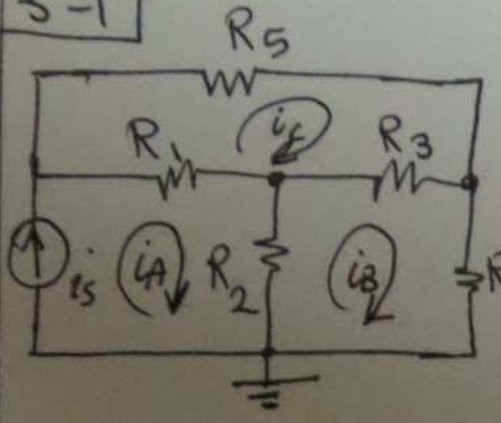
MAE 140 – Linear Circuits – Fall 2012

Homework #4 – Solutions

3-1



We drop the row associated to loop (A) and write the equations for the other two loops by inspection:

$$\begin{pmatrix} \times & \times & \times \\ -R_2 & R_2+R_3+R_4 & -R_3 \\ -R_1 & -R_3 & R_1+R_3+R_5 \end{pmatrix} \begin{pmatrix} i_A \\ i_B \\ i_C \end{pmatrix} = \begin{pmatrix} \times \\ 0 \\ 0 \end{pmatrix}$$

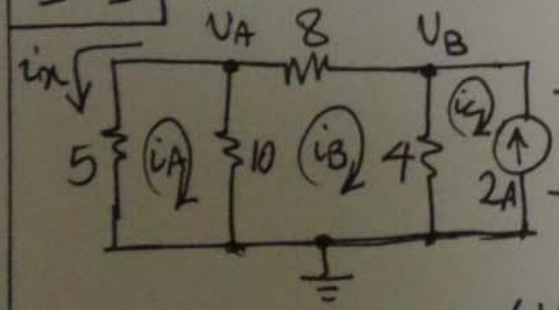
for loop Ⓐ we have:

$$i_A = i_s$$

We can also write $i_A = i_s$ in the matrix form $\begin{pmatrix} 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} i_A \\ i_B \\ i_C \end{pmatrix} = \begin{pmatrix} i_s \\ 0 \\ 0 \end{pmatrix}$ and insert it into the matrix form above

$$\Rightarrow \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ -R_2 & R_2+R_3+R_4 & -R_3 \\ -R_1 & -R_3 & R_1+R_3+R_5 \end{pmatrix}}_{A} \underbrace{\begin{pmatrix} i_A \\ i_B \\ i_C \end{pmatrix}}_{\underline{x}} = \underbrace{\begin{pmatrix} i_s \\ 0 \\ 0 \end{pmatrix}}_{\underline{b}} \Rightarrow A\underline{x} = \underline{b}$$

3-3 (a)



$$\begin{matrix} \text{Ⓐ} \\ \text{Ⓑ} \end{matrix} \begin{pmatrix} 5+10 & -10 & 0 \\ -10 & 10+8+4 & -4 \end{pmatrix} \begin{pmatrix} i_A \\ i_B \\ i_C \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

Ⓒ: $i_C = -2\,A$

Gaussian Elimination:

$$\left(\begin{array}{ccc|c} 15 & -10 & 0 & 0 \\ -10 & 22 & -4 & 0 \end{array}\right) \xrightarrow{\frac{2}{3}\text{row1}+\text{row2}} \left(\begin{array}{ccc|c} 15 & -10 & 0 & 0 \\ 0 & \frac{46}{3} & -4 & 0 \end{array}\right)$$

$$\Rightarrow \frac{46}{3} i_B - 4 i_C = 0 \Rightarrow i_B = \frac{3\times 4}{46} i_C = \frac{3\times 4}{46}(-2) = -\frac{12}{23}\,A$$

$$15 i_A - 10 i_B = 0 \Rightarrow i_A = \frac{2}{3} i_B = -\frac{8}{23}\,A$$

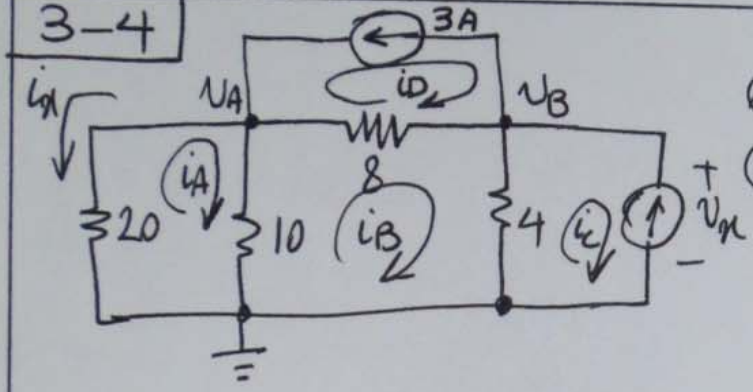
(b) $v_A = 10(i_A - i_B) = 10\left(\frac{-8}{23} + \frac{18}{23}\right) = \frac{40}{23} = 1.7391\ V$

$v_B = 4(i_B - i_C) = 4\left(-\frac{12}{23} + 2\right) = \frac{136}{23} = 5.913\ V$

(c) $v_x = v_B = 5.913\ V$

$i_x = -i_A = \frac{8}{23} = 0.347826\ A = 347.826\ mA$

3-4



$$\begin{pmatrix} 20+10 & -10 & 0 & 0 \\ -10 & 10+4+8 & -4 & -8 \end{pmatrix} \begin{pmatrix} i_A \\ i_B \\ i_C \\ i_D \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$i_C = -1\ A$

$i_D = -3\ A$

Gaussian Elimination $\Rightarrow$ $i_A = -0.5A$, $i_B = -1.5A$

(b) $v_A = -20\, i_A = -20(-0.5) = 10V$

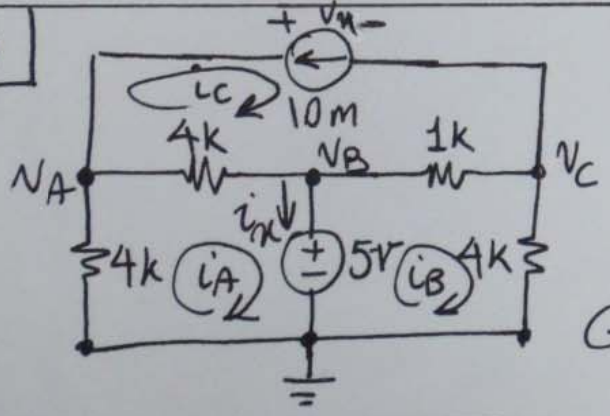
$v_B = 4(i_B - i_C) = 4(-1.5+1) = -2\ V$

(c) $v_x = v_B = -2\ V$

$i_x = -i_A = 0.5A$

3-5

(a)



$$\begin{pmatrix} 4+4 & 0 & -4 \\ 0 & 1+4 & -1 \end{pmatrix} \begin{pmatrix} i_A \\ i_B \\ i_C \end{pmatrix} = \begin{pmatrix} -5 \\ 5 \end{pmatrix}$$

$i_C = -10\ mA$

G.E. $\Rightarrow$ $i_A = -5.625mA$

$i_B = -1\ mA$

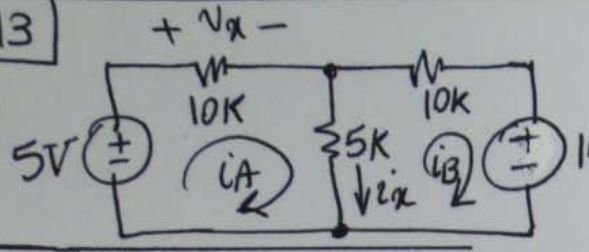
(b) $v_A = -4\, i_A = 22.5\ V$

$v_C = 4\, i_B = -4\ V$

(c) $v_x = v_A - v_C = 22.5+4 = 26.5\ V$, $i_x = i_A - i_B = -4.625mA$

3-13

(a)



$$\begin{pmatrix} 10+5 & -5 \\ -5 & 10+5 \end{pmatrix}\begin{pmatrix} i_A \\ i_B \end{pmatrix} = \begin{pmatrix} 5 \\ -10 \end{pmatrix}$$

G.E. $\Rightarrow i_A = \frac{1}{8}\,mA$, $i_B = -\frac{5}{8}\,mA$

$= 125\,\mu A$ $= -625\,\mu A$

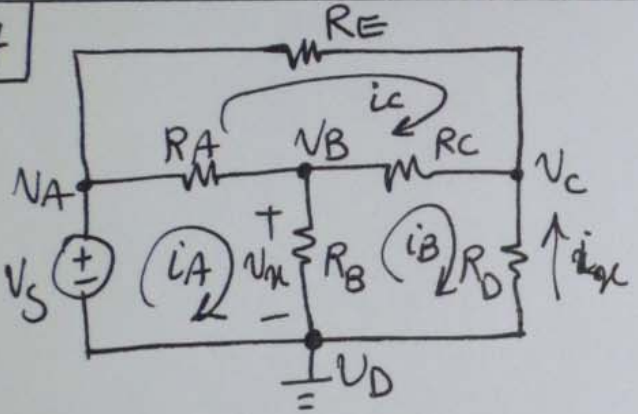
(b)

(c) $v_x = 10 i_A = 10 \times \frac{1}{8} = \frac{5}{4} = \underline{1.25\,V}$

$i_x = i_A - i_B = \frac{1}{8} + \frac{5}{8} = \frac{3}{4}\,mA = 750\,\mu A$

3-17

(a)



$$\begin{pmatrix} R_A+R_B & -R_B & -R_A \\ -R_B & R_B+R_C+R_D & -R_C \\ -R_A & -R_C & R_A+R_E+R_C \end{pmatrix}\begin{pmatrix} i_A \\ i_B \\ i_C \end{pmatrix} = \begin{pmatrix} v_S \\ 0 \\ 0 \end{pmatrix}$$

(b) (B)

(C)

$$\begin{pmatrix} -\frac{1}{R_A} & \frac{1}{R_A}+\frac{1}{R_B}+\frac{1}{R_C} & -\frac{1}{R_C} \\ -\frac{1}{R_E} & -\frac{1}{R_C} & \frac{1}{R_C}+\frac{1}{R_D}+\frac{1}{R_E} \end{pmatrix}\begin{pmatrix} v_A \\ v_B \\ v_C \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$v_A = v_S$

(c) Node-Voltag Analysis:

number of nodes (excluding ground) = 3

number of voltage sources = 1

$\Rightarrow$ 3−1 = 2 unknowns → 2 equations required

Mesh-Current Analysis:

number of meshes = 3

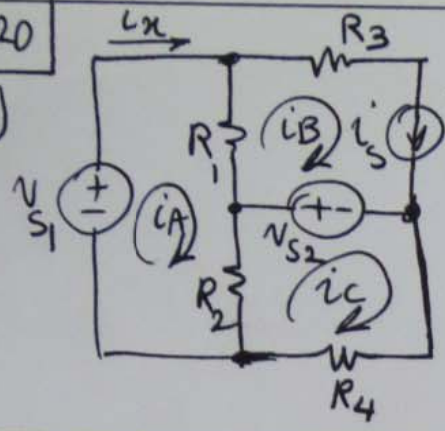
number of current sources = 0

$\Rightarrow$ 3−0 = 3 unknowns → 3 equations required

$\Rightarrow$ Node-voltage analysis is easier to solve because there are fewer unknowns / equations

3-20

(a)



$$\textcircled{A}\begin{pmatrix} R_1+R_2 & -R_1 & -R_2 \\ -R_2 & 0 & R_2+R_4 \end{pmatrix}\begin{pmatrix} i_A \\ i_B \\ i_C \end{pmatrix}=\begin{pmatrix} v_{S_1} \\ -v_{S_2} \end{pmatrix}$$

$$\textcircled{C}\quad i_B = i_S$$

(b) Note: It is easier to solve this problem by node-voltage analysis, because there is only one unknown.

(c) $R_1=R_2=5.6k$, $R_3=2.2k$, $R_4=1.5k$,
$i_S=3mA$, $v_{S_1}=15V$, $v_S=5V$

G.E. $\Rightarrow i_A=4.11mA$, $i_B=3mA$, $i_C=2.53\,mA$

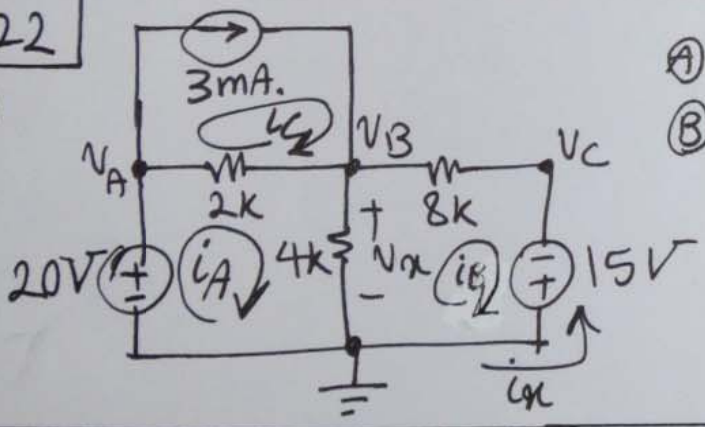
$i_x=i_A=\boxed{4.11\,mA}$

$v_x=v_{S_2}-R_1(i_B-i_A)-R_3 i_B = 5-5.6(3-4.11)-2.2\times3=\boxed{4.6V}$

(d) $p=v_{S_1}\cdot i_x=15\times4.11=61.6W$

3.22

(a)



$$\textcircled{A}\begin{pmatrix} 2+4 & -4 & -2 \\ -4 & 4+8 & 0 \end{pmatrix}\begin{pmatrix} i_A \\ i_B \\ i_C \end{pmatrix}=\begin{pmatrix} 20 \\ 15 \end{pmatrix}$$

$i_C=3\,mA$

(b) $$\textcircled{B}\quad \begin{pmatrix} -\frac{1}{2} & \frac{1}{2}+\frac{1}{4}+\frac{1}{8} & -\frac{1}{8} \end{pmatrix}\begin{pmatrix} v_A \\ v_B \\ v_C \end{pmatrix}=(3)$$

$v_A=20\,V$

$v_C=-15\,V$

(c) Node-Voltag:

number of nodes (excluding ground) = 3

number of voltage sources = 2

$\Rightarrow$ number of unknown/equations = 3 − 2 = 1

mesh-current:

number of meshes = 3

number of current sources = 1

$\Rightarrow$ number of unknowns / equations = 3 − 1 = 2

$\Rightarrow$ Node-Voltage equations are easier to solve

(d) Part (b): $-\frac{1}{2}v_A + (\frac{1}{2}+\frac{1}{4}+\frac{1}{8})v_B - \frac{1}{8}v_C = 3$

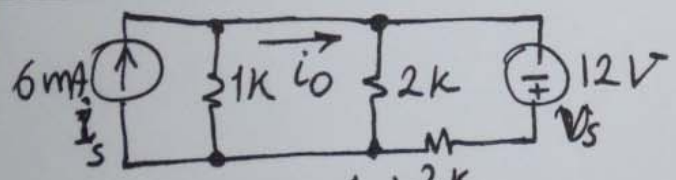
$\Rightarrow -\frac{1}{2}(20)\ \frac{7}{8}v_B - \frac{1}{8}(-15) = 3$

$\Rightarrow v_B = \frac{89}{7} = 12.71\ V$

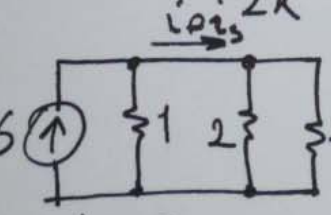
$v_x = v_B = 12.71\ V$

$i_x = \frac{v_C - v_B}{8} = \frac{-15 - \frac{89}{7}}{8} = -\frac{194}{7\times 8} = -3.46mA$

3-37



response to i_s: (turnoff V_s)



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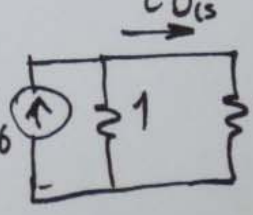
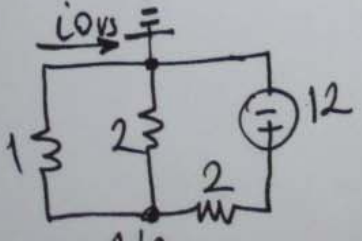


$\Rightarrow i_{o_{is}} = 3mA$

response to V_s: (turn off i_s)



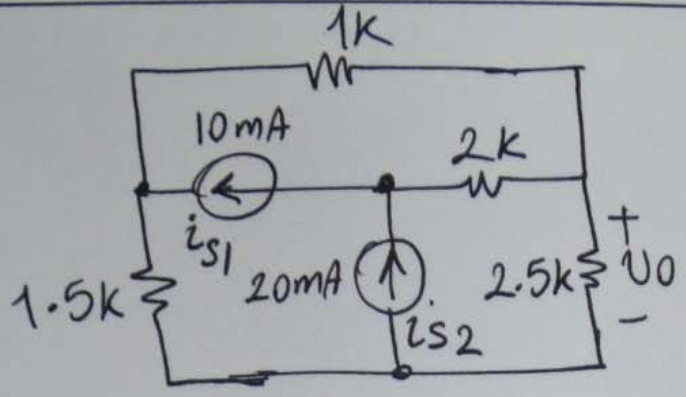
KCL at (A): $\frac{V_A - 12}{2} + \frac{V_A}{2} + \frac{V_A}{1} = 0$

$\Rightarrow V_A = 3$

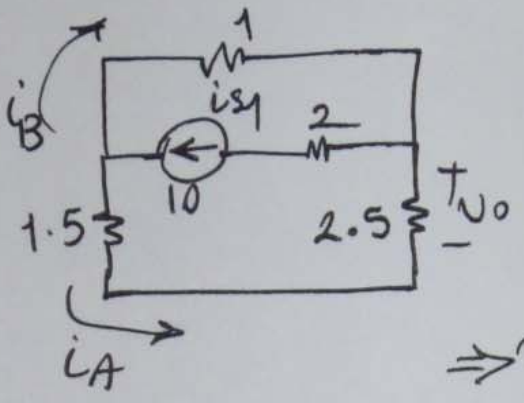
$\Rightarrow i_{o_{V_s}} = \frac{V_A}{1} = 3mA$

$$\Rightarrow i_o = i_{o_{is}} + i_{o_{vs}} = 3+3 = \boxed{6\ mA}$$

3-39



response to i_{s1}: (turn off i_{s2})

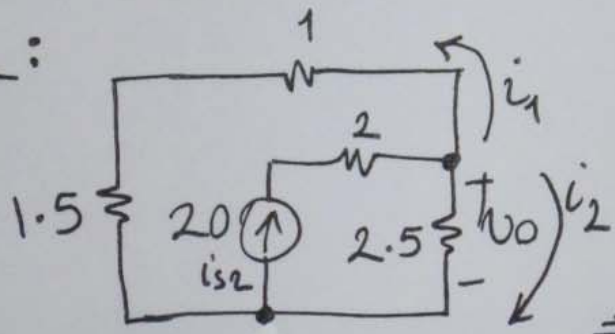


current division rule:

$$i_A' = 10\,\frac{1}{1.5+2.5+1} = 2\ mA$$

$$\Rightarrow v_{o_{is_1}} = -i_A' \times 2.5 = -5\ V$$

response to i_{s2}: (turn off i_{s1})



current division rule:

$$i_2' = 20\,\frac{1+1.5}{1+1.5+2.5} = 10$$

$$\Rightarrow v_{o_{is_2}} = 2.5\,i_2' = 25\ V$$

$$\Rightarrow v_o = v_{o_{is_1}} + v_{o_{is_2}} = -5+25 = \boxed{20\ V}$$

End