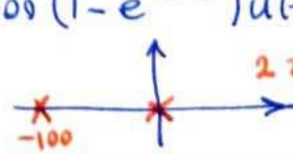
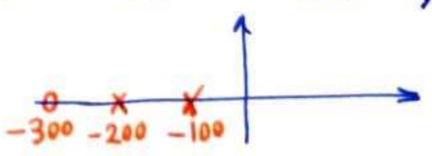
9-1 $f(t) = 500(1 - e^{-100t})u(t) \Rightarrow F(s) = 500\left(\frac{1}{s} - \frac{1}{s+100}\right) = \frac{50000}{s(s+100)}$

2 zeros at infinity

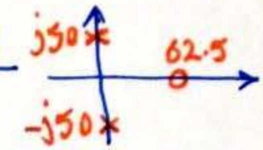


9-4 $f(t) = 20(e^{-200t} - 2e^{-100t})u(t) \Rightarrow F(s) = 20\left(\frac{1}{s+200} - \frac{2}{s+100}\right) = \frac{-(s+300)}{(s+100)(s+200)}$

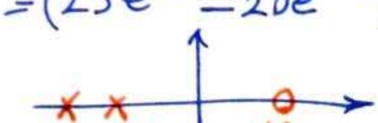


9-7 $f(t) = 5[4\cos(50t) - 5\sin(50t)]u(t)$

$$\Rightarrow F(s) = 5\left[4\frac{s}{s^2+50^2} - 5\frac{50}{s^2+50^2}\right] = \frac{20(s-62.5)}{s^2+50^2}$$



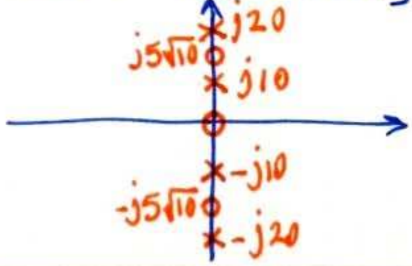
9-10
(a) $f_1(t) = (25e^{-15t} - 20e^{-10t})u(t) \Rightarrow F_1(s) = \frac{25}{s+15} - \frac{20}{s+10} = \frac{5(s-10)}{(s+10)(s+15)}$



(b) $f_2(t) = 10[\cos(10t) + \cos(20t)]u(t) \Rightarrow F_2(s) = 10\left[\frac{s}{s^2+10^2} + \frac{s}{s^2+20^2}\right]$

$$= 10\,\frac{s(2s^2+500)}{(s^2+10^2)(s^2+20^2)}$$

$$= \frac{20\,s\,(s^2+(5\sqrt{10})^2)}{(s^2+10^2)(s^2+20^2)}$$



9-12
(a) $f_1(t) = 5\delta(t) + (625t\,e^{-25t})u(t)$

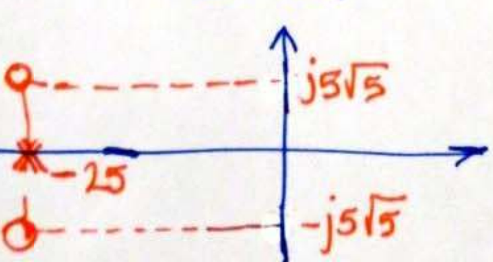
We know: $\boxed{t f(t) \overset{\mathcal{L}}{\longleftrightarrow} -\frac{d}{ds}F(s)}$

Proof: $\mathcal{L}\{t f(t)\} = \int_{0^-}^{\infty} t f(t) e^{-st}\,dt$
$= \int_{0^-}^{\infty} f(t)(t e^{-st})\,dt$
$= \int_{0^-}^{\infty} f(t)\left(-\frac{d}{ds} e^{-st}\right)dt$
$= -\frac{d}{ds}\int_{0^-}^{\infty} f(t) e^{-st} = -\frac{d}{ds}F(s)$

$$\Rightarrow F_1(s) = 5 + 625\left(-\frac{d}{ds}\frac{1}{s+25}\right) = 5 + \frac{625}{(s+25)^2}$$

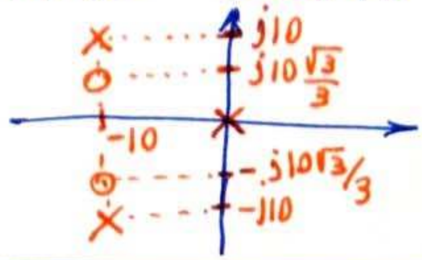
$$= \frac{5(s+25)^2 + 625}{(s+25)^2} = 5\,\frac{(s+25)^2 + 125}{(s+25)^2}$$

$$= 5\,\frac{(s+25)^2 + (5\sqrt{5})^2}{(s+25)^2}$$



(b) $f_2(t) = \left[10 + 5e^{-10t}(\cos 10t + \sin 10t)\right]u(t)$

$$\Rightarrow F(s) = \frac{10}{s} + 5\left(\frac{s+10}{(s+10)^2+10^2} + \frac{10}{(s+10)^2+10^2}\right) = \frac{15\left(s^2+20s+\frac{400}{3}\right)}{s\left((s+10)^2+10^2\right)}$$



9-20

(a) $$F_1(s) = \frac{50}{s(s+50)} = \frac{k_1}{s} + \frac{k_2}{s+50}$$

$$k_1 = \lim_{s\to 0} sF_1(s) = 1$$

$$k_2 = \lim_{s\to -50} (s+50)F_1(s) = -1$$

$$\Rightarrow F_1(s) = \frac{1}{s} - \frac{1}{s+50}$$

$$\Rightarrow \boxed{f_1(t) = (1 - e^{-50t})u(t)}$$

(b) $$F_2(s) = \frac{s+1}{(s+2)(s+3)} = \frac{k_1}{s+2} + \frac{k_2}{s+3}$$

$$k_1 = \lim_{s\to -2}(s+2)F_2(s) = \lim_{s\to -2}\frac{s+1}{s+3} = -1$$

$$k_2 = \lim_{s\to -3}(s+3)F_2(s) = \lim_{s\to -3}\frac{s+1}{s+2} = 2$$

$$\Rightarrow F_2(s) = \frac{-1}{s+2} + \frac{2}{s+3}$$

$$\Rightarrow \boxed{f_2(t) = (2e^{-3t} - e^{-2t})u(t)}$$

9-21

(a) $$F_1(s) = \frac{s+30}{s(s+40)} = \frac{k_1}{s} + \frac{k_2}{s+40}$$

$$k_1 = \lim_{s\to 0} sF_1(s) = \lim_{s\to 0}\frac{s+30}{s+40} = \frac{3}{4}$$

$$k_2 = \lim_{s\to -40}(s+40)F_1(s) = \lim_{s\to -40}\frac{s+30}{s} = \frac{1}{4}$$

$$\Rightarrow F_1(s) = \frac{\frac{3}{4}}{s} + \frac{\frac{1}{4}}{s+40}$$

$$\Rightarrow \boxed{f_1(t) = \frac{1}{4}(3 + e^{-40t})u(t)}$$

(b) $$F_2(s) = \frac{(s+10)(s+20)}{s(s+50)(s+100)} = \frac{k_1}{s} + \frac{k_2}{s+50} + \frac{k_3}{s+100}$$

$$k_1 = \lim_{s\to 0}\frac{(s+10)(s+20)}{(s+50)(s+100)} = \frac{10\times 20}{50\times 100} = \frac{1}{25}$$

$$k_2 = \lim_{s\to -50}\frac{(s+10)(s+20)}{s(s+100)} = \frac{-40\times(-30)}{-50\times 50} = -\frac{12}{25}$$

$$k_3 = \lim_{s\to -100}\frac{(s+10)(s+20)}{s(s+50)} = \frac{-90\times(-80)}{-100\times(-50)} = \frac{36}{25}$$

$$\Rightarrow F_2(s) = \frac{1}{25}\left(\frac{1}{s} - \frac{12}{s+50} + \frac{36}{s+100}\right)$$

$$\Rightarrow \boxed{f_2(t) = \frac{1}{25}(1 - 12e^{-50t} + 36e^{-100t})\,u(t)}$$

9-23

(a) $F_1(s)=\dfrac{900}{(s+10)^2+30^2}=30\,\dfrac{30}{\underbrace{(s+10)}_{\text{frequency shift by 10}}{}^2+30^2}\Rightarrow f_1(t)=30e^{-10t}\sin(30t)\,u(t)$

(b) $F_2(s)=\dfrac{3(s+10)}{\underbrace{(s+10)}_{\text{Frequecy shift by 10}}{}^2+30^2}\Rightarrow \boxed{f_2(t)=3e^{-10t}\cos(30t)\,u(t)}$

9-25

(a) $F_1(s)=\dfrac{\alpha^2}{s^2(s+\alpha)}=\dfrac{k_1}{s}+\dfrac{k_2}{s^2}+\dfrac{k_3}{s+\alpha}$

$k_3=\lim\limits_{s\to-\alpha}(s+\alpha)F_1(s)=\lim\limits_{s\to-\alpha}\dfrac{\alpha^2}{s^2}=1$

$k_2=\lim\limits_{s\to0}s^2F_1(s)=\lim\limits_{s\to0}\dfrac{\alpha^2}{s+\alpha}=\alpha$

$k_3=\lim\limits_{s\to0}\dfrac{d}{ds}(s^2F_1(s))=\lim\limits_{s\to0}\dfrac{-\alpha^2}{(s+\alpha)^2}=-1$

$\Rightarrow F_1(s)=\dfrac{-1}{s}+\dfrac{\alpha}{s^2}+\dfrac{1}{s+\alpha}$

$\Rightarrow \boxed{f_1(t)=(-1+\alpha t+e^{-at})u(t)}$

(b) $F_2(s)=\dfrac{\alpha^2}{s(s+\alpha)^2}=\dfrac{k_1}{s}+\dfrac{k_2}{s+\alpha}+\dfrac{k_3}{(s+\alpha)^2}$

$k_3=\lim\limits_{s\to-\alpha}(s+\alpha)^2F_2(s)=\lim\limits_{s\to-\infty}\dfrac{\alpha^2}{s}=-\alpha$

$k_2=\lim\limits_{s\to-\alpha}\dfrac{d}{ds}((s+\alpha)^2F_2(s))=\lim\limits_{s\to-\infty}\left(\dfrac{-\alpha^2}{s^2}\right)=-1$

$k_1=\lim\limits_{s\to0}sF_2(s)=\lim\limits_{s\to0}\dfrac{\alpha^2}{(s+\alpha)^2}=1$

$\Rightarrow F_2(s)=\dfrac{1}{s}-\dfrac{1}{s+\alpha}-\dfrac{\alpha}{(s+\alpha)^2}$

$\Rightarrow \boxed{f_2(t)=(1-e^{-\alpha t}-\alpha te^{-\alpha t})u(t)}$

9-32

Pole: $s=-20\Rightarrow$ denominator $=(s+20)$

Zero: $s=-\gamma\Rightarrow$ numerator $=(s+\gamma)$

Skale factor $k=1$

$\Rightarrow F(s)=1\,\dfrac{s+\gamma}{s+20}=1+\dfrac{\gamma-20}{s+20}$

$\Rightarrow \boxed{f(t)=\delta(t)+(\gamma-20)e^{-20t}u(t)}$ *

(a) $f(t)=\delta(t)-5e^{-20t}\xrightarrow[*]{\text{Compare with}}\boxed{\gamma=15}$

(b) $f(t)=\delta(t)\longrightarrow\boxed{\gamma=20}$

(c) $f(t)=\delta(t)+5e^{-20t}\longrightarrow\boxed{\gamma=25}$