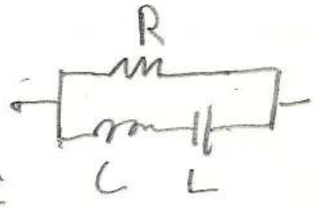
HW 7 | Solutions | Due 11/23/10

10. 3, 7, 9, 11, 13, 19, 21, 23, 30

10.3 Find Z_{eq} of 

$R = 2k\Omega$, $L = .4$, $C = 100nF$

$$Z_{eq} = (Z_C + Z_L) \| R$$

$$= \left(\frac{1}{Cs} + Ls\right) \| R$$

$$= \frac{Cs + LCs^2}{Cs} \| R$$

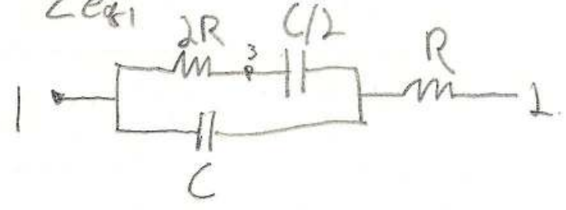
$$\boxed{= \frac{R(CLs^2 + 1)}{CLs^2 + CRs + 1} = \frac{2{,}000(s^2 + 25\times10^6)}{s^2 + 5{,}000s + 25\times10^6}}$$

Zeros: $s = \sqrt{-25\times10^6} \Rightarrow \boxed{s = \pm 5{,}000j}$

poles: $\boxed{s = -2{,}500 \pm 4{,}330j}$

10.7

For Z_{eq1}



$$Z_{eq1} = \left(\frac{1}{Cs}\right) \parallel \left(2R + \frac{2}{Cs}\right) + R$$

$$= \frac{\frac{1}{Cs} \cdot \frac{2RCs+2}{Cs}}{\frac{1}{Cs} + \frac{2RCs+2}{Cs}} + R$$

$$= \frac{2RC^{\cancel{2}}s^{\cancel{2}} + 2\cancel{C}\cancel{s}}{\cancel{C^2s^2} + 2RC^{\cancel{3}}s^{\cancel{3}} + 2C^{\cancel{2}}s^{\cancel{2}}} + R$$

$$= \frac{2RCs + 2}{Cs + 2RC^2s^2 + 2Cs} + \frac{CsR(2RCs+3)}{Cs(2RCs+3)}$$

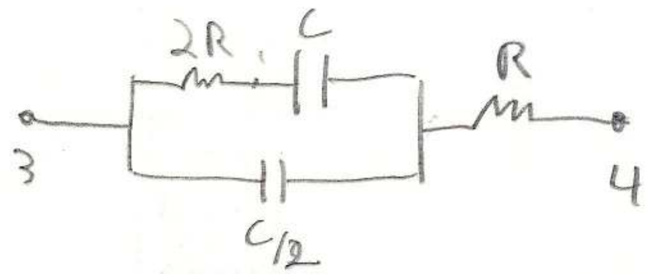
$$= \frac{CRs(2CRs+5) + 2}{Cs(2RCs+3)}$$

$$\boxed{= \frac{(2CRs+1)(CRs+2)}{Cs(2RCs+3)}}$$

$$\boxed{\begin{array}{ll} \text{Zeros: } & s = -\frac{1}{2CR},\ s = -\frac{2}{CR} \\ \text{poles: } & s = 0,\ s = -\frac{3}{2RC} \end{array}}$$

10.7 Cont.

Z_{eq_2}:



$$Z_{eq_2} = \left(\frac{2RCs + 1}{Cs}\right) \parallel \frac{2}{Cs} + R$$

$$= \frac{\cancel{Cs}(4RCs + 2)}{\cancel{C^2s^2}(2RCs + 3)} + \frac{CsR(2RCs + 3)}{Cs(2RCs + 3)}$$

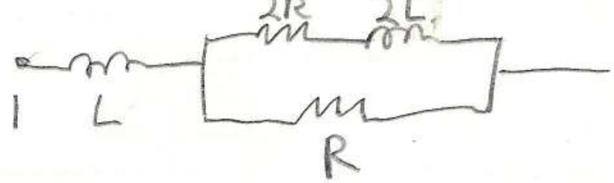
$$\boxed{= \frac{2R^2C^2s^2 + 7RCs + 2}{Cs(2RCs + 3)}}$$

Zeros: $s = \dfrac{-7RC \pm \sqrt{49R^2C^2 - 16R^2C^2}}{4}$

$$\boxed{s = \frac{-7RC \pm RC\cdot\sqrt{33}}{4}}$$

$$\boxed{\text{poles} \quad s = 0, \; s = -\frac{3}{2RC}}$$

10.9 Z_{eq1}:



$$Z_{eq1} = Ls + 2(R+Ls) \parallel R$$

$$= Ls + \frac{2R^2 + 2RLs}{3R + 2Ls}$$

$$= \frac{3RLs + 2L^2s^2 + 2R^2 + 2RLs}{3R+2Ls}$$

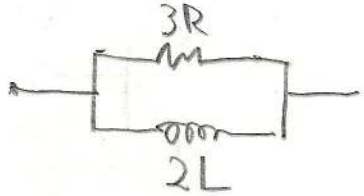
$$\boxed{= \frac{(2Ls + R)(Ls + 2R)}{3R + 2Ls}}$$

$$\boxed{\text{Zeros: } s = \frac{-R}{2L}, \frac{-2R}{L}}$$

$$\boxed{\text{poles: } s = \frac{-3R}{2L}}$$

10.9 cont.

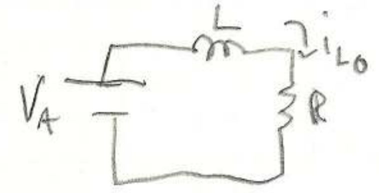
Z_{eq_L}:



$$Z_{eq_L} = 3R \parallel 2Ls = \boxed{\frac{6RLs}{3R + 2Ls}}$$

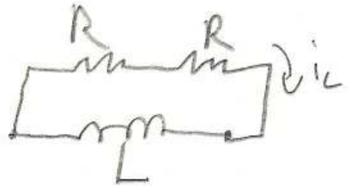
$$\boxed{\begin{array}{l} \text{Zeros: } s = 0 \\ \text{poles: } s = -\dfrac{3R}{2L} \end{array}}$$

10.11 When switch is at A for a long time



$i_{L0} = \frac{V_A}{R}$ ← this is your initial condition $i_L(0)$

At $t=0$:



KVL: $iR + iR + L\dot{i}_L = 0$

$$\Rightarrow 0 = I_L(s)\cdot 2R + L\left(s I_L(s) - i_L(0)\right)$$

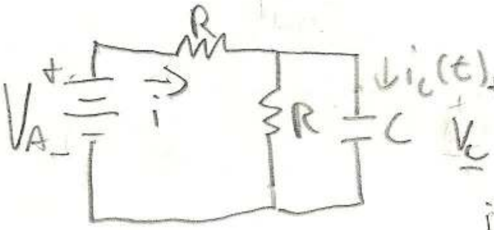
$$0 = I_L(s)(2R + Ls) - L V_A / R$$

$$\Rightarrow \boxed{I_L(s) = \frac{L V_A}{LRs + 2R^2}}$$

$$I_L(s) = \frac{L V_A}{LR}\cdot\left(\frac{1}{s + 2R/L}\right)$$

$$\boxed{i_L(t) = \mathcal{L}^{-1}\left[I_L(s)\right] = \frac{V_A}{R}\cdot e^{-(2R/L)t}}$$

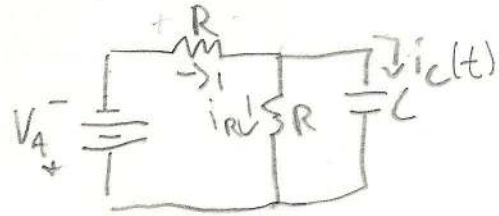
10.13 When switch is at A:

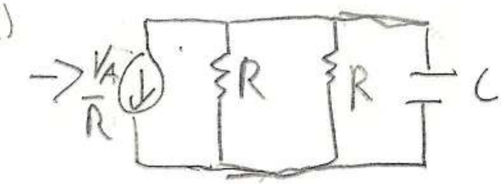


$i_c(0) = 0$

$i = \frac{V_A}{2R}, \Rightarrow V_c(0) = \frac{V_A}{2}$

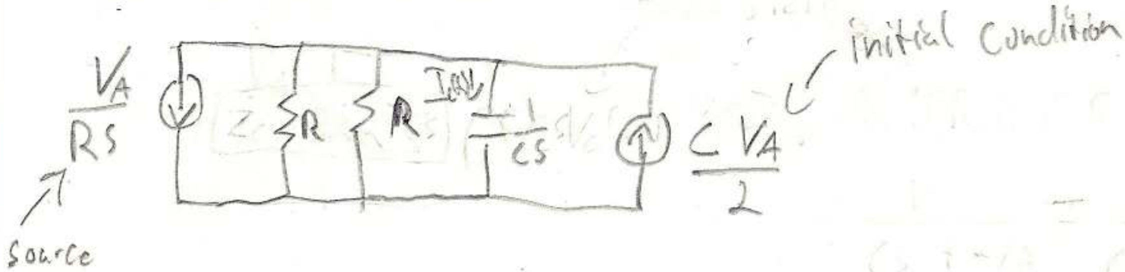
At $t=0$, switched to B





Use Superposition for 0 input / 0 state

In S domain



Zero Source,

$$I_{czs}(s) = \frac{Cs}{Cs + \frac{1}{R} + \frac{1}{R}} \cdot \frac{CV_A}{2} = \frac{C^2 V_A R s}{2(CRs + 2)}$$

Zero Initial Condition

$$I_{czi}(s) = \frac{-Cs}{Cs + \frac{1}{R} + \frac{1}{R}} \cdot \frac{V_A}{R \cdot s} = \frac{-CV_A}{CRs + 2}$$

10.13 cont. $I_c(s) = I_{cZS}(s) + I_{cZI}(s)$

$$\boxed{= \frac{C^2 V_A R s - 2CV_A}{2(CRs+2)}}$$

Note $I_c(s)$ is not proper so divide

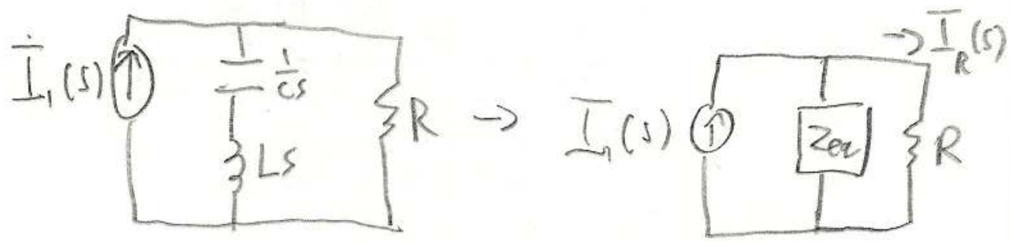
$$\begin{array}{r|l} & \tfrac{1}{2}CV_A \\ 2CRs+4 & C^2V_ARs - 2CV_A \\ & -C^2V_ARs + 2CV_A \\ \hline & \quad -4CV_A \end{array}$$

$$\Rightarrow I_c(s) = \frac{CV_A}{2} - \frac{2CV_A}{CRs+2}$$

$$\Rightarrow i_c(t) = \mathcal{L}^{-1}[I_c(s)]$$

$$\boxed{i_c(t) = \frac{CV_A}{2}\delta(t) - \frac{2V_A}{R} \cdot e^{-\left(\frac{2}{CR}\right)t}}$$

10.19. In S domain



$$Z_{eq} = \frac{1}{Cs} + Ls = \frac{CLs^2+1}{Cs}$$

Using Current Division:

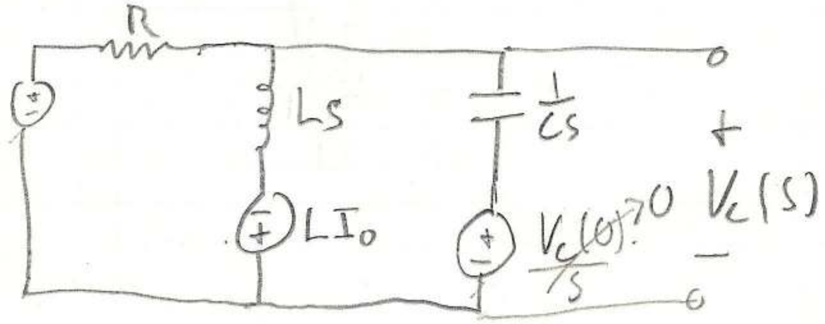
$$I_R(s) = \frac{\frac{1}{R}}{\frac{1}{R} + \frac{1}{Z_{eq}}} I_1(s)$$

$$\Rightarrow \frac{I_R(s)}{I_1(s)} = \frac{\frac{1}{R}}{\frac{1}{R} + \frac{Cs}{CLs^2+1}} = \frac{\frac{1}{R}}{\frac{CLs^2+1+RCs}{R(CLs^2+1)}}$$

$$\boxed{= \frac{\cancel{R}(CLs^2+1)}{\cancel{R}(CLs^2+RCs+1)}}$$

10.21

In s domain



Zero State (inductor section is O.C.)

$$V_{cZS}(s) = \frac{\frac{1}{Cs}}{R+\frac{1}{Cs}} V_s(s) = \frac{Cs}{Cs(RCs+1)} V_s(s)$$

$$\boxed{= \frac{1}{RCs+1} V(s)}$$

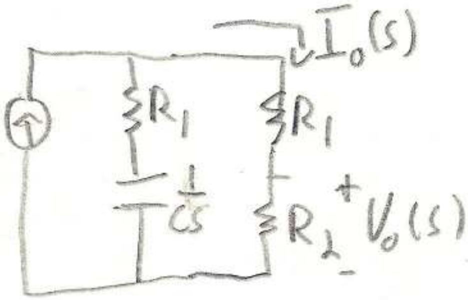
Zero Input (Source section is O.C)

$$V_{cZI}(s) = -\frac{\frac{1}{Cs}}{\frac{1}{Cs}+Ls} = \frac{-Cs}{Cs(1+LCs^2)}$$

$$\boxed{= -\frac{1}{LCs^2+1}}$$

10.23

In S Domain



$$V_o(s) = I_o(s) R_2$$

$$I_o(s) = \frac{\frac{1}{R_1+R_2}}{\frac{1}{R_1+R_2} + \frac{1}{R_1+\frac{1}{Cs}}} \cdot I_s(s) = \frac{\frac{1}{R_1+R_2}}{\frac{1}{R_1+R_2} + \frac{Cs}{R_1Cs+1}} \cdot I_s(s)$$

$$= \frac{\frac{1}{R_1+R_2}}{\frac{RCs+1+Cs(R_1+R_2)}{(R_1+R_2)(R_1Cs+1)}} I_s(s) = \frac{(R_1+R_2)(R_1Cs+1)}{(R_1+R_2)(R_1Cs+1+Cs(R_1+R_2))} \cdot I_s(s)$$

Sub in and Manipulate to get

$$I_o(s) = \frac{2(s+1000)}{5(s+400)} \cdot I_s(s) \ ; \quad I_s(s) = \frac{5}{2} \cdot \frac{1}{(s+200)}$$

$$\Rightarrow I_o = \frac{(s+1000)}{(s+400)(s+200)}$$

$$\Rightarrow \boxed{V_o(s) = 5k\Omega \cdot \frac{(s+1000)}{(s+400)(s+200)}}$$

Forced pole: $s = -200$
Natural pole: $s = -400$

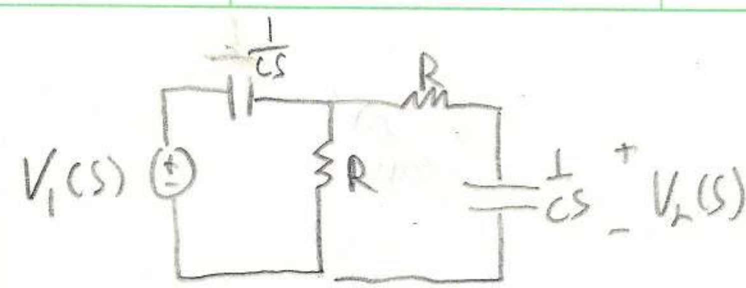
10.23 cont. $I_o(s) = \frac{a}{s+400} + \frac{b}{s+200}$

$$a = \left.\frac{s+1000}{s+200}\right|_{s=-400} = -3$$

$$b = \left.\frac{s+1000}{s+400}\right|_{s=-200} = 4$$

$$\Rightarrow \boxed{V(t) = 5{,}000 \cdot (4 \cdot e^{-200t} - 3e^{-400t})}$$

10.30



Zeq (as seen from $V_1(s)$)

$$Z_{eq} = \frac{1}{Cs} + R \,||\, \left(R + \frac{1}{Cs}\right) \qquad \frac{RCs+1}{Cs}$$

$$= \frac{1}{Cs} + \frac{\frac{R(RCs+1)}{\cancel{Cs}}}{\frac{RCs + RCs + 1}{\cancel{Cs}}}$$

$$= \frac{1}{Cs} + \frac{R(RCs+1)}{2RCs+1} = \frac{2RCs+1 + RCs(RCs+1)}{2RC^2s^2 + Cs}$$

$$= \frac{1 + RCs(RCs+3)}{2RCs^2 + Cs}$$

$$\frac{V_2(s)}{V_1(s)} = \frac{1/Cs}{Z_{eq}} = \frac{2CRs+1}{C^2R^2s^2 + 3 \cdot CRs + 1}$$

poles $s = \dfrac{-3CR \pm \sqrt{9C^2R^2 - 4C^2R^2}}{2 \cdot C^2R^2}$

$$= \frac{-3 \pm \sqrt{5}}{2 \cdot C \cdot R}$$

10.30 cont.

Want $\frac{-3-\sqrt{5}}{2CR} = -2{,}618$ rad/s

$$\Rightarrow CR = \frac{+3+\sqrt{5}}{2 \cdot 2{,}618}$$

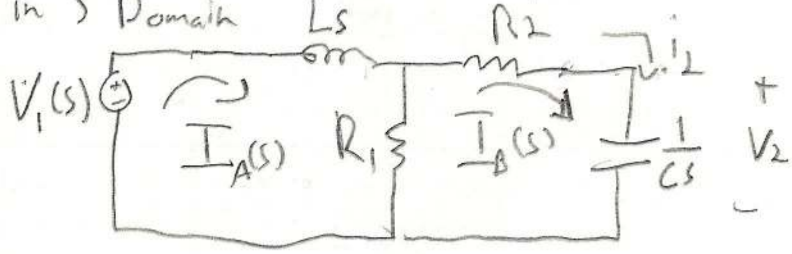
Also $\frac{-3+\sqrt{5}}{2CR} = -382$ rad/s

$$\Rightarrow CR = \frac{3-\sqrt{5}}{2 \cdot 382}$$

Either way $\boxed{CR = .001}$

~~So~~ For example $R = 1\,k\Omega$, $C = 1\,\mu F$

10.31 in S Domain



a. Mesh A: $-V_1(s) + Ls\,I_A(s) + R_1\left(I_A(s) - I_B(s)\right) = 0$

Mesh B $\quad R_1\left(I_B(s) - I_A(s)\right) + R_2 I_B(s) + \dfrac{I_B(s)}{Cs} = 0$

b. $I_2(s) = I_B(s)$

Solve
$$\begin{bmatrix} Ls + R_1 & -R_1 \\ -R_1 & R_1 + R_2 + \frac{1}{Cs} \end{bmatrix} \begin{bmatrix} I_A \\ I_B \end{bmatrix} = \begin{bmatrix} V_1 \\ 0 \end{bmatrix}$$

$$\Rightarrow \boxed{I_2(s) = \frac{V_1(s)\,CR_1 s}{CL(R_1+R_2)s^2 + (CR_1R_2 + L)s + R_1}}$$

Zeros: $s = 0$

poles: $s = \dfrac{-(CR_1R_2 + L) \pm \sqrt{(CR_1R_2+L)^2 - 4R_1CL(R_1+R_2)}}{2CL(R_1+R_2)}$

Part C. $R_1 = 1K\Omega$, $R_2 = 2K\Omega$, $L = 2H$

$C = 500nF$, $V(t) = 50$

$\Rightarrow V_1(s) = 50/s.$

$\Rightarrow$ poles $s: -500 \pm \frac{500\sqrt{3}}{3} j$

$$\Rightarrow I_2(s) = \frac{a}{(s + 500 - \frac{500\sqrt{3}}{3} j)} + \frac{a^*}{(s + 500 + \frac{500\sqrt{3}}{3} j)}$$

$$a = I_2(s) \cdot (s + 500 - \frac{500\sqrt{3}}{3} j)\Big|_{s = -500 + \frac{500\sqrt{3}}{3} j}$$

$$= \frac{.025}{s + 500 + \frac{500\sqrt{3}}{3} j}\Big|_{s = -500 + \frac{500\sqrt{3}}{3} j} = -4.33 \times 10^{-5} j$$

$|a| = 4.33 \times 10^{-5}$ $\qquad \angle a = -\pi/2$

$$\Rightarrow i_2(t) = 2 \cdot |a| t e^{-500t} \cdot \cos\left(\frac{500\sqrt{3}}{3} t - \angle a\right)$$

$$\boxed{i_2(t) = .0866 t e^{-500t} \cos\left(\frac{500\sqrt{3}}{3} t - \frac{\pi}{2}\right) mA}$$