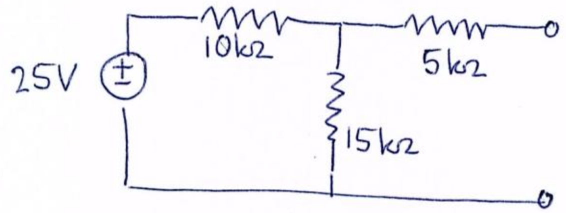
MAE 140: HW3 solutions

fall 2012

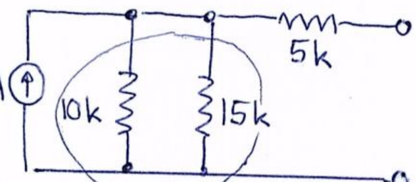
3.47, 3.52, 3.58, 3.1, 3.3, 3.4, 3.5, 3.13, 3.17 b)+c), 3.22 b)+d)

3.47 Find its Thevenin and Norton equivalent circuit



↓ source transformation

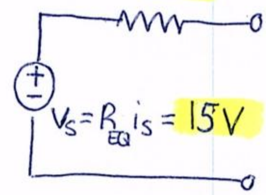
$i_S = \frac{25V}{10k\Omega} = 2.5\,mA$



$R_{EQ} = 10k \| 15k = 6k\Omega$

≡

$6k + 5k = 11\,k\Omega$

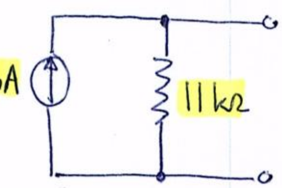


$V_S = R_{EQ}\, i_S = 15V$

Thevenin equivalent circuit

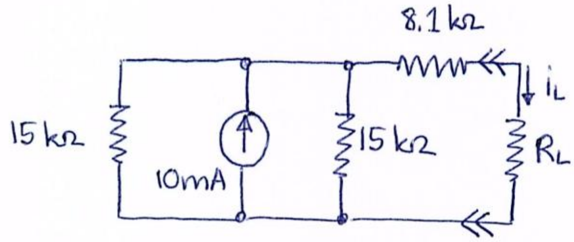
≡

$i_S = \frac{15V}{11k\Omega} = 1.36\,mA$

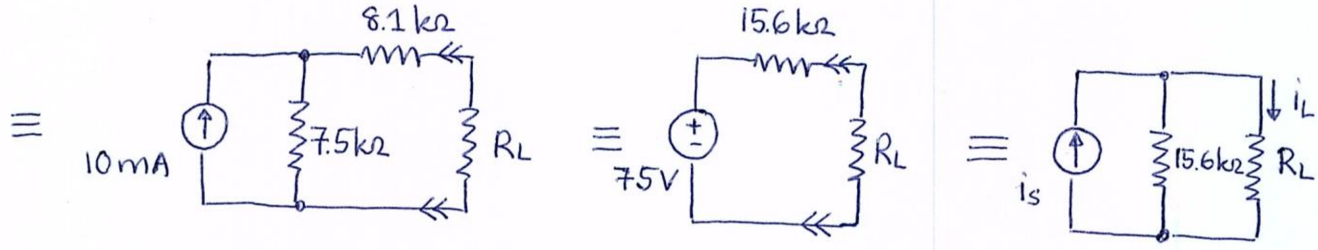


Norton equivalent circuit

3.52 Norton equivalent circuit and current i_L



Norton equivalent circuit



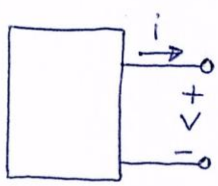
$$i_s = \frac{75V}{15.6k\Omega} = 4.8mA$$

Current i_L via current division

$$i_L = \frac{15.6\,k\Omega}{15.6k\Omega + R_L}\, i_s$$

$i_L(R_L = 4.7\,k\Omega) = 3.69\ mA$
$i_L(R_L = 15\,k\Omega) = 2.45\ mA$
$i_L(R_L = 68\,k\Omega) = 0.99\ mA$

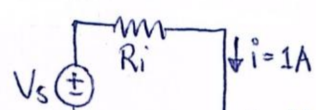
3.58



- $i = 1A$ for $V = 0V$ — case A
- $V = 12V$ when connected to 20Ω resistor — case B
- current when charging a 12V battery ?

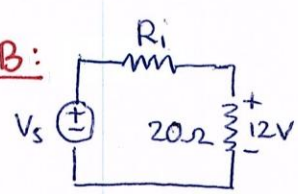
Use Thevenin equivalent circuit

case A:



$$i = \frac{V_s}{R_i}$$

case B:



voltage division

$$12V = \frac{20\Omega}{20\Omega + R_i} \cdot V_s$$

$$\rightarrow 12V = \frac{20\Omega}{20\Omega + R_i} \cdot V_s \qquad V_s = 1A \cdot R_i$$

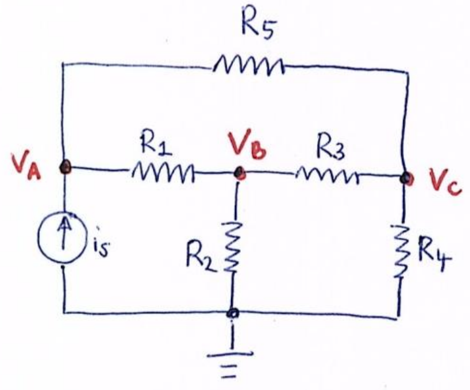
$$12V \cdot 20\frac{V}{A} + 12V \cdot R_i = 20\frac{V}{A} \cdot 1A \cdot R_i$$

$$240\frac{V^2}{A} = 8V \cdot R_i$$

$$R_i = 30\Omega \quad \rightarrow \quad V_s = 30V$$

$$i = \frac{V_s}{R_{total}} = \frac{30V}{30\Omega + 20\Omega} = \underline{\underline{\frac{3}{5}A}}$$

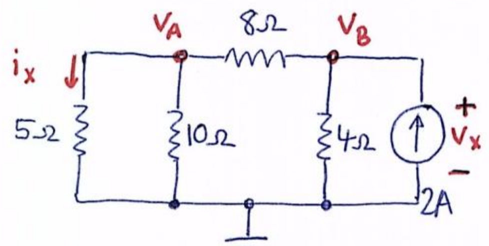
3.1 Node-voltage equations



By inspection we can write

$$\begin{bmatrix} \frac{1}{R_1}+\frac{1}{R_5} & -\frac{1}{R_1} & -\frac{1}{R_5} \\ -\frac{1}{R_1} & \frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3} & -\frac{1}{R_3} \\ -\frac{1}{R_5} & -\frac{1}{R_3} & \frac{1}{R_3}+\frac{1}{R_4}+\frac{1}{R_5} \end{bmatrix} \begin{Bmatrix} V_A \\ V_B \\ V_C \end{Bmatrix} = \begin{Bmatrix} i_s \\ 0 \\ 0 \end{Bmatrix}$$

3.3 Node-voltage equations



node A: $\left(\frac{1}{5\Omega}+\frac{1}{10\Omega}\right)V_A - \frac{1}{8\Omega}(V_A - V_B) = 0$

node B: $-\frac{1}{8\Omega}(V_B - V_A) - \frac{1}{4\Omega}V_B + 2A = 0$

By inspection we can write

$$\begin{bmatrix} \frac{1}{5\Omega}+\frac{1}{10\Omega}+\frac{1}{8\Omega} & -\frac{1}{8\Omega} \\ -\frac{1}{8\Omega} & \frac{1}{8\Omega}+\frac{1}{4\Omega} \end{bmatrix} \begin{Bmatrix} V_A \\ V_B \end{Bmatrix} = \begin{bmatrix} 0 \\ 2A \end{bmatrix} \iff \underset{A}{\begin{bmatrix} 0.425 & -0.125 \\ -0.125 & 0.375 \end{bmatrix}} \underset{x}{\begin{Bmatrix} V_A \\ V_B \end{Bmatrix}} = \underset{b}{\begin{Bmatrix} 0 \\ 2 \end{Bmatrix}}$$

$$V_A = \frac{\begin{vmatrix} 0 & -0.125 \\ 2 & 0.375 \end{vmatrix}}{|A|} = \frac{0.25}{0.14375} = \underline{\underline{1.74 \text{ V}}}$$

$$i_x = \frac{V_A}{5\Omega} = \underline{\underline{347.8\text{mA}}}$$

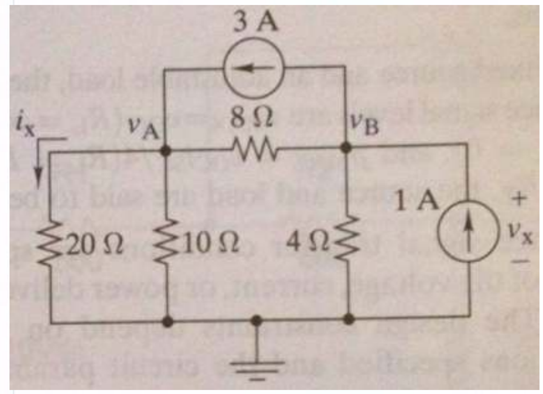
$$V_B = \frac{\begin{vmatrix} 0.425 & 0 \\ -0.125 & 2 \end{vmatrix}}{|A|} = \frac{0.85}{0.14375} = \underline{\underline{5.91 \text{ V}}}$$

$$V_x = V_B = \underline{\underline{5.91 \text{ V}}}$$

3.4 Node voltage equations

$$\begin{bmatrix} \frac{1}{20\Omega}+\frac{1}{10\Omega}+\frac{1}{8\Omega} & -\frac{1}{8\Omega} \\ -\frac{1}{8\Omega} & \frac{1}{4\Omega}+\frac{1}{8\Omega} \end{bmatrix} \begin{Bmatrix} V_A \\ V_B \end{Bmatrix} = \begin{Bmatrix} 3A \\ -3A+1A \end{Bmatrix} \Leftrightarrow \underbrace{\begin{bmatrix} 0.275 & -0.125 \\ -0.125 & 0.375 \end{bmatrix}}_{A} \underbrace{\begin{Bmatrix} V_A \\ V_B \end{Bmatrix}}_{x} = \underbrace{\begin{Bmatrix} 3. \\ -2 \end{Bmatrix}}_{b}$$

$$V_A = \frac{\begin{vmatrix} 3 & -0.125 \\ -2 & 0.375 \end{vmatrix}}{|A|} = \frac{0.875}{0.0875} = \underline{\underline{10\ V}}$$

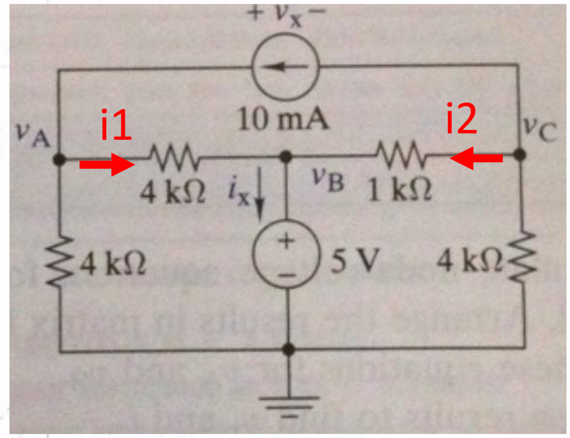


$$V_B = \frac{\begin{vmatrix} 0.275 & 3 \\ -0.125 & -2 \end{vmatrix}}{|A|} = \frac{-0.175}{0.0875} = \underline{\underline{-2\ V}}$$

$$i_x = \frac{V_A}{20\Omega} = \underline{\underline{0.5\,A}}$$

$$V_x = V_B = \underline{\underline{-2\,V}}$$

3.5 Node-voltage analysis



The circuit contains a voltage source! Since one terminal of the source is connected to ground we know that $V_B = V_{source} = 5V$.

Using method 2 we can write by inspection

$$\begin{array}{l} \\ \text{node A} \\ \text{node C} \end{array} \begin{array}{c} \begin{array}{ccc} V_A & \qquad V_B & \qquad V_C \end{array} \\ \begin{bmatrix} \frac{1}{4k\Omega}+\frac{1}{4k\Omega} & -\frac{1}{4k\Omega} & 0 \\ 0 & -\frac{1}{4k\Omega} & \frac{1}{1k\Omega}+\frac{1}{4k\Omega} \end{bmatrix} \end{array} \begin{Bmatrix} V_A \\ V_B \\ V_C \end{Bmatrix} = \begin{Bmatrix} 10mA \\ -10mA \end{Bmatrix}$$

Rearrange

$$\begin{bmatrix} \frac{1}{2k\Omega} & 0 \\ 0 & \frac{5}{4k\Omega} \end{bmatrix} \begin{Bmatrix} V_A \\ V_C \end{Bmatrix} = \begin{Bmatrix} 11.25\,mA \\ -5mA \end{Bmatrix} \rightarrow \begin{array}{l} V_A = \underline{\underline{22.5V}} \\ V_B = \underline{\underline{-4V}} \end{array}$$

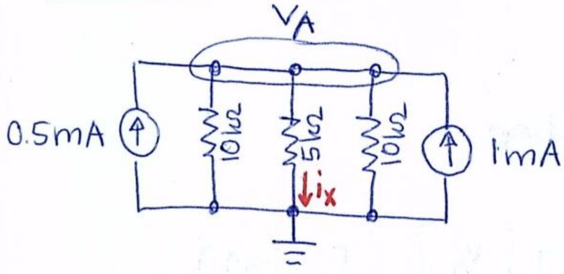
$$V_x = V_A - V_C = \underline{\underline{26.5\,V}}$$

$$i_x = i_1 + i_2 = \frac{1}{4k\Omega}(V_A - V_B) + \frac{1}{1k\Omega}(V_C - V_B) = \frac{1}{4k\Omega}\,17.5V + \frac{1}{1k\Omega}(-9\Omega) = \underline{\underline{-4.625\,mA}}$$

3.13 Current voltage analysis

You can use method 1 or method 2.

Method 1: Source transformation



$$\left(\frac{1}{10k\Omega}+\frac{1}{5k\Omega}+\frac{1}{10k\Omega}\right)V_A = 0.5mA + 1mA$$

$$\frac{4}{10k\Omega}V_A = 1.5mA$$

$$V_A = \underline{\underline{3.75V}}$$

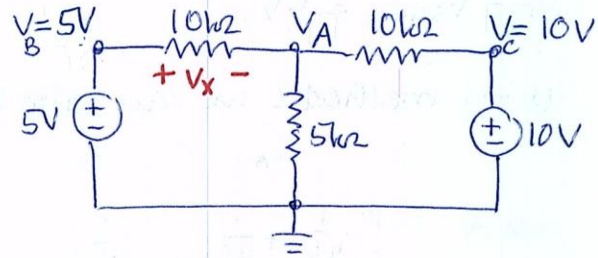
$$i_x = \frac{1}{5k\Omega}\cdot V_A = 0.75mA = \underline{\underline{750\mu A}}$$

$$V_x = 5V - V_A = 5V - 3.75V = \underline{\underline{1.25V}}$$

$$i_A = \frac{1}{10k\Omega}(V_B - V_A) = \frac{1}{10k\Omega}1.25V = \underline{\underline{125\mu A}}$$

$$i_B = \frac{1}{10k\Omega}(V_A - V_C) = \frac{1}{10k\Omega}\cdot(-6.25V) = \underline{\underline{-625\mu A}}$$

Method 2:



$$\left[\frac{1}{10k\Omega}+\frac{1}{5k\Omega}+\frac{1}{10k\Omega} \quad -\frac{1}{10k\Omega} \quad -\frac{1}{10k\Omega}\right]\begin{Bmatrix} V_A \\ V_B \\ V_C \end{Bmatrix} = 0$$

$$\Leftrightarrow \frac{4}{10k\Omega}V_A = 1.5mA$$

...

3.17 b) Select node D as the reference node $\rightarrow V_A = V_S$

By inspection we obtain

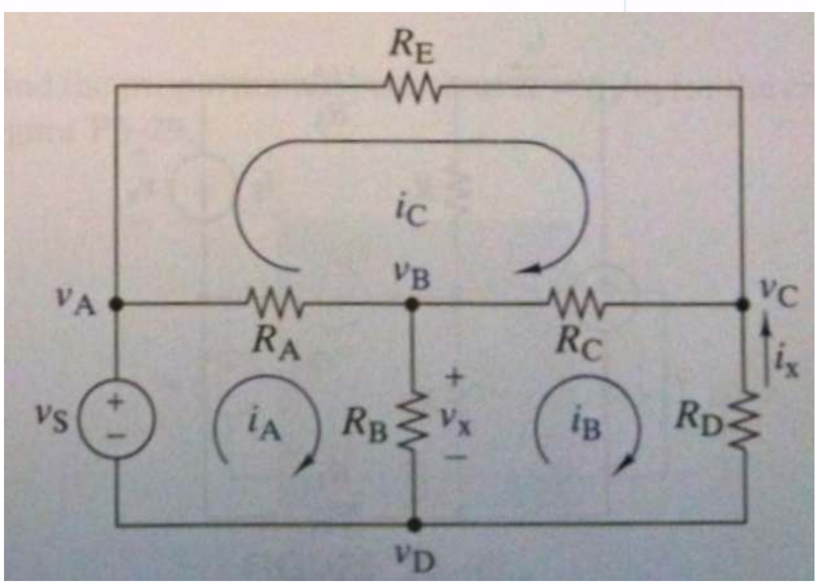
$$\begin{array}{c|c} & \begin{array}{ccc} V_A & V_B & V_C \end{array} \\ \begin{array}{c} \text{node B} \\ \text{node C} \end{array} & \begin{bmatrix} -\frac{1}{R_A} & \frac{1}{R_A}+\frac{1}{R_B}+\frac{1}{R_C} & -\frac{1}{R_C} \\ -\frac{1}{R_E} & -\frac{1}{R_C} & \frac{1}{R_C}+\frac{1}{R_D}+\frac{1}{R_E} \end{bmatrix} \end{array} \begin{Bmatrix} V_A \\ V_B \\ V_C \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \qquad V_A = V_S$$

Rarrange

$$\begin{bmatrix} \frac{1}{R_A}+\frac{1}{R_B}+\frac{1}{R_C} & -\frac{1}{R_C} \\ -\frac{1}{R_C} & \frac{1}{R_C}+\frac{1}{R_D}+\frac{1}{R_E} \end{bmatrix} \begin{Bmatrix} V_B \\ V_C \end{Bmatrix} = \begin{Bmatrix} \frac{V_S}{R_A} \\ \frac{V_S}{R_E} \end{Bmatrix}$$

e) $$V_x = V_B = \frac{\begin{vmatrix} \frac{V_S}{R_A} & -\frac{1}{R_C} \\ \frac{V_S}{R_E} & \frac{1}{R_C}+\frac{1}{R_D}+\frac{1}{R_E} \end{vmatrix}}{\begin{vmatrix} \frac{1}{R_A}+\frac{1}{R_B}+\frac{1}{R_C} & -\frac{1}{R_C} \\ -\frac{1}{R_C} & \frac{1}{R_C}+\frac{1}{R_D}+\frac{1}{R_E} \end{vmatrix}}$$

$$i_x = \frac{1}{R_D}\cdot(-V_C) = -\frac{1}{R_D}\cdot \frac{\begin{vmatrix} \frac{1}{R_A}+\frac{1}{R_B}+\frac{1}{R_C} & \frac{V_S}{R_A} \\ -\frac{1}{R_C} & \frac{V_S}{R_E} \end{vmatrix}}{\begin{vmatrix} \frac{1}{R_A}+\frac{1}{R_B}+\frac{1}{R_C} & -\frac{1}{R_C} \\ -\frac{1}{R_C} & \frac{1}{R_C}+\frac{1}{R_D}+\frac{1}{R_E} \end{vmatrix}}$$



3.22 b) + d) Node-voltage equation

b) NOTE: we only have to look at node B since V_A and V_C are determined by the voltage sources.

$V_A = 20\,V$

$V_C = -15\,V$

For node B we can write

$$3mA - \frac{1}{2k\Omega}(V_B - 20V) - \frac{1}{4k\Omega}V_B - \frac{1}{8k\Omega}(V_B + 15V) = 0$$

$$\left(\frac{1}{2k\Omega} + \frac{1}{4k\Omega} + \frac{1}{8k\Omega}\right) V_B = 3mA + 10mA - \frac{15}{8}mA$$

$$\frac{0.875}{1000} V_B = 11.125\,mA$$

$$V_B = \underline{\underline{12.71\,V}}$$

d) $i_x = \frac{1}{8k\Omega}(V_C - V_B) = \frac{1}{8k\Omega}(-27.71\,V) = \underline{\underline{-3.46V}}$

$V_x = V_B = \underline{\underline{12.71\,V}}$