9: 1, 5, 7, 9, 10, 15, 17, 19, 25, 27.

9.1 $\mathcal{L}[10(e^{-50t} - 2e^{-100t})u(t)]$

$$= 10\cdot\left[\frac{1}{s+50} - \frac{2}{s+100}\right]$$

$$= 10\cdot\left[\frac{s+100-2s-100}{(s+50)(s+100)}\right]$$

$$\boxed{= -10\left[\frac{s}{(s+50)(s+100)}\right]}$$

Zeros: $s=0$

poles: $s = -50, -100.$

9.5 $\mathcal{L}[2\delta(t) - 100e^{-10t}\cdot\cos(10t)\cdot u(t)]$

$$= 2 - 100\cdot\left(\frac{s+10}{(s+10)^2+100}\right)$$

$$= \boxed{\frac{2s^2-60s-600}{s^2+20s+200}}$$

=> Zeros: $s = 37.913, -7.913$

poles: $s = -10 \pm 10j$

9.5 Cont.

If you did the problem as printed in the book it looks like this:

For $t>0$ $u(10t) = u(t)$

$$\Rightarrow \mathcal{L}\left[2\delta(t) - 100e^{-10t}\cdot\cos(t)\, u(t)\right]$$

$$= 2 - 100\cdot\frac{s+10}{(s+10)^2+1} = 2 - \frac{100s+1000}{s^2+20s+101}$$

$$\boxed{= \frac{2(s^2-30s-399)}{s^2+20s+101}}$$

$$\boxed{\begin{array}{l}\text{Zeros: } s = 32.34,\ -12.34 \\ \text{poles: } s = -10 \pm j\end{array}}$$

9.7 a. $\mathcal{L}[(15e^{-5t} - 20e^{-10t})u(t)]$

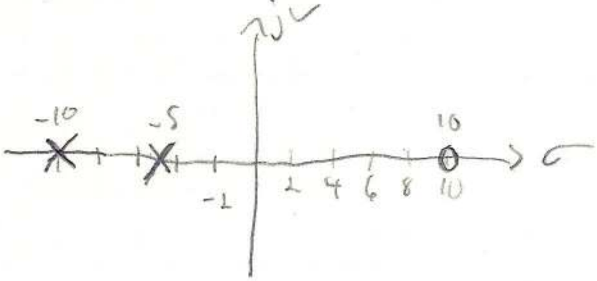
$$= \frac{15}{s+5} - \frac{20}{s+10}$$

$$= \frac{15s + 150s - 20s - 100}{(s+5)(s+10)}$$

$$= \frac{-5s + 50}{(s+5)(s+10)}$$

Zero: $s = 10$

pole: $s = -5, -10$



b. $\mathcal{L}[5 \cdot \cos(10t) + 10 \cdot \cos(20t)]$

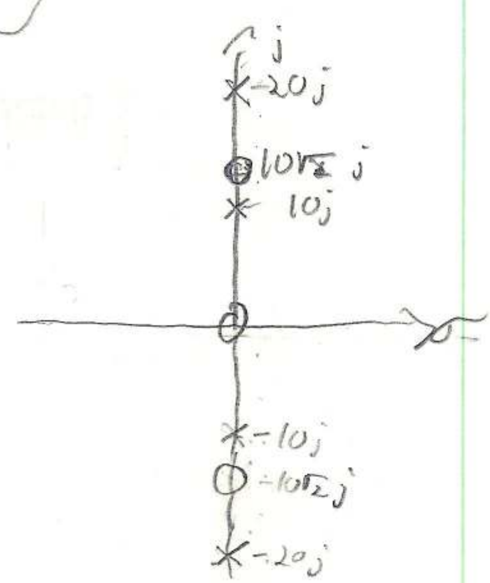
$$= \frac{5s}{s^2+100} + \frac{10s}{s^2+400}$$

$$= \frac{15 \cdot s \cdot (s^2 + 200)}{(s^2+100)(s^2+400)}$$

Comes from $s = \sqrt{-200}$

Zeros: $s = 0$, $s = \pm 10 \cdot \sqrt{2} \cdot j$

poles: $s = \pm 10j$, $s = \pm 20j$



9.9a. $\mathcal{L}[\delta(t) + 625te^{-25t}u(t)]$

$$= \frac{625}{(s+25)^2} + 1$$

$$= \frac{s^2+50s+1250}{(s+25)^2}$$

Zeros: $s = -25 \pm 25j$

poles: $s = \pm 5j$

b. $\mathcal{L}[(10 + e^{-10t} + 5\cos(10t) + 5\sin(10t))u(t)]$

$$= \frac{10}{s} + \frac{1}{s+10} + \frac{5s}{s^2+100} + \frac{50}{s^2+100}$$

$$= \frac{16s^3 + 200s^2 + 1600s + 1000}{s^4 + 10s^3 + 100s^2 + 1000s}$$

Use calc, to find zeros of numirator and denominator

Zeros: $s = -9.07, -1.71 \pm 8.12 \pm i$

poles: $s = 0, -10, \pm 10j$

9.10. Recall: $[f(t-a)]u(t-a) \to e^{-as}F(s)$

a. $\mathcal{L}[3\delta(t-2)] = 3 \cdot e^{-2s}$

b. $\mathcal{L}[10 \cdot e^{-50(t-1)} \cdot u(t-1)] = \dfrac{10e^{-s}}{s+50}$

c. $\mathcal{L}[10e^{-50(t-2)}u(t-2)] = \dfrac{10e^{-2s}}{s+50}$

9.15 Recall $tf(t) \xrightarrow{\mathcal{L}} -\dfrac{dF(s)}{ds}$

$$\mathcal{L}[(500 + 75\underbrace{t \cdot e^{-200t}\sin(200t))}_{\text{Here we see } t \cdot f(t)}u(t)]$$

$$\mathcal{L}[e^{-200t} \cdot \sin(200t)] = \frac{200}{(s+200)^2 + 200^2}$$

$$\Rightarrow tf(t) = -\frac{d\left(\frac{200}{(s+200)^2+200^2}\right)}{ds}$$

$$tf(t) = \frac{200(2s+400)}{((s+200)^2+200^2)^2}$$

Sub in $t \cdot f(t)$

So Now: $\mathcal{L}[(500 + 75 \cdot tf(t))u(t)]$

$$\boxed{= \frac{500}{s} + \frac{75 \cdot 200(2s+400)}{((s+200)^2+200^2)^2}}$$

b. (opt) Matlab! syms t;

f = laplace(500 + 75*exp(-200t)*t*sin200t)

$$F = \frac{15000(2s+400)}{((s+200)^2+200^2)^2} + \frac{500}{s}$$

9.17. a. $F_1(s) = \dfrac{50(s+10)}{(s+5)(s+20)} = \left[\dfrac{a}{s+5} + \dfrac{b}{s+20}\right]$

poles: $s = -5, -20$

$a = (s+5)F(s)\big|_{s=-5} = \dfrac{50(s+10)}{s+20}\Big|_{s=-5} = \dfrac{50\cdot 5}{15} = \dfrac{50}{3}$

$b = (s+20)F(s)\big|_{s=-20} = \dfrac{50(s+10)}{s+5}\Big|_{s=-20} = 50\cdot\dfrac{10}{15} = \dfrac{100}{3}$

$\Rightarrow \mathcal{L}^{-1}[F_1(s)] = \boxed{\dfrac{50}{3}\cdot e^{-5t} + \dfrac{100}{3}\cdot e^{-20t}}$

b. $F(s) = \dfrac{2s^2}{(s+200)(s+500)}$

Improper fraction so divide through

$$\begin{array}{r|l} & \;2 \\ s^2+700s+100{,}000 & 2s^2 \\ & -2s^2+1400s+200{,}000 \\ \hline & -1400s-200{,}000 \end{array}$$

$\Rightarrow F(s) = 2 - \underbrace{\left(\dfrac{1400s+200{,}000}{(s+200)(s+500)}\right)}$

Now Expand this to $\dfrac{a}{s+200} + \dfrac{b}{s+500}$

$a = \dfrac{1400s+200{,}000}{s+500}\Big|_{s=-200} = \underline{-800/3}$

$b. = \dfrac{1400s+200{,}000}{s+200}\Big|_{s=-500} = 5000/3$

9.17b. Con't

Now $F(s) = 2 + \frac{800}{3} \cdot \frac{1}{s+200} - \frac{5,000}{3} \frac{1}{s+500}$

$$\boxed{\mathcal{L}^{-1}[F(s)] = 2\delta(t) + \frac{800}{3} e^{-200t} - \frac{5,000}{3} \cdot e^{-500t}}$$

9.14.a. $F(s) = \beta \cdot \left[\frac{s+\beta}{s(s^2+\beta^2)}\right] = \frac{a}{s} + \frac{b}{s+\beta j} + \frac{b^*}{s-\beta j}$

$a = sF(s)\Big|_{s=0} = \frac{\beta s + \beta^2}{(s^2+\beta^2)}\Big|_{s=0} = 1$

$b = (s+\beta j)F(s)\Big|_{s=-\beta j} = \frac{-1+j}{2}$

$\Rightarrow b^* = \frac{-1-j}{2}$

$\Rightarrow |b| = \sqrt{1/4 + 1/4} = \sqrt{2}/2$

$\angle b = -3\pi/4$

$\Rightarrow \mathcal{L}[F(s)] = 1 + 2|b|e^{-\alpha t} \cos(\beta t + \angle b)$

$\alpha = \mathrm{Re}(\text{Complex poles of } Z) = \mathrm{Re}(\pm \beta j) = 0$

$$\Rightarrow \boxed{\mathcal{L}[F(s)] = 1 + \sqrt{2} \cos(\beta t - 3\pi/4)}$$

9.19 b. $F(s) = \dfrac{s^2 + \beta s}{s^2 + \beta^2}$ is improper so divide through

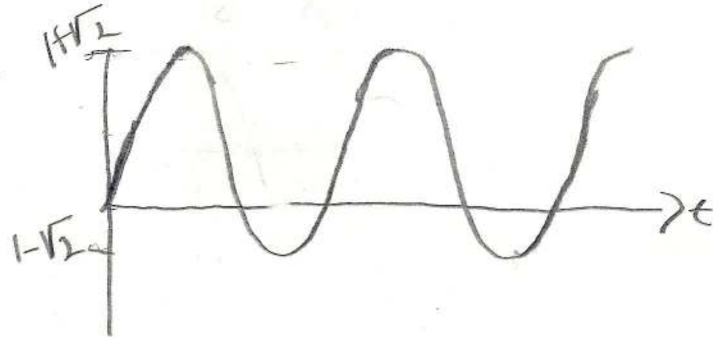
$$\begin{array}{r|l} & 1 \\ \hline s^2+\beta^2 & s^2 + \beta s \\ & -s^2 + \beta^2 \\ \hline & \beta s - \beta^2 \end{array}$$

$$\Rightarrow F(s) = 1 + \frac{\beta s - \beta^2}{s^2+\beta^2} = 1 + \beta\left(\frac{s}{s^2+\beta^2} - \frac{\beta}{s^2+\beta^2}\right)$$

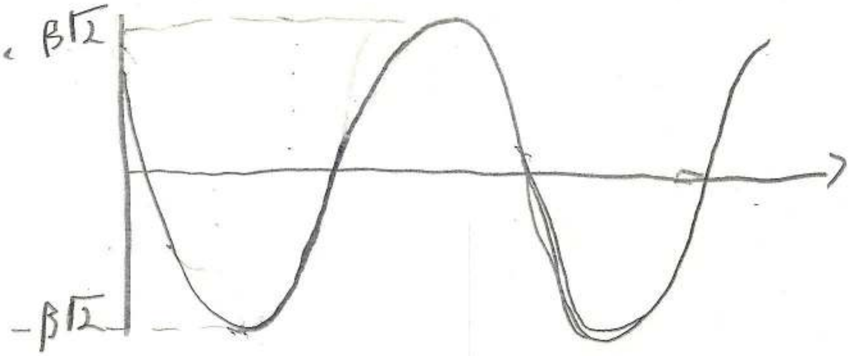
$$\Rightarrow \boxed{\mathcal{L}^{-1}[F(s)] = \delta(t) + \beta\left(\cos(\beta t) - \sin(\beta t)\right)}$$

9.19 cont Wave forms

For a.



b.



9.25 a. $F(s) = \dfrac{4(s^2+16)}{s\cdot(s+4+4j)(s+4-4j)}$

$$= \frac{a}{s} + \frac{b}{s+4+4j} + \frac{b^*}{s+4-4j}$$

$$a = \left.\frac{4(s^2+16)}{s^2+8s+32}\right|_{s=0} = \frac{64}{32} = 2$$

$$b = \left.\frac{4s^2+16}{s\cdot(s+4-4j)}\right|_{s=-4-4j} = 1-3j$$

$$\Rightarrow b^* = 1+3j$$

$$|b| = \sqrt{10}\ ,\ \Theta = a\tan(3/1)$$

$$\Rightarrow \mathcal{L}^{-1}[F(s)] = \mathcal{L}^{-1}[2/s] + \mathcal{L}^{-1}\left[\frac{1-3j}{s+4+4j} + \frac{1+3j}{s+4-4j}\right]$$

$$= \underline{2 + 2\cdot\sqrt{10}\,e^{-4t}\cdot\cos(4t+\tan^{-1}(3))}$$

b. $F(s) = \dfrac{2(s^2+30s+800)}{s(s+40)(s+10)} = \dfrac{a}{s} + \dfrac{b}{s+40} + \dfrac{c}{s+10}$

$$a = \left.\frac{2(s^2+30s+800)}{(s+40)(s+10)}\right|_{s=0} = 4$$

$$b = \left.\frac{2(s^2+30s+800)}{s(s+10)}\right|_{s=-40} = 2$$

9.25 b. Cont

$$C = \left. \frac{2(s^2+30s+800)}{s(s+40)} \right|_{s=-10} = -4$$

$$\Rightarrow \mathcal{L}^{-1}[F(s)] = \boxed{4 + 2e^{-40t} - 4e^{-10t}}$$

9.27. $F(s) = \dfrac{s+r}{s+20}$

a. $\mathcal{L}[f(t)] = 1 - \dfrac{5}{s+20}$

$$= \frac{s+20-5}{s+20} = \frac{s+15}{s+20}$$

$$\Rightarrow \boxed{r = 15}$$

b. $\mathcal{L}[f(t)] = 1 = \dfrac{s+20}{s+20} \Rightarrow \boxed{r = 20}$

c. $\mathcal{L}[f(t)] = 1 + \dfrac{5}{s+20} = \dfrac{s+25}{s+20}$

$$\Rightarrow \boxed{r = 25}$$