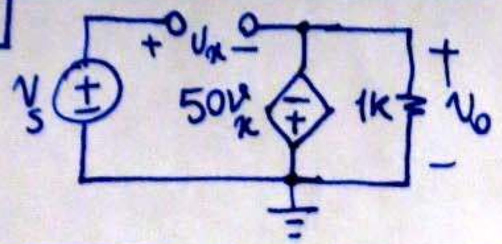
MAE 140: Linear Systems – Fall 2012

Homework #5 – Solutions

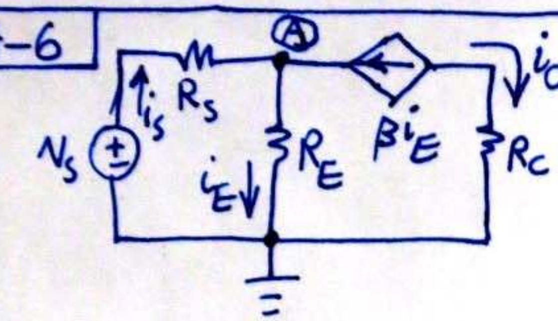
4-5



$$v_x = V_s - (-50v_x) = V_s + 50v_x \Rightarrow v_x = -\frac{V_s}{49}$$

$$v_o = -50v_x = -50\left(-\frac{V_s}{49}\right) = \frac{50}{49}V_s \Rightarrow \boxed{\frac{v_o}{v_s} = \frac{50}{49}}$$

4-6



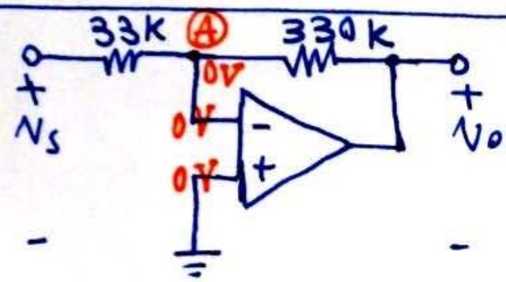
KCL at A: $i_E - \beta i_E - i_s = 0$

$$\Rightarrow \frac{i_E}{i_s} = \frac{1}{1-\beta}$$

$$i_o = -\beta i_E \Rightarrow \frac{i_o}{i_s} = \frac{-\beta i_E}{i_s} = \frac{-\beta}{1-\beta} = \boxed{\frac{\beta}{\beta-1}}$$

4-23

(a)

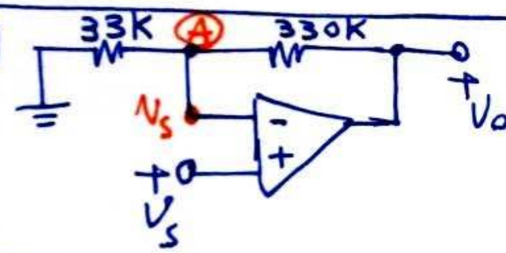


$V_A = 0$ due to the virtual short-circuit between OpAmp input terminals

KCL at A: $\frac{0 - V_s}{33} + \frac{0 - V_o}{330} = 0$

$$\Rightarrow \frac{V_o}{V_s} = -\frac{330}{33} = \boxed{-10}$$

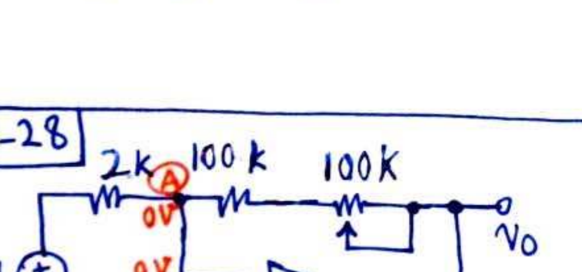
(b)



$V_A = V_s$

KCL at A: $\frac{V_s}{33} + \frac{V_s - V_o}{330} = 0 \Rightarrow \boxed{\frac{V_o}{V_s} = 11}$

4-27



$v_B = 0V$

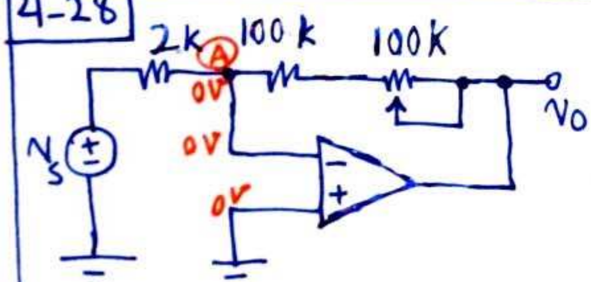
KCL at A: $\frac{VA}{68} + \frac{VA - V_s}{22} + \frac{VA - 0}{33} = 0$

$$\Rightarrow V_A = 0.5025\,V_s$$

KCL at B: $\frac{0 - VA}{33} + \frac{0 - V_o}{330} = 0 \Rightarrow V_o = -10V_A$

$$\Rightarrow \frac{V_o}{V_s} = -10\frac{VA}{Vs} = \boxed{-5.025}$$

4-28



$V_A = 0$

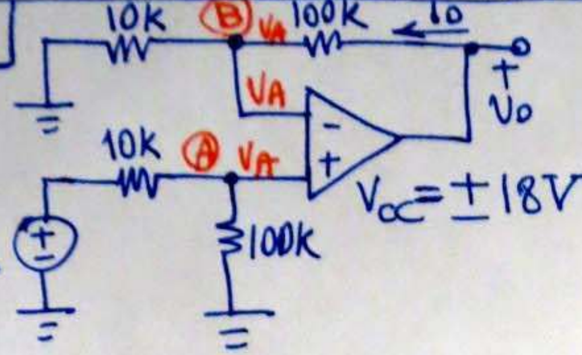
KCL at A $\frac{0 - V_s}{2} + \frac{0 - V_o}{100 + R} = 0 \Rightarrow \frac{V_o}{V_s} = -\frac{100 + R}{2}$

$$0 \leqslant R \leqslant 100K \Rightarrow \frac{100 + 0}{2} \leqslant -\frac{V_o}{V_s} \leqslant \frac{100 + 100}{2}$$

$$\Rightarrow \boxed{50 \leqslant -\frac{V_o}{V_s} \leqslant 100}$$

4-31

(a)



$V_A = V_B$

Voltage division rule: $V_A = V_s \frac{100}{100+10} = \frac{10}{11} V_s$

KCL at (B): $\frac{V_A}{10} + \frac{V_A - V_o}{100} = 0 \Rightarrow V_o = 11 V_A$

$$\Rightarrow V_o = 11\left(\frac{10}{11} V_s\right) = 10 V_s$$

Note: $V_o = 10 V_s$ provided the OpAmp is not saturated, i.e., $-V_{cc} \leq V_o \leq V_{cc}$

$$\Rightarrow -V_{cc} \leq 10 V_s \leq V_{cc} \Rightarrow -18 \leq 10 V_s \leq 18 \Rightarrow \underbrace{-1.8 \leq V_s \leq 1.8}$$

range of values V_s without saturating the OpAmp

$$\Rightarrow V_o = \begin{cases} -18V, & V_s < -1.8 \\ 10 V_s, & -1.8 \leq V_s \leq 1.8 \\ 18V, & 1.8 < V_s \end{cases}$$

(b) $V_s = 0.5V \Rightarrow -1.8 \leq V_s \leq 1.8 \rightarrow$ OpAmp is not saturated

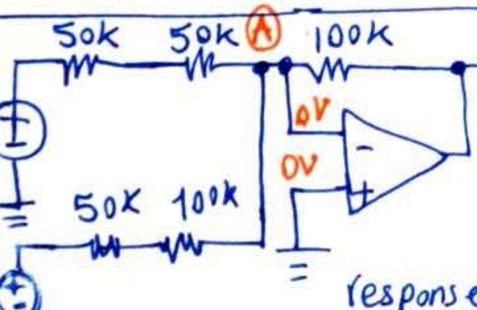
$$i_o = \frac{V_o - V_A}{100} = \frac{10V_s - \frac{10}{11}V_s}{100} = \frac{1}{11}V_s = \frac{1}{11}(0.5)\,mA = \boxed{45.45\,\mu A}$$

$V_s = 2V \Rightarrow V_s > 1.8 \rightarrow$ Op Amp is saturated $\rightarrow V_o = 18V$

$$i_o = \frac{V_o - V_A}{100} = \frac{18 - \frac{10}{11}V_s}{100} = \frac{18 - \frac{10}{11}\times 2}{100} = 0.161818\,mA = \boxed{161.818\,\mu A}$$

4.34

(a)



We can find V_o in terms of V_1 and V_2 by writing a KCL at (A). However, it is easier to find using superposition principle.

response to V_1: Inverting Amp $\rightarrow V_{o_{V_1}} = -\frac{100}{50+50} V_1 = -V_1$
(V_2: OFF)

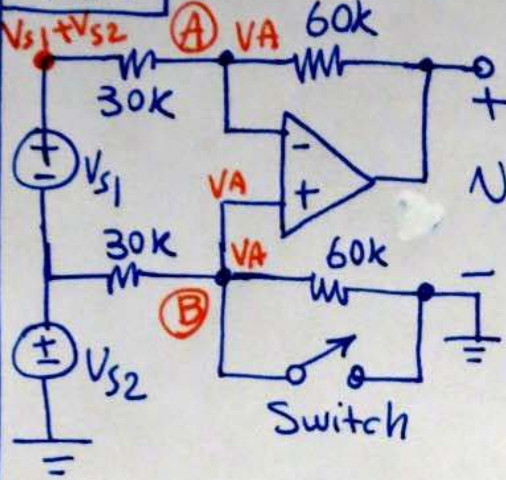
response to V_2: Inverting Amp $\rightarrow V_{o_{V_2}} = -\frac{100}{100+50} V_2 = -\frac{2}{3} V_2$

$$\Rightarrow V_o = V_{o_{V_1}} + V_{o_{V_2}} = \boxed{-V_1 - \frac{2}{3} V_2}$$

(b) $-V_{cc} \leq V_0 \leq V_{cc} \Rightarrow -15 \leq -V_1 - \frac{2}{3}V_2 \leq 15$

$V_1 = 1V \Rightarrow -14 \leq -\frac{2}{3}V_2 \leq 16 \Rightarrow \boxed{-24 \leq V_2 \leq 21}$

4.37



switch open:

KCL at Ⓐ: $\frac{V_A - (V_{s1} + V_{s2})}{30} + \frac{V_A - V_0}{60} = 0$

KCL at Ⓑ: $\frac{V_A - V_{s2}}{30} + \frac{V_A - 0}{60} = 0$

$\Rightarrow V_A = \frac{2}{3}V_{s2}$

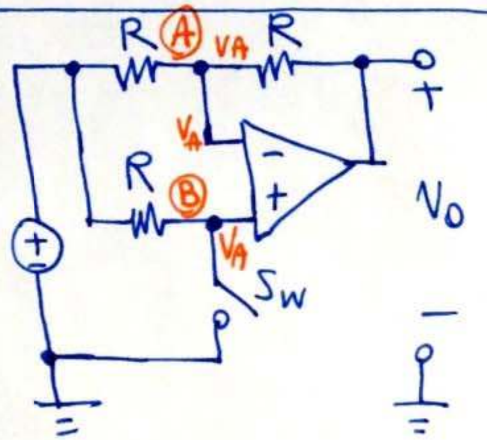
Substituting V_A in KCL Ⓐ $\Rightarrow \boxed{V_0 = -2V_{s1}}$

switch closed: $\Rightarrow V_A = 0$

KCL at Ⓐ: $\frac{0 - (V_{s1} + V_{s2})}{30} + \frac{0 - V_0}{60} = 0$

$\Rightarrow \boxed{V_0 = -2(V_{s1} + V_{s2})}$

4.39



Switch open:

KCL at Ⓐ: $\frac{V_A - V_s}{R} + \frac{V_A - V_0}{R} = 0$

$\Rightarrow V_0 = 2V_A - V_s$

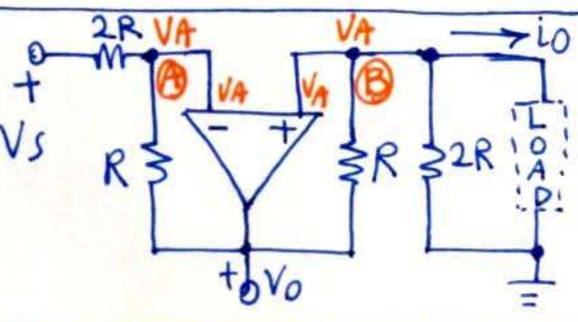
KCL at Ⓑ: $\frac{V_A - V_s}{R} = 0 \Rightarrow V_A = V_s$

Substituting V_A in KCL Ⓐ $\Rightarrow \boxed{V_0 = V_s}$

Switch Closed: $V_A = 0$

KCL at Ⓐ: $\frac{0 - V_s}{R} + \frac{0 - V_0}{R} = 0 \Rightarrow \boxed{V_0 = -V_s} \longrightarrow$ The claim is false

4.42



$V_B = V_A$

KCL at Ⓐ: $\frac{V_A - V_s}{2R} + \frac{V_A - V_0}{R} = 0$

KCL at Ⓑ: $\frac{V_A - V_0}{R} + \frac{V_A}{2R} + i_0 = 0$

Ⓑ−Ⓐ $\Rightarrow \frac{V_s}{2R} + i_0 = 0 \Rightarrow \boxed{i_0 = \frac{-V_s}{2R}}$