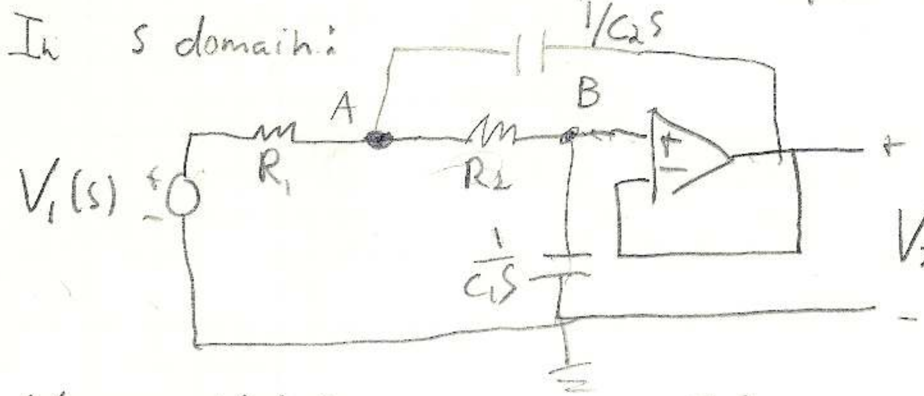
Solutions HW 8

10: 55, 11: 5, 7, 9, 27, 51, 59, 12: 3, 15, 57

10.55 Note Zero States so $V_{C_1}(t=0) = V_{C_2}(t=0) = 0$

In s domain:



$$V_2(s) = V_B(s) \qquad (1)$$

$$\frac{V_A(s) - V_B(s)}{R_2} = V_B(s) \cdot C_1 \cdot s \qquad (2)$$

$$\frac{V_1(s) - V_A(s)}{R_1} = V_B(s) \cdot C_1 \cdot s + (V_A(s) - V_2(s))\, C_2 \cdot s \qquad (3)$$

Eq 2 gives $V_A(s) = R_2 C_1 V_2(s) \cdot s + V_2(s)$

$= V_2(s) \cdot (R_2 C_1 s + 1)$

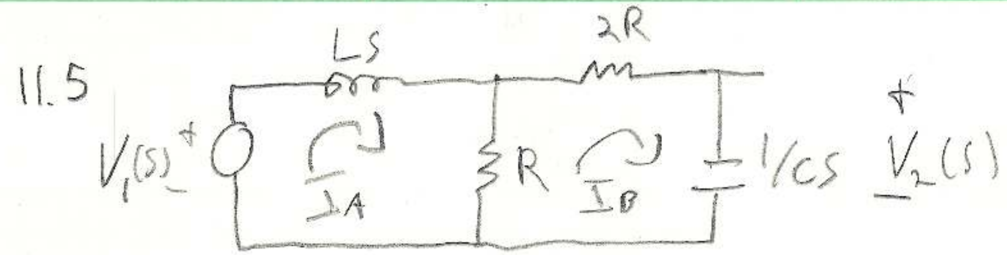
Sub into eq 3

$$V_2(s)\,(R_2 C_1 s + 1 + R_1 C_1 s + R_1 R_2 C_1 s^2 \cdot C_2 \cancel{- C_2 R_1 s} + \cancel{R_1 C_2 s})$$

$$= V_1(s)$$

$$\Rightarrow \boxed{\frac{V_2(s)}{V_1(s)} = \frac{1}{R_1 R_2 C_1 C_2 s^2 + C_1 (R_2 + R_1) s + 1}}$$

11.5



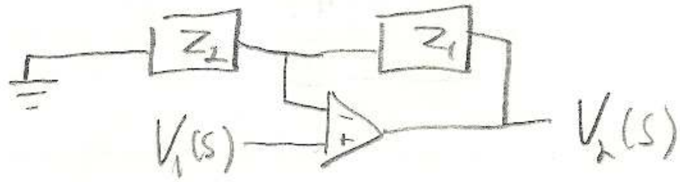
$$\begin{bmatrix} Ls+R & -R \\ -R & 3R+\frac{1}{Cs} \end{bmatrix} \begin{bmatrix} I_A \\ I_B \end{bmatrix} = \begin{bmatrix} V_1(s) \\ 0 \end{bmatrix}$$

$$I_B = \frac{C \cdot R \cdot V_1(s) \cdot s}{3CLR \cdot s^2 + (2CR^2+L)\cdot s + R}$$

$$V_2(s) = I_B(s) \cdot \frac{1}{Cs}$$

$$\Rightarrow \boxed{\frac{V_2(s)}{V_1(s)} = \frac{R\,V_1(s)}{3CLR \cdot s^2 + (2CR^2+L)\cdot s + R}}$$

11.7.



Noniverting Amp.

$$T_V = \frac{Z_1 + Z_2}{Z_2}, \quad Z_2 = R, \quad Z_1 = \frac{R/cs}{R + 1/cs}$$

$$= \frac{R}{Rcs + 1}$$

$$\Rightarrow T_V = \frac{R + \frac{R}{Rcs+1}}{R}$$

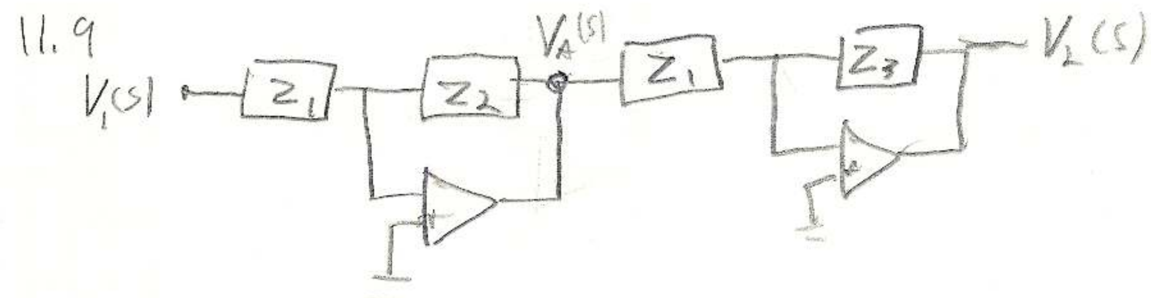
$$\Rightarrow \boxed{T_V(s) = \frac{CRs + 2}{CRs + 1}}$$

poles at $s = -1/CR$

$$\Rightarrow \boxed{C \cdot R = .001}$$

Let $C = 1\,\mu F$, $R = 1\,K\Omega$ for example

11.9



2 - inverting Amps

$$\frac{V_A}{V_1} = -\frac{Z_2}{Z_1} \quad ; \quad \frac{V_2}{V_A} = \frac{-Z_3}{Z_1}$$

$$\Rightarrow \frac{V_2}{V_1} = \frac{Z_2 \cdot Z_3}{Z_1^2}$$

$$\Rightarrow \boxed{T_V = \frac{V_2}{V_1} = \frac{10^6}{(s+100)(s+1000)}}$$

$$\boxed{\text{Poles: } s = -100, -1000}$$

$$Z_1 = 10k\Omega$$

$$Z_2 = \frac{10k\Omega}{10k\Omega \cdot 100nF s + 1}$$

$$Z_3 = \frac{100k\Omega}{100k\Omega \cdot 100nF s + 1}$$

11.27 Simple Voltage divider

So.

$$V_2(s) = \frac{Ls}{R + \frac{1}{Cs} + Ls} V_1(s) = \frac{CLs^2}{RCs + LCs^2 + 1} V_1(s)$$

When $V_1(t) = 10\cos(2000t)$, $V_1(s) = \frac{10s}{s^2 + 2000^2}$

Sub in $L = 5mH$, $C = 50\mu F$, $R = 10\Omega$ and $V_1(s)$ to get

$$V_2(s) = \frac{10s^3}{\underbrace{(s^2 + 4\times10^6)}\underbrace{(s^2 + 2000s + 4\times10^6)}}$$

poles: $s = \pm 2000j$, $-1000 \pm 1000\sqrt{3}j$

$$\Rightarrow V_2(s) = \frac{a}{(s + 2000j)} + \frac{a^*}{s - 2000j} + \frac{b}{s + 1000(1 - \sqrt{3}j)} + \frac{b^*}{s + 1000(1 + \sqrt{3}j)}$$

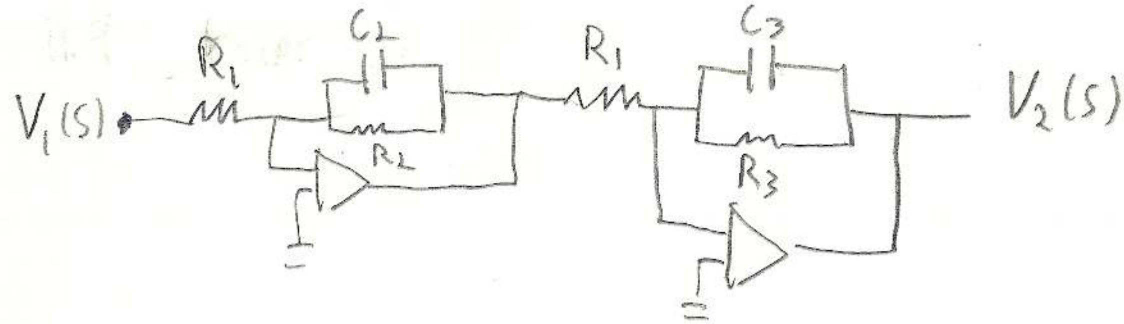
$$a = V_2(s)(s + 2000j)\Big|_{s = -2000j} = -5j$$

$$b = V_2(s)(s + 1000(1 - \sqrt{3})j)\Big|_{s = -1000 + \sqrt{3}j} = 5 - \frac{5}{\sqrt{3}}j = 5 - \frac{5\sqrt{3}}{3}j$$

$\text{abs}(a) = 5$, $\angle a = -\frac{\pi}{2}$; $\text{abs}(b) = \frac{10\sqrt{3}}{3}$; $\angle b = -\frac{\pi}{6}$

$$\Rightarrow \boxed{V_2(t) = 10 \cdot \cos(200t - \pi/2) + \frac{20\sqrt{3}}{3} \cdot \cos(1000\sqrt{3}t - \pi/6)}$$

11.51 11.9 gives some help setting this up.



In 11.9 we see this is:

$$\frac{V_2}{V_1} = \frac{1}{R_1^2} \cdot \left(\frac{R_2}{R_2C_2s+1}\right)\left(\frac{R_3}{R_3C_3s+1}\right)$$

$$= \frac{1}{R_1^2} \cdot \left(\frac{1/C_2}{s+1/(C_2R_2)}\right)\left(\frac{1/C_3}{s+1/C_3R_3}\right)$$

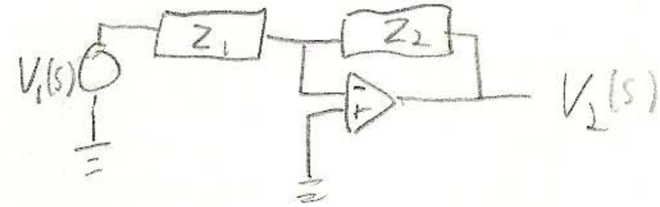
let $\frac{1}{C_2R_2} = 100$ so $\boxed{C_2 = 100nF,\ R_2 = 100k\Omega}$

let $\frac{1}{C_3R_3} = 10{,}000$ so $\boxed{C_3 = 10nF,\ R_3 = 10k\Omega}$

Now $2\times10^5 = \frac{1}{R_1^2C_2C_3} \Rightarrow R_1 = \sqrt{\frac{1}{2\times10^5 C_2 \cdot C_3}}$

$\boxed{R_1 = 70.71k\Omega}$

11.59 First Circuit look like:



$$T_V = -\frac{Z_2}{Z_1} \; ; \quad Z_1 = R_1 + \frac{1}{C_1 s} = \frac{C_1 R_1 s + 1}{C_1 s}$$

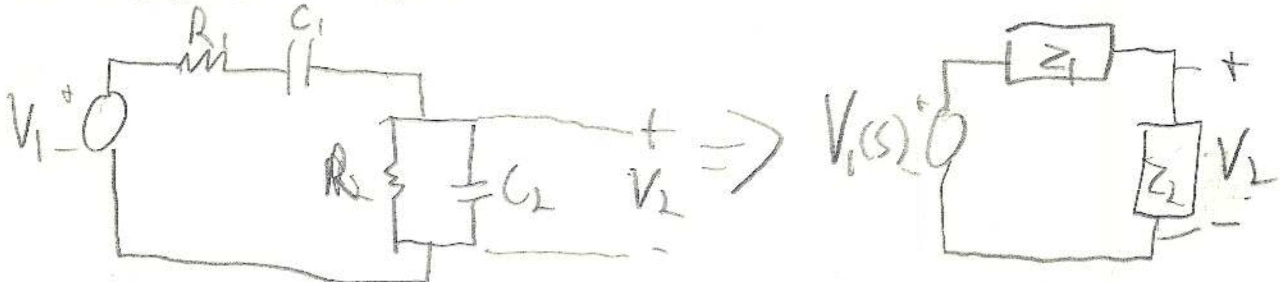
$$Z_2 = \frac{R_2 / C_2 s}{R_2 + 1/C_2 s} = \frac{R_2}{C_2 R_2 s + 1}$$

$$\Rightarrow T_V = -\frac{R_2 C_1 s}{(C_1 R_1 s + 1)(C_2 R_2 s + 1)}$$

$$= -\frac{(R_2 C_1 / (C_1 R_1 R_2 C_2)) \cdot s}{(s + 1/C_1 R_1)(s + 1/R_2 C_2)}$$

$$= -\frac{1000\, s}{(s + 4000)(s + 1000)} \quad \checkmark$$

Second Circuit



Voltage Division gives

$$T_V = \frac{V_2}{V_1} = \frac{Z_2}{Z_1 + Z_2} = \frac{R_2/(C_2 R_2 s + 1)}{\frac{C_1 R_1 s + 1}{C_1 s} + \frac{R_2}{C_2 R_2 s + 1}}$$

$$T_v = \frac{C_1 R_2 s}{C_1 C_2 R_1 R_2 s^2 + (C_1 R_1 + C_1 R_2 + C_2 R_2) s + 1}$$

$$= \frac{(C_1 R_1) \cdot s}{C_1 C_2 R_1 R_2} \cdot \left(\frac{1}{s^2 + \frac{(C_1 R_1 + C_1 R_2 + C_2 R_2)}{C_1 C_2 R_1 R_2} s + \frac{1}{C_1 C_2 R_1 R_2}} \right)$$

$$= \frac{1000\, s}{s^2 + 5000 s + 4{,}000{,}000} \checkmark$$

b. The output impedance for circuit 1 is 0 so use Circuit 1. It is 0 due to the op amp.

c. Circuit 1 input impedance is

$$Z_1 = \frac{s + 1/C_1R_1}{s} \cdot \left(\frac{C_1R_1}{C_1}\right) = 10k\Omega \cdot \left(\frac{s+4000}{s}\right)$$

$\Rightarrow$ 10K Gain

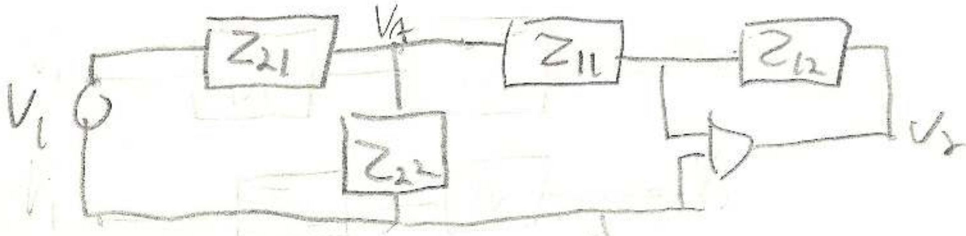
Circuit 2 has an input impedance of

$$Z_1 + Z_2 = \frac{(C_1C_2R_1R_2)s^2 + (C_1R_1 + C_1R_2 + C_2R_2)s + 1}{C_1C_2R_2s^2 + C_1s}$$

$$= 200 \cdot \left(\frac{(s+1000)(s+4000)}{s \cdot (s+2000)}\right)$$

$\rightarrow$ Use Circuit 2. Circuit 1 has a much higher input impedance.

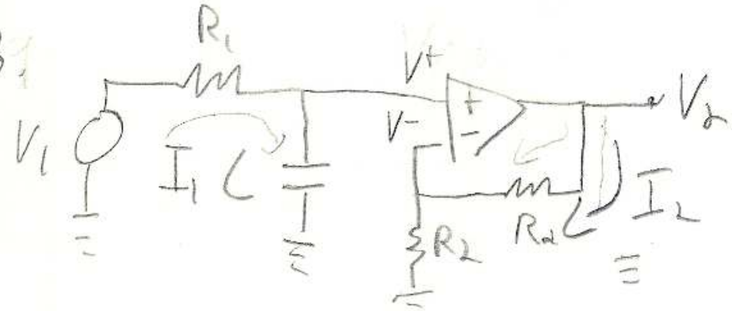
d. For C_2 followed by C_1:



In this set up, $T_2 \left(\frac{V_A}{V_1}\right)$ is different then before so it will not work for this set up.

However C_1 followed by C_2 is fine since the op-amp comes first.

12.3



$$V^+ = \frac{I_1}{Cs} \; ; \; I_1 = V_1 / \left(R_1 + \frac{1}{Cs}\right) = \frac{Cs\,V_1}{CR_1 s + 1}$$

$$\Rightarrow V^+ = \frac{V_1}{CR_1 s + 1}, \quad V^+ = V^-$$

$$I_2 = \frac{V_2}{2R_2} \; ; \; V^- = I_2 R_2 = \frac{V_2}{2}$$

$$\Rightarrow \frac{V_1}{CR_1 s + 1} = \frac{V_2}{2} \Rightarrow \boxed{T_V = \frac{2}{CR_1 s + 1}}$$

a. $$T_V(j\omega) = \frac{400}{j\omega + 200}$$

DC gain $|T_V(0)| = 2 = 6.0 \text{ dB}$

Infinite freq Gain:

$$|T_V(j\omega = \infty)| = 0$$

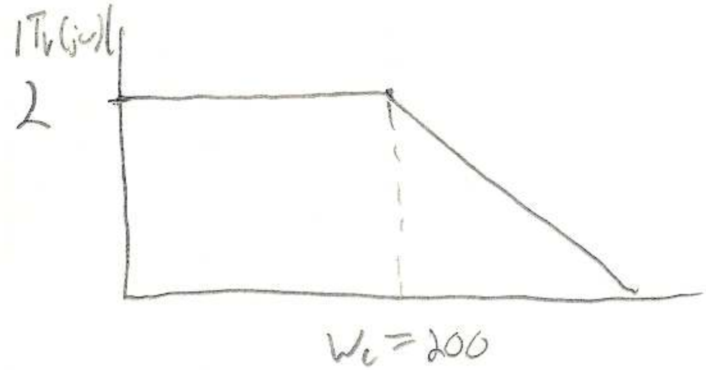
Cut off freq For $T(s) = \dfrac{2/CR_1}{s + \frac{1}{CR_1}}$ Standard low pass form so $\omega_c = \alpha = \dfrac{1}{CR_1}$

$$\Rightarrow \boxed{\omega_c = 200 \text{ rad/s}}$$

12.3 cont.

a. Cont. This is a low pass filter

b.



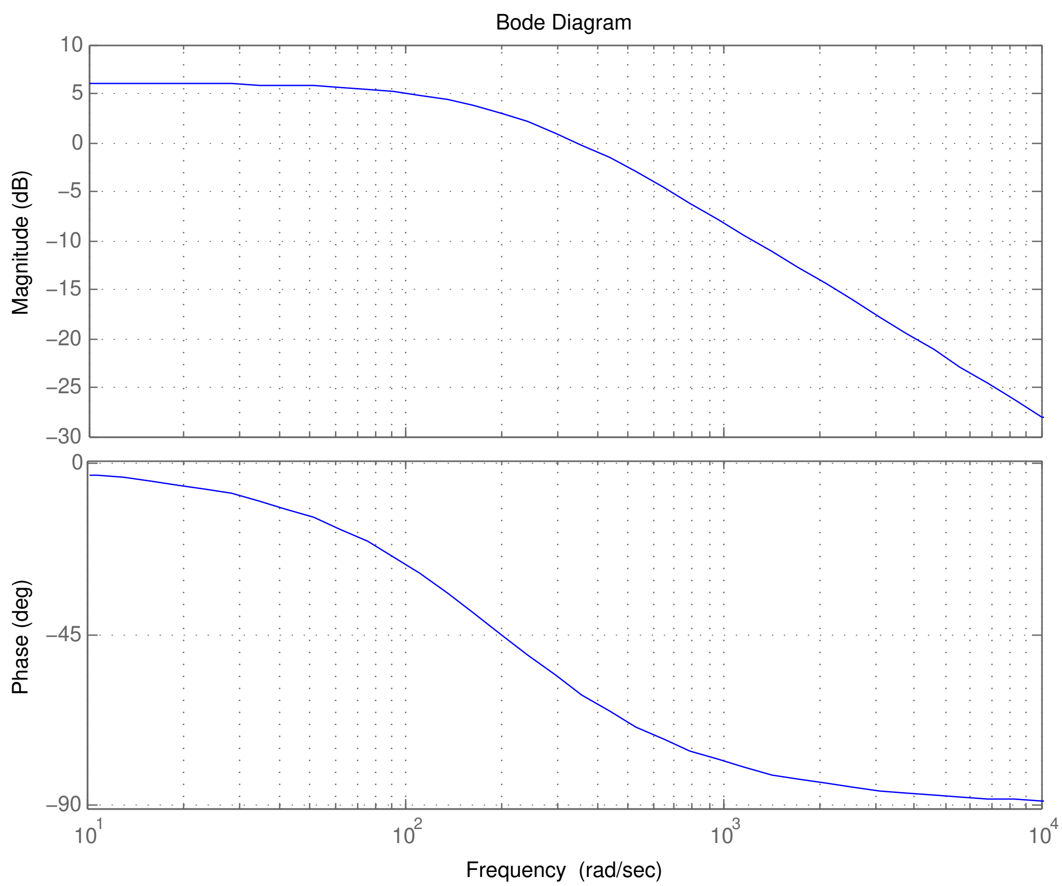
c. $|T_v(j\omega)|\big|_{\omega=.5\omega_c} = \underline{5.05\ dB}$

$|T_v(j\omega)|\big|_{\omega=\omega_c} = \underline{3.01\ dB}$

$|T_v(j\omega)|\big|_{\omega=2\omega_c} = \underline{-.97\ dB.}$

d. Not Nessisary but see next page
Command in Matlab: sys=tf([2],[CR, 1])
Bode(sys)

e. Since W_c is set by the $20K\Omega$ resistor and capacitor, you would adjust the gain by changing the two $30k\Omega$ res.


Bode Diagram
Magnitude (dB)
10
5
0
−5
−10
−15
−20
−25
−30
Phase (deg)
0
−45
−90
10^1
10^2
10^3
10^4
Frequency (rad/sec)

✓

12.15 For the first inverting Amp

$$T_1(s) = \frac{\frac{1}{C_1 s} \| R_2}{R_1} = -\frac{R_2}{R_1(C_1 R_2 s + 1)}$$

And for the second

$$T_2(s) = -\frac{R_4}{\frac{1}{C_2 s} + R_3} = -\frac{R_4 C_2 s}{C_2 R_3 s + 1}$$

For the 1st part, $W_c = \frac{1}{C_1 R_2}$, it is low pass

For the second part, $W_c = \frac{1}{C_2 R_3}$, it is high pass.

For high pass $W_{cH} = \frac{1}{C_2 R_3} = 100$ rad/s

For low pass $W_{cL} = \frac{1}{C_1 R_2} = 2500$ rad/s

The gain is $\frac{|K_1 K_2|}{W_{cL}} = \frac{1}{R_1 \cancel{C_1}} \cdot \frac{R_4}{R_3} \cdot \cancel{C_1} R_2$

$$= \frac{R_4 R_2}{R_1 R_3} = 30$$

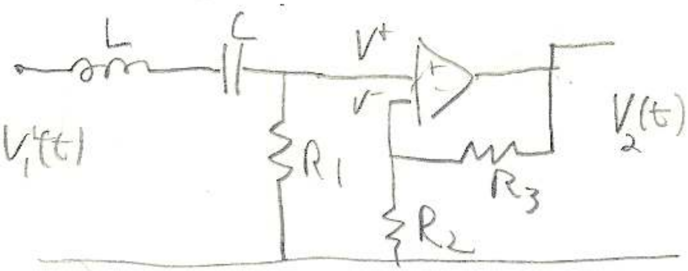
Let $C_2 = 100nF$, $R_3 = 100k\Omega$

Let $C_1 = 1nF$, $R_2 = 400k\Omega$

Then to meet the gain $R_1 = 10k\Omega$

$R_4 = 75k\Omega$

12.57 For Vendor 1:

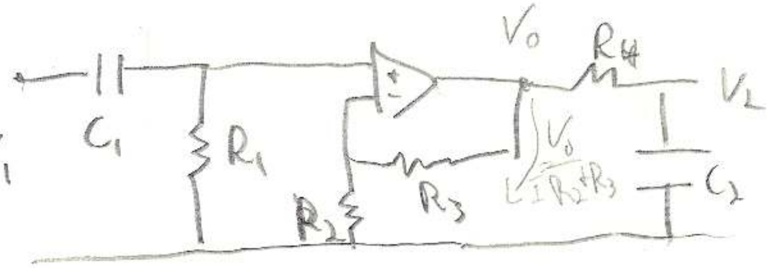


$$V^+ = \frac{R_1}{Ls + \frac{1}{Cs} + R_1} V_1(s) \qquad \text{(Voltage Divider)}$$

$$V^- = \frac{R_2}{R_3 + R_2} V_2(s) \qquad \text{(Voltage Division)}$$

$V^+ = V^-$ gives: $T_1(s) = \frac{V_2}{V_1} = \frac{R_1(R_2+R_3)C\cdot s}{R_2(CLs^2 + CR_1 s + 1)}$

Vendor 2



$I = V_2 \cdot C_2 s$

$$V^+ = \frac{R_1}{R_1 + \frac{1}{C_1 s}} V_1 \; ;$$

$V^+ = V^-$

Gives

$$T_2(s) = \frac{R_1 C_1 (R_2 + R_3) s}{R_2(C_1 R_1 s + 1)(C_2 R_4 s + 1)}$$

$$V_2 + V_2 C_2 s R_4 = V_0$$

$$\frac{V_0 - V^-}{R_3} = \frac{V^-}{R_2}$$

$$\Rightarrow V_0 = V^-\left(\frac{R_3}{R_2} + 1\right)$$

$$\Rightarrow V^- = \frac{V_2(1 + C_2 s R_4)}{\left(\frac{R_3}{R_2} + 1\right)}$$

12.57 Cont: Both denominators can be written in the form $s^2 + 2\zeta\omega_0 s + \omega_0^2$

ω_0

For T_1 $\omega_0 = \sqrt{1/CL} = 5$ Krad/s

T_2 $\omega_0 = \sqrt{1/C_1R_1C_2R_4} = 5.0023$ K rad/s

both meet requirments

B: for T_1: $B = R/L = 10{,}000$ rad/s

T_2: $B = \frac{1}{C_2R_4} - \frac{1}{C_1R_1} = 10{,}074$ rad/s

Passband Gain K

For T_1: Non inverting Amp so $K = 1 + \frac{R_3}{R_2} = 10$

For T_2: $\frac{K_1 \cdot K_2}{\alpha_2} = \frac{R_1C_1(R_2+R_3)}{R_2(C_1R_1)(C_2R_4)} \cdot C_2R_4 = \underline{\underline{11.7}}$

low pass pole

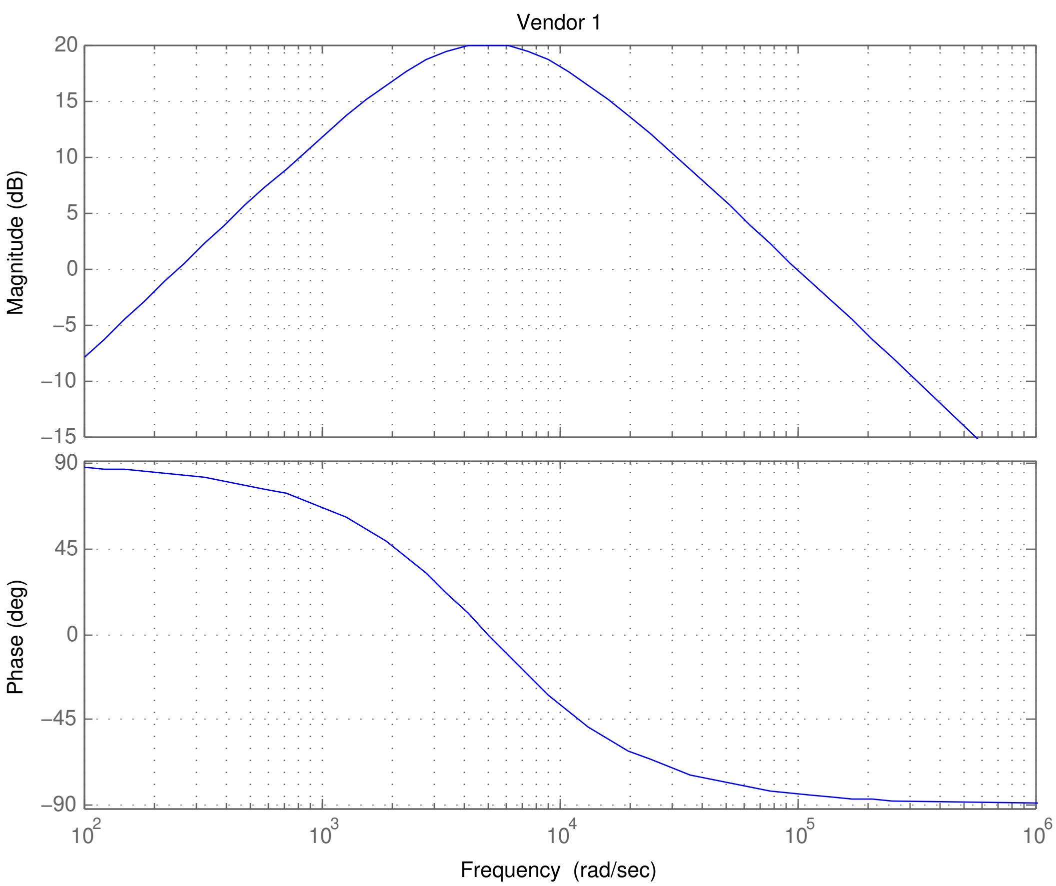
Not Good

For ω_{cL}

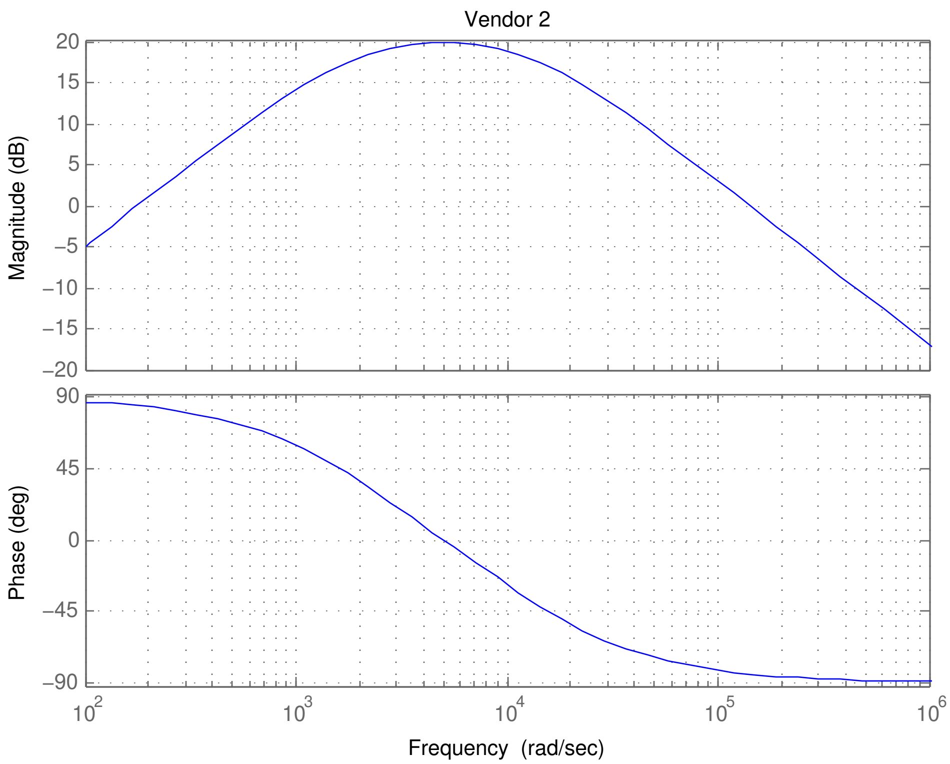
For T_1 $\omega_{cL} = -\frac{R_1}{2L} + \sqrt{\left(\frac{R_1}{2L}\right)^2 + \frac{1}{LC}} = 2{,}071$ rad/s

T_2. $\omega_{cL} = \frac{1}{C_1R_1} = 2{,}062$ rad/s

$$\boxed{\text{Pick Circuit 1, it meets all Criterion. Circuit 2 Fails Passband Gain}}$$



Vendor 1
Magnitude (dB)
Phase (deg)
Frequency (rad/sec)



Vendor 2
Magnitude (dB)
20
15
10
5
0
−5
−10
−15
−20
Phase (deg)
90
45
0
−45
−90
10^2
10^3
10^4
10^5
10^6
Frequency (rad/sec)