MAE 140: Linear Circuits Fall 2012

Solution HW2

PROBLEM 2-30:

a) Voltages and currents

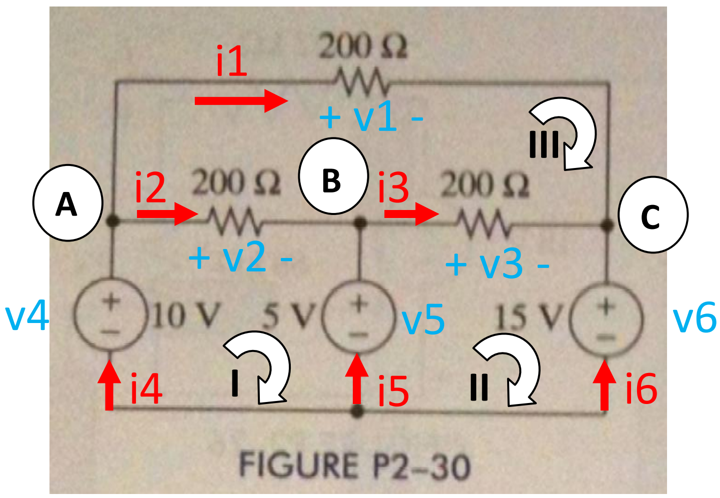


FIGURE P2–30

b) Voltage across each resistor

Loop I:	$-v_4 + v_2 + v_5 = 0$	$v_2 = 5V$
Loop II:	$-v_5 + v_3 + v_6 = 0$	$v_3 = -10V$
Loop III:	$v_1 - v_2 - v_3 = 0$	$v_1 = -5V$

c) Current through each resistor: $i = \frac{v}{R}$

$$i_1 = -\frac{5}{200A} = -25\text{ mA}$$

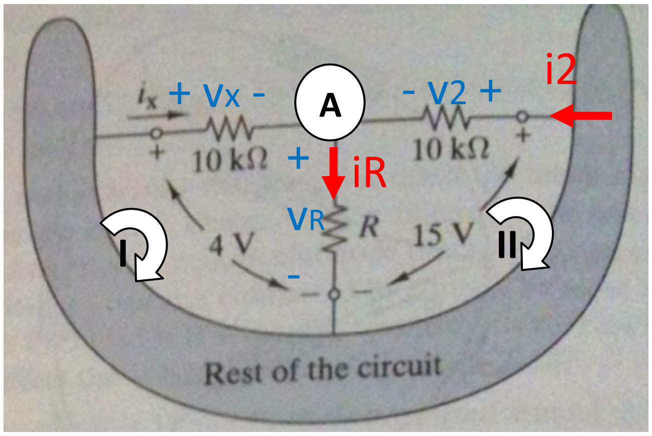
$$i_2 = \frac{5}{200A} = 25\text{ mA}$$

$$i_3 = -\frac{10}{200A} = -50\text{ mA}$$

d) Current through each voltage source

Node A:	$i_4 = i_1 - i_2 = 0$
Node B:	$i_5 = i_3 - i_2 = -75\text{ mA}$
Node C:	$i_6 = -i_1 - i_3 = 75\text{ mA}$

PROBLEM 2-33



NOTE:

There is a typo in the problem, i_x should be -0.5mA. If, i_x=0.5 mA, R cannot be a resistor (see solution below for more details). We will give points for both solutions.

CASE 1: $\mathbf{i_x = -0.5mA}$

$R = \frac{v_R}{i_R}$ **loop I:** $v_R = 4V - v_x = 4V - 10k\Omega * (-0.5mA) = 9V$

$i_R = i_x + i_2$ **loop II:** $v_2 = 15V - v_R = 6V$

$i_2 = \frac{v_2}{10\ k\Omega} = 0.6\ mA$

$i_R = i_x + i_2 = -0.5\ mA + 0.6\ mA = 0.1\ mA$

$R = \frac{v_R}{i_R} = \frac{9V}{0.1mA} = 90\ k\Omega$

CASE 2: $\mathbf{i_x = 0.5mA}$

$R = \frac{v_R}{i_R}$ **loop I:** $v_R = 4V - v_x = 4V - 10k\Omega * 0.5mA = -1V$

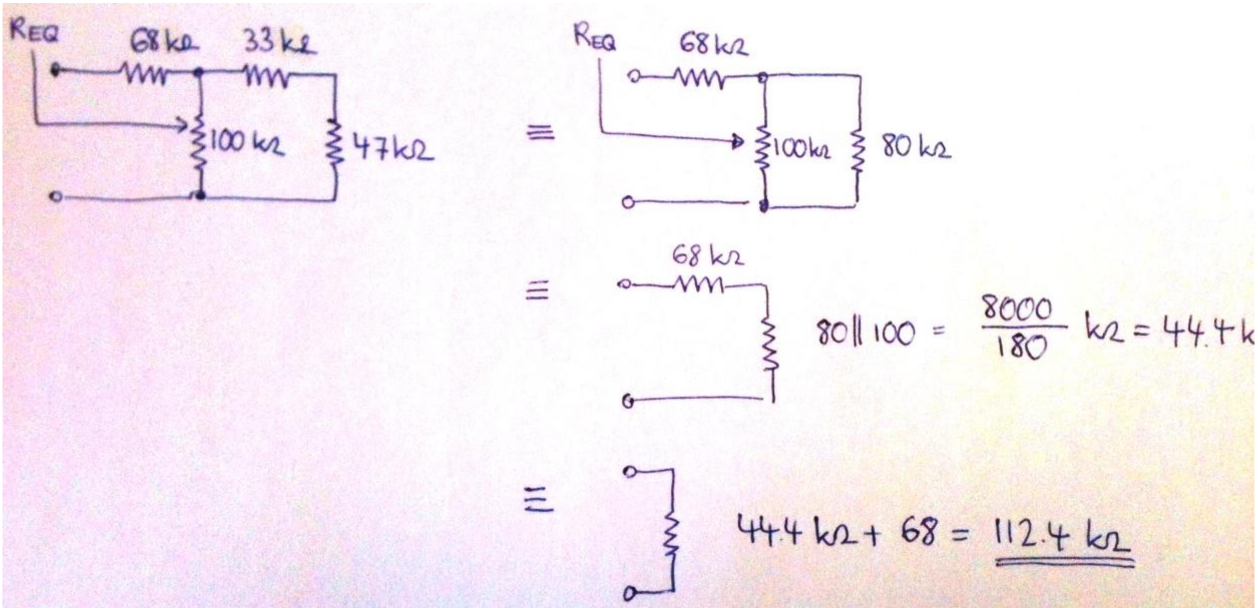
$i_R = i_x + i_2$ **loop II:** $v_2 = 15V - v_R = 16V$

$i_2 = \frac{v_2}{10\ k\Omega} = 1.6\ mA$

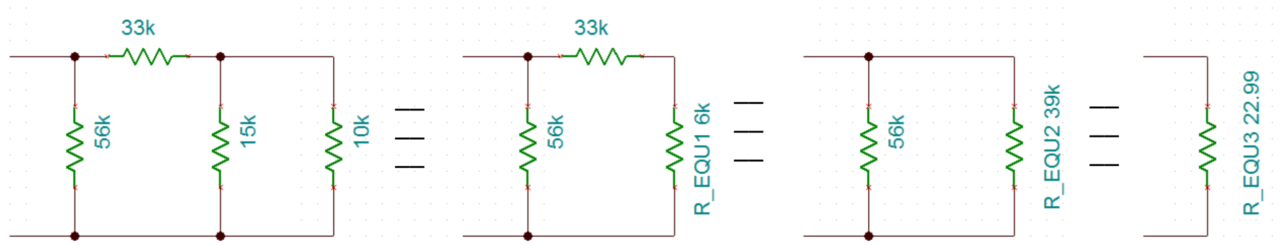
$i_R = i_x + i_2 = 2.1\ mA$

$R = \frac{v_R}{i_R} = \frac{-1V}{2.1mA} = -476.2\Omega$ Note that R cannot be a resistor/passive element since the voltage is negative but we can still compute a resistance!

PROBLEM 2-36



PROBLEM 2-37



$R_{EQU1} = 15k||10k = 6k$

$R_{EQU2} = 33k + R_{EQU1} = 39k$

$R_{EQU1} = 56k||R_{EQU2} = 22.99k$

PROBLEM 2-54

Resistance for which you want to determine the voltage

$$v_x = \frac{4\ k\Omega}{4\ k\Omega + 2k\Omega + 8k\Omega} * 12V = 3.43\ V$$

Total resistance of the loop

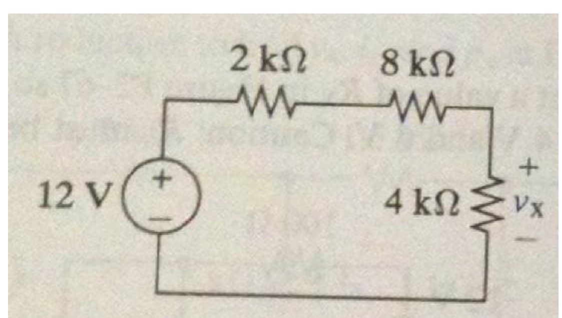
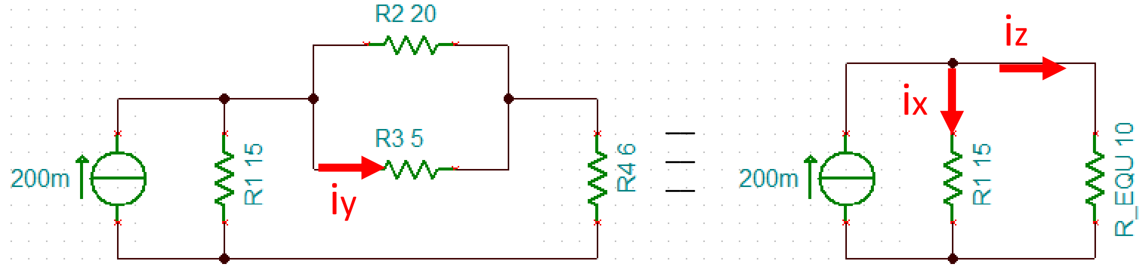


FIGURE P2–54

PROBLEM 2-58

First, we determine $i_x$and $i_z$using the current division rule. For that it is convenient to transform the circuit into an equivalent circuit that has two resistances in parallel.



where $$R_{EQU} = R_2||R_3 + R_4 = \frac{20*5}{20+5} + R_4 = 4 + 6\Omega = 10\ \Omega$$

Using the current division rule we find

$$i_x = \frac{10\ \Omega}{10\Omega+15\Omega} * 200\text{mA} = 0.08\ \text{A}$$

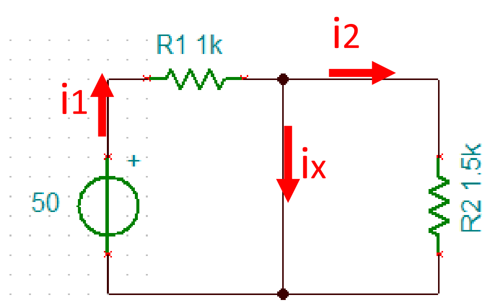
$$i_z = \frac{15\ \Omega}{10\Omega+15\Omega} * 200\text{mA} = 0.12\ \text{A}$$ or using KCL: $i_z = 200\ \text{mA} - i_x$

$i_y$can be determined from $i_z$using the current division rule

$$i_y = \frac{20\ \Omega}{20\Omega+5\Omega} * 120\text{mA} = 0.096\ \text{A} = 96\ \text{mA}$$

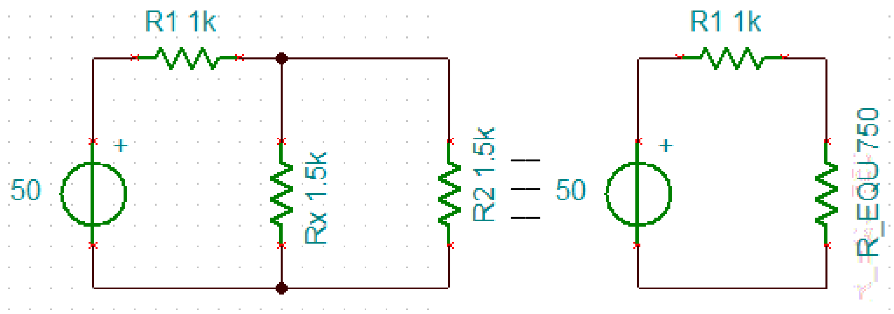
PROBLEM 2-61

There are two extreme cases: R = 0 Ω and R = 1500 Ω



(A) **R=0Ω:** If R=0, the current through R2 will be zero and ix = i1 (the current follows the path with the smallest resistance!).
Consequently, v2 will be zero as well.

(B) R=1500Ω:
Note that Rx and R2 are in parallel and thus, the voltage across both elements is equal. To calculate v2 we proceed as follows: We first define an equivalent resistance and then calculate the voltage across element R2 using the voltage division rule:



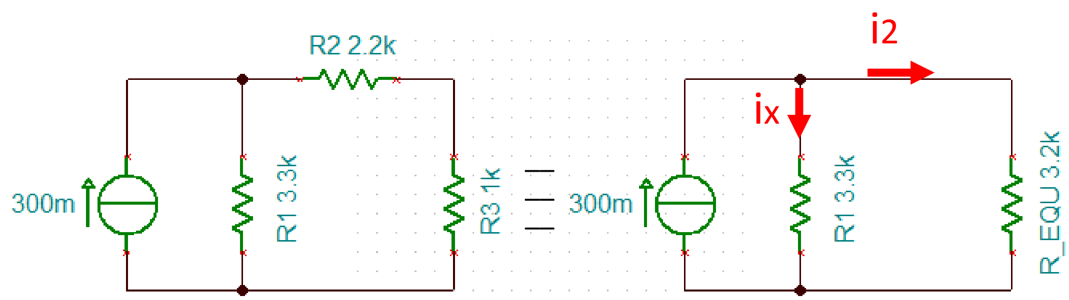
$$R_{EQU} = R_x||R_2 = \frac{1.5 * 1.5}{1.5 + 1.5} = 750\ \Omega$$

$$v_2 = \frac{750\ \Omega}{1000\Omega + 750\Omega} * 50V = 21.43\ V$$

Solution: v_o ranges from 0 V to 21.43 V.

PROBLEM 2-68

Note that R_2 and R_3 are in series. As a consequence the current through both resistances is equal. Thus we can replace R_2 and R_3 with an equivalent resistance R_{EQU}.



i_2 can be determined using the current division rule

$$i_2 = \frac{3.3k\Omega}{3.3k\Omega+3.2k\Omega} * 300mA = 152.3\ mA$$

Using KCL we obtain for i_x

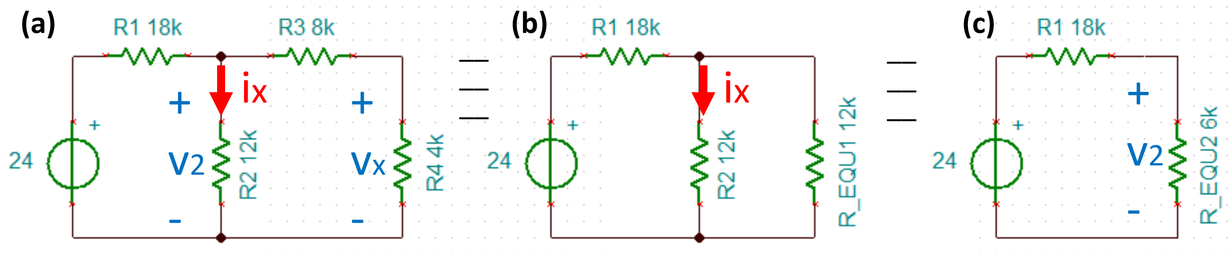
$$i_x = 300\ mA - i_2 = 147.7\ mA$$

Thus v_x can be calculated via Ohm's law

$$v_x = Ri_x = 3300\ \Omega * .1477A = 487.4\ V$$

PROBLEM 2-72

First, we determine v_x using the voltage division rule. Note that the voltage v_2 over resistor R2 is equal to the voltage over resistor R3 and R4 together. We therefore reduce the circuit as follows:



Using the voltage division rule we obtain for v_2:

$$v_2 = \frac{6k\Omega}{6k\Omega + 18k\Omega} * 24V = 6\ V$$

v_x follows from v_2 apllying the voltage division rule one more time:

$$v_x = \frac{4k\Omega}{4k\Omega + 8k\Omega} * v_2 = \frac{1}{3} * 6V = 2V$$

To calculate i_x we use the current division rule. First we need to know the current through the voltage source

$$i = \frac{24V}{R_{total}} = \frac{24V}{18k\Omega + 6k\Omega} = \frac{24}{24} mA = 1\ mA$$

Using current division rule we obtain

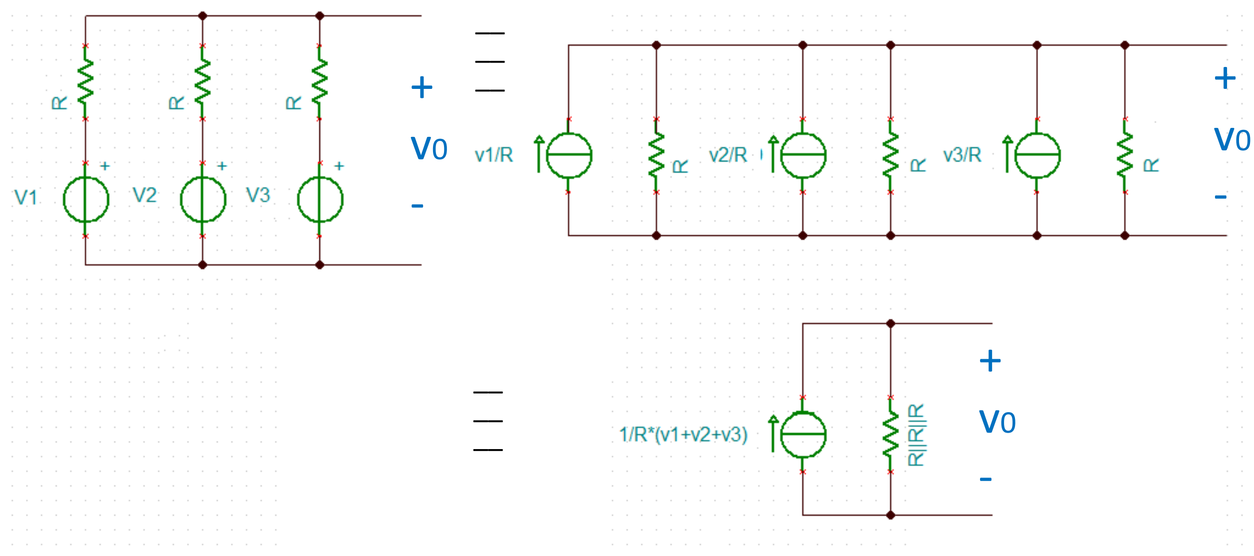
$$i_x = \frac{12k\Omega}{12k\Omega + 12k\Omega} * 1mA = 0.5\ mA$$

Alternatively, we can calculate i_x from v_x using Ohm's law

$$i_x = \frac{v_2}{12k\Omega} = \frac{6V}{12k\Omega} = 0.5\ mA$$

PROBLEM 2-75

First we transform each voltage source into a current source. As a second step replace the three current sources with a single equivalent current source by adding up the currents and computing the equivalent resistance.



The equivalent resistor value of all resistors that are in series with the voltage sources is R1||R2||R3 that is

$$R_{EQU} = R||R||R = \frac{1}{R} + \frac{R}{2} = \frac{R}{3}$$

The voltage v_0 can be computed via Ohm's law

$$v_x = R_{EQU} * i = \frac{R}{3} * \frac{v_1 + v_2 + v_3}{R} = \frac{v_1 + v_2 + v_3}{3}$$