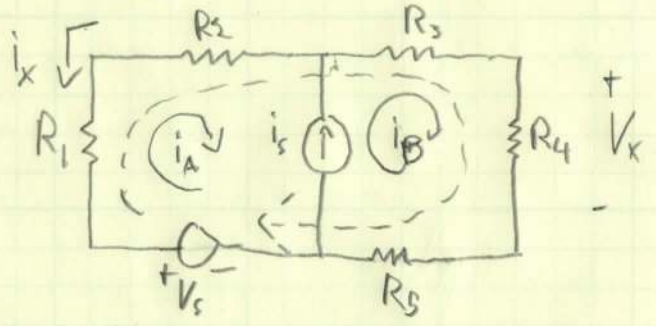
MAE 140 | P.S. 4 Solutions

3. 12, 13, 14, 20, 23, 27, 32
4. 1, 5, 11

3.12.a. <u>Step 1.</u> Identify Mesh



<u>Step 2 (and 3)</u> Go around each loop

For the Super mesh (dashed line)

$$\underbrace{i_A R_1}_{\substack{\text{Voltage} \\ \text{across } R_1}} + \underbrace{i_A R_2}_{\substack{\text{Voltage} \\ \text{across } R_2}} + i_B R_3 + i_B R_4 + i_B R_5 = V_s \quad (1)$$

... etc.

Our Super node gives: $i_B - i_A = i_s$ (2)

So put (1)* and (2) in $A\vec{x} = \vec{b}$

$$\begin{bmatrix} R_1+R_2 & R_3+R_4+R_5 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} i_A \\ i_B \end{bmatrix} = \begin{bmatrix} V_s \\ i_s \end{bmatrix} \quad (3)$$

*Note (1) can be written as $(R_1+R_2)i_A + (R_3+R_4+R_5)i_B = V_s$

Solving (3) for: $\vec{x} = A^{-1} \cdot \vec{b}$ gives:

$$\begin{bmatrix} i_A \\ i_B \end{bmatrix} = \begin{bmatrix} -25 \\ 75 \end{bmatrix} mA$$

3.12 Cont.

b. $\boxed{i_x = -i_A = 25\,mA}$

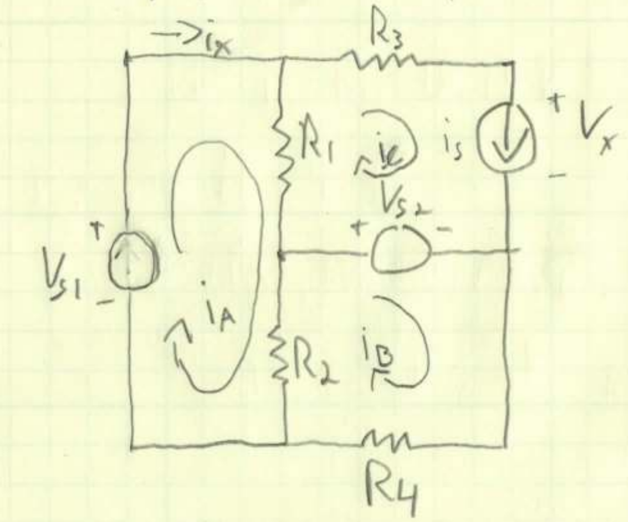
$\boxed{V_x = i_B \cdot R_4 = 18.75\,V}$

c. $P = iV = i^2 R$

$$= \underbrace{i_A^2(R_1+R_2+R_3) + i_B^2(R_4+R_5)}_{\text{Power dissipated in Resistors}} + \underbrace{i_x V_s}_{\substack{\text{power dissipated} \\ \text{in Voltage Source}}} = \underline{\underline{3.75\,W}}$$

3.13 a.

Step 1. Indentify Mesh



Step(2+3) Analyze each loop using KCL + KVL

First we not $i_C = i_s$, We only need to examine loops A and B since loop C is solved!

Loop A: $R_1(i_A - i_C) + R_2(i_A - i_B) = V_{S1}$ Collect terms + Sub $i_C = i_s$

$(R_1+R_2)i_A \quad - R_2\, i_B \quad = V_{S1} + R_1 i_s$

Loop B: $R_2(i_B - i_A) + R_4\, i_B \quad = -V_{S2}$ Collect terms

$-R_2 i_A \quad + (R_2+R_4) i_B \quad = -V_{S2}$

Since $i_C = i_s$ is solved We have a 2×2 System

$$\begin{bmatrix} R_1+R_2 & -R_2 \\ -R_2 & R_2+R_4 \end{bmatrix}\begin{bmatrix} i_A \\ i_B \end{bmatrix} = \begin{bmatrix} V_{S1} + R_1 i_s \\ -V_{S2} \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} i_A \\ i_B \end{bmatrix} = \begin{bmatrix} 4.06 \\ 3 \end{bmatrix} mA$$

3.13 b. Skip

c. To find V_x we need the loop for C

Loop C: $V_{S2} = R_1(i_C - i_A) + R_3\, i_C + V_x$

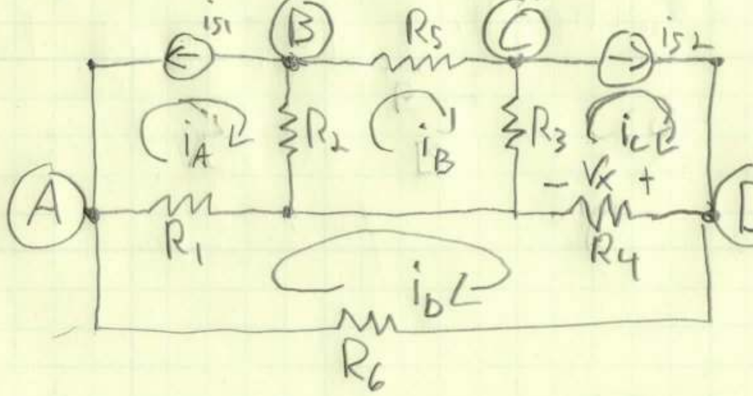
$$\Rightarrow V_x = V_{S2} - R_1(i_C - i_A) - R_3 i_C = \underline{.4 \text{ Volts}}$$

$$i_x = i_A = \underline{4.06\, mA}$$

d. $P = iV \Rightarrow P_{VS1} = i_x V_{S1} = i_A V_{S1} = \underline{40.64\, mW}$

3.14

a. We want to move R_6 so it's not in a loop with either current sources.
Lets slide R_1 and R_4 to the bottom and let nodes A and D follow. Then just move the R_6 branch to the bottom:



Nothing has change but the picture!

b. Loop A: $i_A = -i_{S1}$ (Thats all you need to know)

Loop B: $R_2(i_B - i_A) + R_5 i_B + R_3(i_B - i_C) = 0$ just rearranching

$-R_2 i_A + (R_2 + R_5 + R_3) i_B - R_3 i_C = 0$

Loop C: $i_C = i_{S2}$

Loop D: $R_1(i_D - i_A) + R_4(i_D - i_C) + R_6 i_D$

$-R_1 i_A - R_4 i_C + (R_1 + R_4 + R_6) i_D = 0$

Now put in matrix forms subbing in $i_A = -i_{S1}$, $i_C = i_{S2}$

c.
$$\begin{bmatrix} R_2 + R_5 + R_3 & 0 \\ 0 & R_6 + R_1 + R_4 \end{bmatrix} \begin{bmatrix} i_B \\ i_D \end{bmatrix} = \begin{bmatrix} R_3 i_{S2} - R_2 i_{S1} \\ -R_1 i_{S1} + R_4 i_{S2} \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} i_B \\ i_D \end{bmatrix} = \begin{bmatrix} -8 \\ -8 \end{bmatrix} \text{mA} \qquad V_x = R_4(i_C - i_D) = \underline{56V}$$

3.20 a. By inspection:

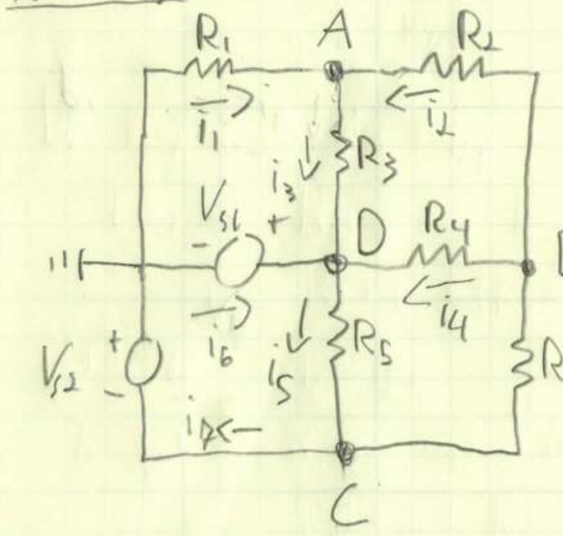
$$10^3 \cdot \underbrace{\begin{bmatrix} 4+10 & -10 & 0 & 0 \\ -10 & 10+1+2 & -2 & 0 \\ 0 & -2 & 2+6+8 & -6 \\ 0 & 0 & -6 & 6 \end{bmatrix}}_{A} \cdot \underbrace{\begin{bmatrix} i_A \\ i_B \\ i_C \\ i_D \end{bmatrix}}_{\vec{x}} = \underbrace{\begin{bmatrix} -10 \\ 0 \\ 0 \\ 5+10 \end{bmatrix}}_{\vec{b}}$$

This is all you need for credit.

But can use Matlab to get:

$$\vec{x} = A^{-1} \cdot \vec{b} \longrightarrow \vec{x} = \begin{bmatrix} i_A \\ i_B \\ i_C \\ i_D \end{bmatrix} = \begin{bmatrix} -1.256 \\ -.759 \\ 1.348 \\ 3.85 \end{bmatrix} \text{mA}$$

Node Voltage



Note we moved the ground to simplify. V_D and V_C are known

KCL gives

A $i_3 = i_1 + i_2$

B $0 = i_2 + i_4 + i_6$

Sub in KVL

(A) $(V_A - V_D) G_3 = -V_A G_1 + G_2(V_B - V_A)$

$(G_2 + G_3 + G_1) V_A - G_2 V_B = G_3 V_D$

(B) $(V_B - V_A) G_2 + (V_B - V_D) G_4 + (V_B - V_C) G_6 = 0$

$(-G_2) V_A + (G_2 + G_4 + G_6) V_B = G_6 V_C + G_4 V_D$

3.20 cont

Sub in $V_D = V_{S1}$, $V_C = -V_{S2}$

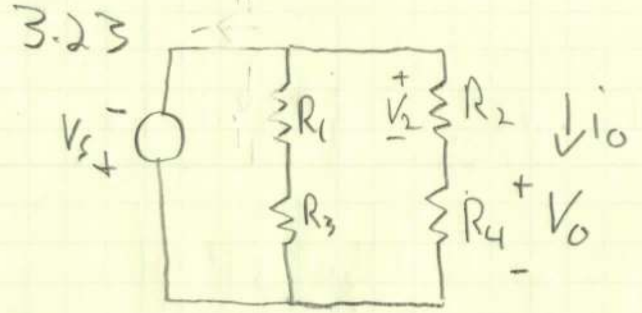
$$\begin{bmatrix} G_1+G_2+G_3 & -G_2 \\ -G_2 & G_2+G_4+G_6 \end{bmatrix} \begin{bmatrix} V_A \\ V_B \end{bmatrix} = \begin{bmatrix} G_3 V_{S1} \\ G_4 V_{S1} - G_6 V_{S2} \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} V_A \\ V_B \end{bmatrix} = \begin{bmatrix} 5.026 \\ 5.79 \end{bmatrix}$$

$$\boxed{\begin{aligned} i_A &= i_1 = V_A/R_4 = 1.256\,mA \\ i_B &= i_2 = (V_B - V_A)/R_2 = .789\,mA \\ i_C &= i_6 = (V_B - V_C)/R_6 = 1.348\,mA \\ i_D &= i_7 = i_5 + i_6 = (V_D - V_C)/R_5 + (V_B - V_C)/R_6 = 3.85\,mA \end{aligned}}$$

This way is harder...
But, it is easier to solve a 2x2 by hand.

3.23



Use Ohm's law to find i_o:

$$i_o = \frac{V_o}{R_4} = \frac{-V_s}{R_2+R_4} \Rightarrow \frac{V_o}{V_s} = -\frac{R_4}{R_2+R_4}$$ a simple voltage divider

$$K = \frac{V_o}{V_s} = -R_4/(R_2+R_4)$$

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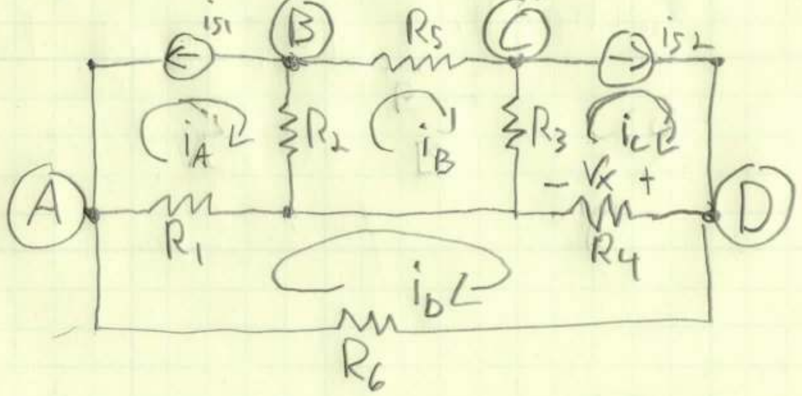

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Loop C: $i_C = i_{s2}$

Loop D: $R_1(i_D - i_A) + R_4(i_D - i_C) + R_6 i_D$

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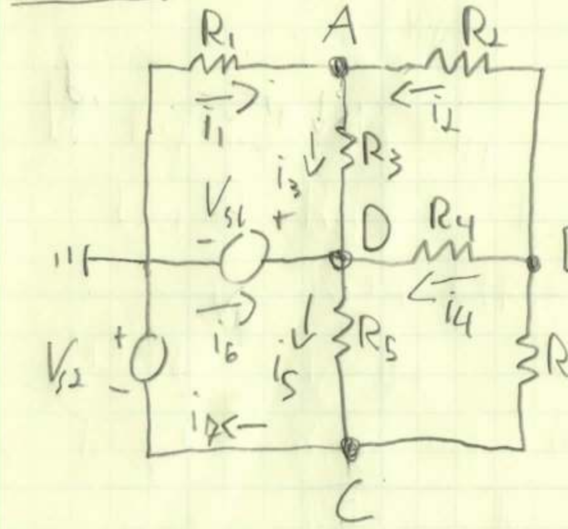

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3.20 cont

Sub in $V_D = V_{S1}$, $V_C = -V_{S2}$

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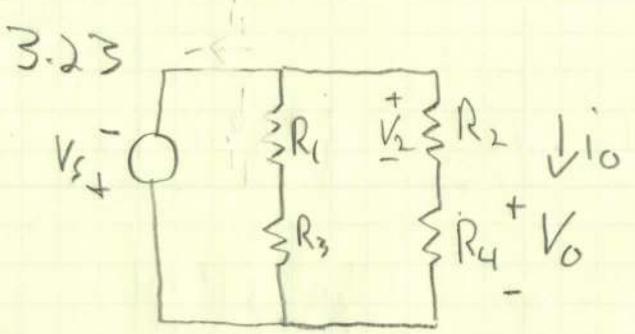

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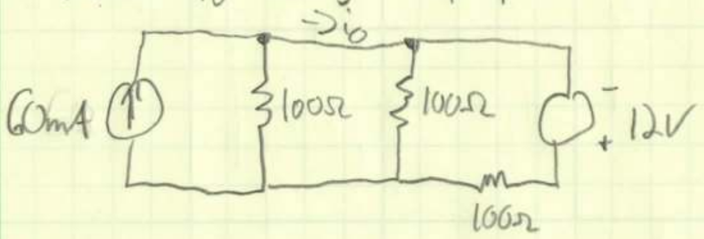
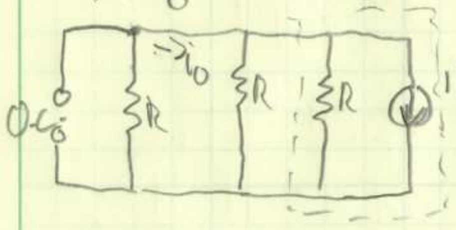
3.27

Find i_0 Using Superposition



1. First let the 60mA sourse be an open circuit

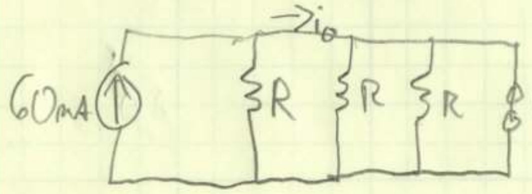
A quick Source transformation Shows its a current Divider



$$i_{01} = \frac{(1/R)}{(3/R)} \cdot i_s = \frac{R}{3R} \cdot \frac{12V}{R} = \frac{4V}{R}$$

2. Now let the 12V source be a Short Circuit.

Oh wow! Another Current Divider!

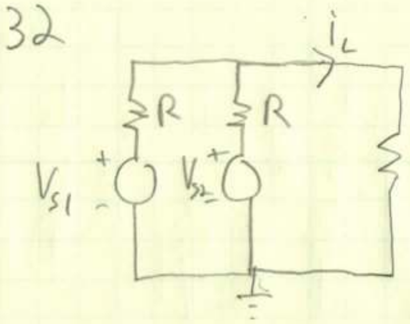


$$i_{02} = \frac{(2/R)}{(3/R)} \cdot i_s = \frac{2}{3} \cdot 60mA$$

$$i_0 = i_{01} + i_{02} = \frac{4V}{100\Omega} + \frac{2}{3} \cdot 60mA = \underline{\underline{80mA}}$$

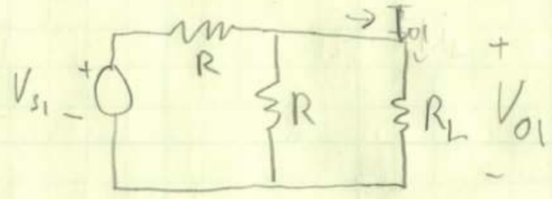
Law of Superposition

3.32



The Circuit drawn here is arbitrary. It is not used to Solve for the power but it helps to give intuition about Superposition. Current Sources could have also been used.

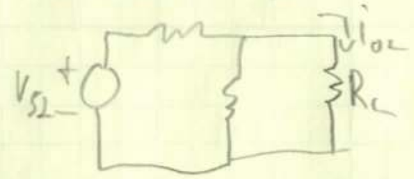
Source 1: Let Source 2 be a Short circuit



Source 1 delivers 250mW

$$i_{01} = \sqrt{\frac{P_1}{R_L}} = \sqrt{\frac{250mW}{100\Omega}} = 50mA$$

Source 2: Let Source 1 be a Short circuit



$$i_{02} = \sqrt{\frac{P_2}{R_L}} = \sqrt{\frac{4W}{100\Omega}} = 200mA$$

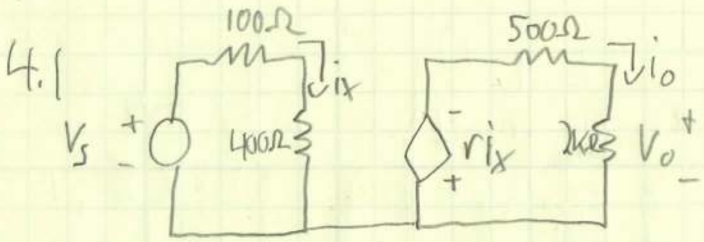
Sum Using Superposition Principle.

$$i_L = i_{01} + i_{02} = 250mA$$

$$P = i_L^2 R_L = (250mA)^2 \cdot 100\Omega = \underline{\underline{6.25W}}$$

The answer is not 4.25W becuse you cannot use Superposition with power, <u>Power is not linear.</u>

4.1



Find Votage and Current gains

$i_x = V_s/(100\Omega + 400\Omega)$; $i_o = V_o/2k\Omega$

$-ri_x = i_o \cdot 500\Omega + V_o$ (KVL)

Sub in i_o and i_x into above eq.

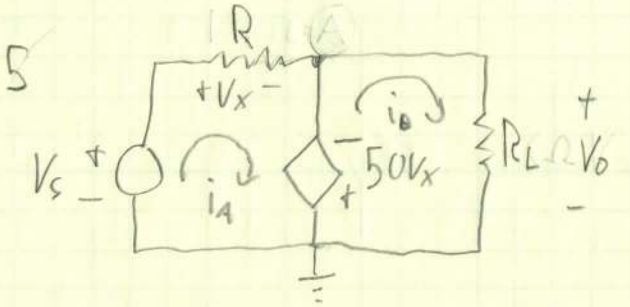
$$-\frac{V_s \cdot r}{(100\Omega + 400\Omega)} = \frac{V_o \cdot 500\Omega}{2k\Omega} + V_o = V_o \cdot \left(\frac{500\Omega + 2k\Omega}{2k\Omega}\right)$$

$$\Rightarrow \frac{V_o}{V_s} = -\left(\frac{r}{100\Omega + 400\Omega}\right) \cdot \left(\frac{2k\Omega}{500\Omega + 2k\Omega}\right) = -16$$ when $r = 10k\Omega$

Now divid i_o/i_x to get:

$$\frac{i_o}{i_x} = -\frac{V_o}{2k\Omega} \cdot \left(\frac{100 + 400\Omega}{V_s}\right) = -\frac{V_o}{V_s} \cdot \frac{1}{4} = -4$$

4.5



-500

$R = 10K\Omega$

$R_L = 1K\Omega$

Find Voltage Gain.

Use Mesh-Current

A: $V_s - V_x + 50V_x = 0$

note that $V_x = i_A R$

$V_o = i_B R_L$

B: $50V_x + V_o = 0$

$\Rightarrow V_x = -\frac{V_o}{50}$

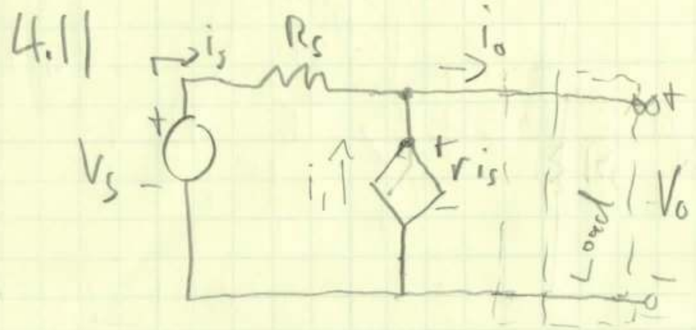
Substitute V_x into A:

$V_s + \frac{V_o}{50} - V_o = 0$

$\Rightarrow V_s = V_o(1 - 1/50) = V_o\left(\frac{49}{50}\right)$

$V_o/V_s = \frac{50}{49}$

4.11



We want something of the form

$V_o = \_\_\_ - i_o \_\_\_$

$r i_s = V_o = V_s - i_s R_s \; ; \quad V_s - i_s R = r i_s$

$\Rightarrow i_s = \frac{V_s}{R+r}$

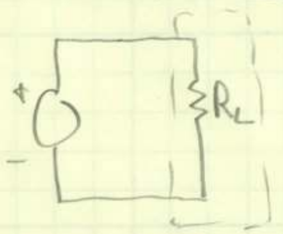
$V_o = r i_s = \frac{r V_s}{R_s + r}$

$\Rightarrow V_o = \frac{r V_s}{R_s + r} - i_o \cdot 0$

In thévenin $V_o = V_T - iR_T$

$\Rightarrow V_T = \frac{r V_s}{R_s + r}$ and $R_T = 0$

$V_T = \frac{r V_s}{R + r}$



4.11 cont.

We can also approach this problem using the method in chapter 3. (finding V_{oc} and I_{sc})

To find V_{oc} we equate the top to bottom voltages using KVL:

$$V_{oc} = V_s - i_s R_s = r i_s$$

This expression gives $i_s = \frac{V_s}{r + R_s}$

Therefor $V_{oc} = V_{Th} = \frac{r V_s}{R + r}$

Looking at i_{sc} we begin to see a problem immediately. A short circuit implies that the top to bottom voltage $= 0$. Since there is an ideal voltage source between the top and bottom of the circuit it will put out as much power as nessisary to maintain that voltage. $\Rightarrow R_{Th} = 0$.