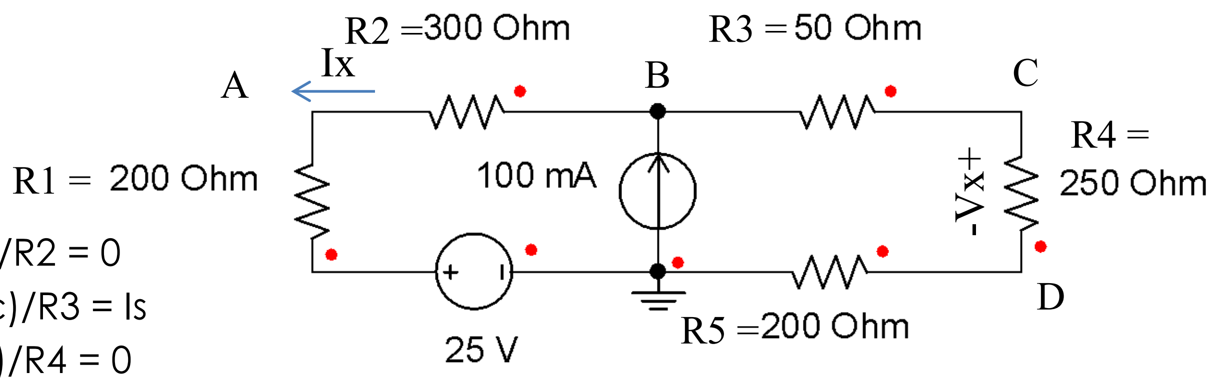
$$\begin{bmatrix} \frac{2}{1000} & \frac{-1}{1000} \\ \frac{1}{1000} & \frac{-3}{1000} \end{bmatrix} \begin{bmatrix} Va \\ Vb \end{bmatrix} = \begin{bmatrix} 0.021 \\ -0.02 \end{bmatrix} \longrightarrow \begin{bmatrix} Va \\ Vb \end{bmatrix} = \begin{bmatrix} 16.6V \\ 12.2V \end{bmatrix}$$

See solution 3-2 for help inverting a 2x2 matrix

Vx = Vb-Vc = 12.2V – 20V = -7.8V
Ix = (Vc-Va)/R3 = (20-16.6)/1k = 3.4mA

MAE140 HW3 Solutions

3.12)



We have 4 unknown nodes

Node A: (Va-Vs)/R1 - (Vb-Va)/R2 = 0

Node B: (Vb-Va)/R2 + (Vb-Vc)/R3 = Is

Node C: (Vb-Vc)/R3 - (Vc-Vd)/R4 = 0

Node D: (Vc-Vd)/R4 - Vd/R5 = 0

To solve this circuit for Vx and Ix, we don't need to put this into a 4x4 matrix form. Instead, all we really need to find is Vb, and the whole thing will fall apart.

To do this, combine R1 and R2, and combine R3, R4 and R5. Then we get (at node B)

(Vb-Vs)/(R1+R2) + (Vb-0)/(R3+R4+R5) = Is We can now solve for Vb directly

(Vb-25)/500 + Vb/500 = 100mA → Vb = 37.5V

Now we can compute Ix (It is flowing through R2 and R1) as

(Vb-Vs)/(R1+R2) = Ix → (37.5-25)/500 = 25 mA = Ix

To solve for Vx, note the current from B to C is Is-Ix = 75 mA. Now just compute the I*R voltage drop for each resistor starting from Vb.

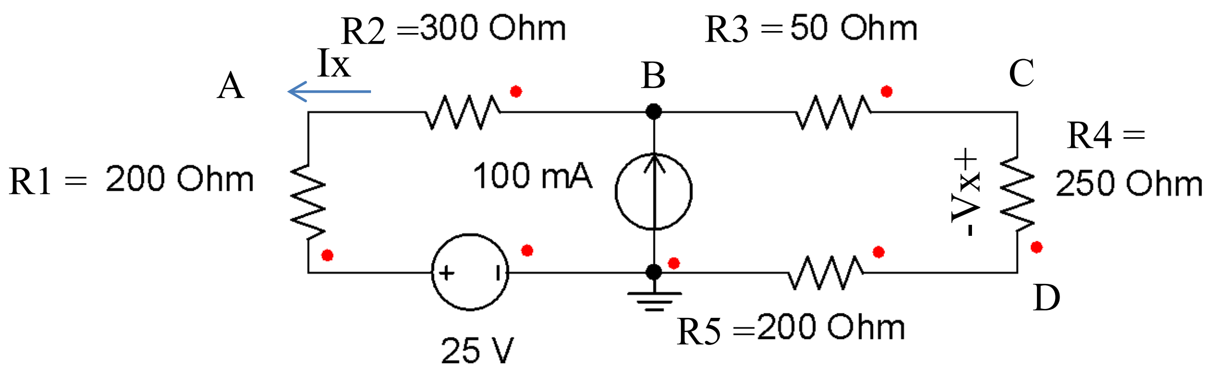
Vc = Vb – (Is-Ix)R3 = 37.5 - 75mA*50 = 33.75V

Vd = Vc – (Is-Ix)R4 = 33.75-(75mA)*250 = 15V →Vx = Vc-Vd = 18.75V

*You can also see that Vx must be Vb/2, since half the voltage is dropped across R3+R5 and the other half is dropped across R4

MAE140 HW3 Solutions

3.12)continued



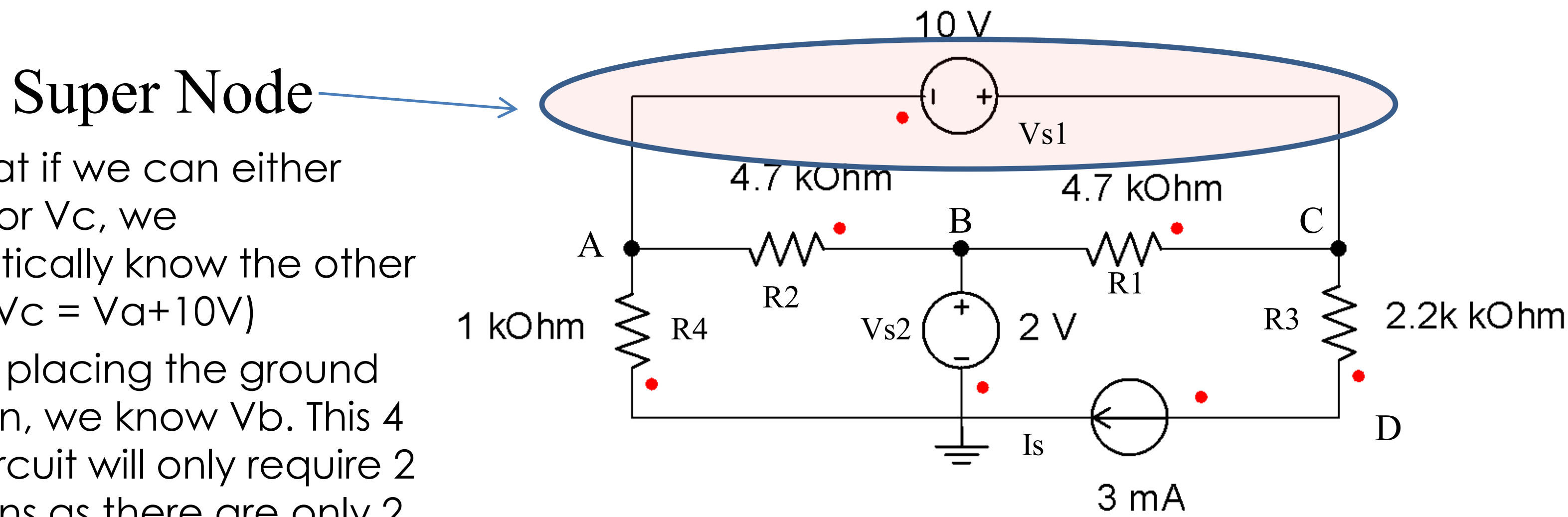
To Find the total power dissipated in the circuit, sum the I^2R drops across each resistor plus the power dissipated by the voltage source (Current entering positive terminal means the voltage supply is dissipating power)

Power = $Ix^2(R1+R2) + (Is-Ix)^2(R3+R4+R5) + Ix*Vs$

$= .025^2*500 + .075^2*500 + .025*25 =$ 3.75 Watts

MAE140 HW3 Solutions

3.13)



Note that if we can either find Va or Vc, we automatically know the other one (ie Vc = Va+10V)

Also, by placing the ground as shown, we know Vb. This 4 node circuit will only require 2 equations as there are only 2 unknowns

The super node: (the sum of all currents entering the super node must be zero),

Va/R4 + (Va-Vb)/R2 + (Vc-Vb)/R1 + (Vc-Vd)/R3 = 0

Node D:

(Vc-Vd)/R3 = is = 3mA

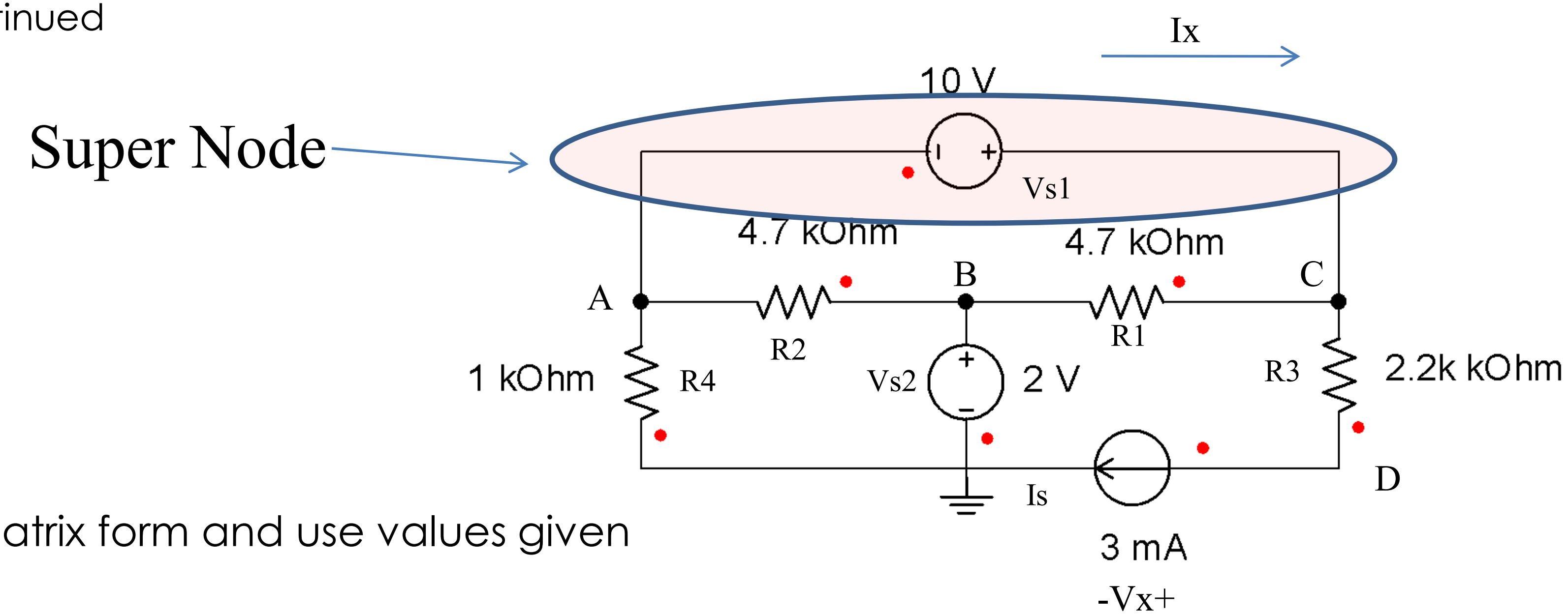
Using Vb = Vs2 = 2V and Vc = Va+10

Va(1/R4 + 1/R2 + 1/R1 + 1/R3) - Vd(1/R3) = Vs2(1/R1 +1/R2) - Vs1(1/R1 + 1/R3)

Va/R3 - Vd/R3 = Is – Vs1/R3

MAE140 HW3 Solutions

3.13) continued



Put into matrix form and use values given

$$\begin{bmatrix} .00188 & -.0004545 \\ .0004545 & -.0004545 \end{bmatrix}\begin{bmatrix} Va \\ Vd \end{bmatrix} = \begin{bmatrix} -.005822 \\ -.001545 \end{bmatrix}$$

$$\begin{bmatrix} Va \\ Vd \end{bmatrix} = \begin{bmatrix} -3 \\ .4 \end{bmatrix}$$

Vd = Vx = 400mV

Ix = -Va/R4 +(Vb-Va)/R2 = 4.0638mA

Power supplied by Vs1 = I*V = 10V*4mA = 40.638mW

MAE140 HW3 Solutions

3.14)

Refer to circuit drawing in T,R&T pg 133

Node Equations

1) Va/R1 + (Va-Vd)/R6 = is1
2) -Vb/R2 + (Vc-Vb)/R5 = is1
3) -Vc/R3 + (Vb-Vc)/R6 = is2
4) Vd/R4 + (Vd-Va)/R6 = = is2

To find Vx we only need Vd (place reference at –Vx)

So use equations 1 and 4 (2 equations, 2 unknowns, equations 2 and 3 don't have the node variables we are interested in, skip them)

Va(1/R1 + 1/R6) + Vd(-1/R6) = Is1

Va(-1/R6) + Vd(1/R4 + 1/R6) = Is2

You should be able to put this into matrix form and solve it by now

Va = 64V

Vd = 56V = Vx