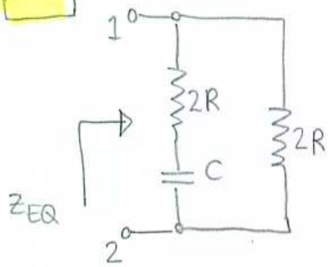
HW7 - MAE 140 fall 2012

10.3, 10.5, 10.9, 10.17, 10.19, 10.25, 10.27, 10.32, 10.61, 10.68

10.3



a) $Z_{EQ} = \left(2R + \frac{1}{sC}\right) \| 2R = \left(\frac{1+2sCR}{sC}\right) \| 2R$

$$= \frac{\frac{(1+2sCR)2R}{sC}}{\frac{1+2sCR}{sC} + 2R} = \frac{1+2sCR}{\frac{1}{2R} + sC + sC}$$

$$\boxed{Z_{EQ} = \frac{sR + \frac{1}{2C}}{s + \frac{1}{4RC}}}$$

zero: $s = -\frac{1}{2CR}$

pole: $s = -\frac{1}{4RC}$

b) Want pole at -640 rad/s

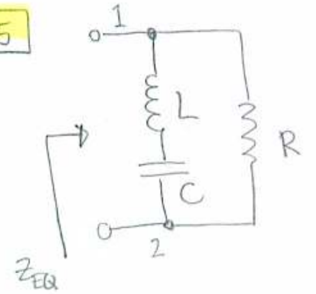
- select $R = 1k\Omega = \boxed{1000\,\Omega}$

$$-640 \text{ rad/s} = -\frac{1}{4RC}$$

$$C = \frac{1}{4R\,640} = 3.9063 \cdot 10^{-7} = 0.391\,\mu F = \boxed{391\,nF}$$

- $s_{zero} = -\frac{1}{2CR} = 2 \cdot s_{pole} = \boxed{-1280 \text{ rad/s}}$

10.5



a) $Z_{EQ} = \left(sL + \frac{1}{sC}\right) \| R = \left(\frac{1+s^2LC}{sC}\right) \| R$

$$Z_{EQ} = \frac{\frac{(1+s^2LC)R}{sC}}{\frac{1+s^2LC}{sC} + R} = \boxed{\frac{R+s^2LCR}{1+s^2LC+sCR}}\ Z_{EQ}$$

$$Z_{EQ} = \frac{s^2R + \frac{R}{LC}}{s^2 + \frac{R}{L}s + \frac{1}{LC}}$$

$$\boxed{s_{1/2} = \pm j\sqrt{\frac{1}{LC}}}$$

zeros

$$\boxed{\begin{array}{l} s_{1/2} = -\frac{R}{2L} \pm \sqrt{\frac{R^2}{4L^2} - \frac{1}{LC}} \\ s_{1/2} = \frac{-RC \pm \sqrt{R^2C^2 - 4LC}}{2LC} \end{array}}$$

poles

b) $R = 2k\Omega$, $C = 0.1\mu F$ } L such that zero at $s = \pm j5000$ rad/s ?

$$(5000)^2 = \frac{1}{LC}$$

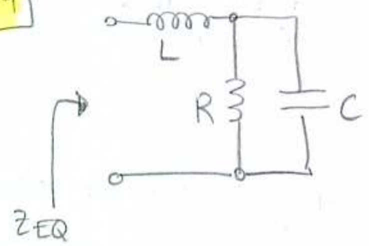
$$L = \frac{1}{C \cdot 5000^2} = \frac{1}{0.1\cdot 10^{-6} \cdot 25 \cdot 10^6} = \frac{1}{2.5} = 0.4\ H = \boxed{400\ mH}$$

c) $s_{1/2} = -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$

$$s_{1/2} = -\frac{2\cdot 10^3}{2\cdot 0.4} \pm \sqrt{\frac{10^6}{0.16} - 5000^2}$$

$$\boxed{s_{1/2} = -2500 \pm j4330.1}$$

10.9



$L = 2H$
$R = 1k\Omega$
$C = 0.5\mu F$

a) $Z_{EQ} = sL + R \| \frac{1}{sC}$

$$= sL + \frac{R}{1+sCR} = \frac{R + sL + s^2LCR}{1+sCR}$$

$$Z_{EQ} = \frac{\frac{1}{C} + s\frac{L}{CR} + s^2L}{s + \frac{1}{CR}}$$

poles: $s = -\frac{1}{CR} = -\frac{1}{10^3 \cdot 0.5\, 10^{-6}} = \boxed{-2000 \text{ rad/sec}}$ pole

zeros: $s^2 + s\frac{1}{CR} + \frac{1}{LC} = 0$

$$s_{1/2} = -\frac{1}{2CR} \pm \sqrt{\left(\frac{1}{2CR}\right)^2 - \frac{1}{LC}}$$

$$s_{1/2} = -\frac{1}{2CR} = \boxed{-1000 \text{ rad/sec}}$$

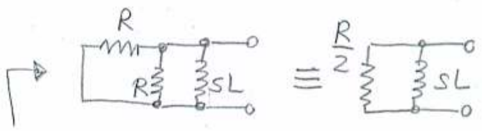
2 zeros at -1000 rad/sec

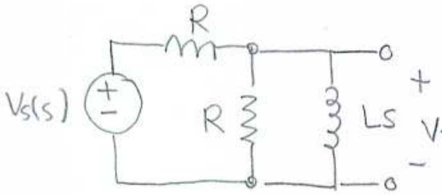
b) $L = 5H$
- poles don't change: $s = -2000$ rad/sec
- zeros:

$$s_{1/2} = -1000 \pm \sqrt{10^6 - 0.4\, 10^6} = -1000 \pm 774.6 \text{ rad/sec}$$

$$\boxed{\begin{array}{l} s_1 = -225.4 \text{ rad/sec} \\ s_2 = -1774.6 \text{ rad/sec} \end{array}}$$

10.17 Thevenin equivalent





1. For Z_T set $V_s(s) = 0$

$$Z_T = sL \| (R \| R) = sL \| \frac{R}{2} = \frac{sL\frac{R}{2}}{\frac{R}{2} + sL}$$

$$\boxed{Z_T = \frac{sLR}{R + 2sL}}$$

2. For V_T open circuit

$$V_T(s) = \frac{(R \| sL)\, V_s(s)}{R \| sL + R} = \frac{\left(\frac{sLR}{sL+R}\right) V_s(s)}{\frac{sLR}{sL+R} + R} = \frac{sLR}{sLR + sLR + R^2} V_s(s) = \boxed{\frac{sL}{2sL+R} V_s(s)}$$

3. Want pole at $s = -12$ k rad/s:

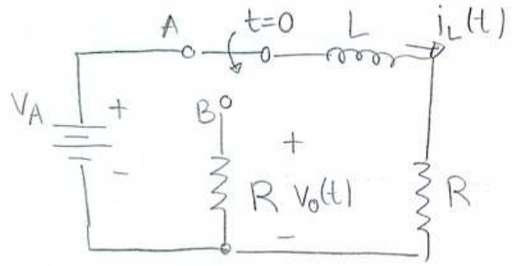
$$2sL + R = 0$$

$$s = -\frac{R}{2L} = -12k \quad \text{select } \boxed{R = 3k\Omega}$$

$$L = \frac{3k}{2 \cdot 12k} = \frac{1}{8} H = \boxed{125 \text{ mH}}$$

NOTE: Students can select different values for R, resulting L will be $L = \frac{R}{24000}$

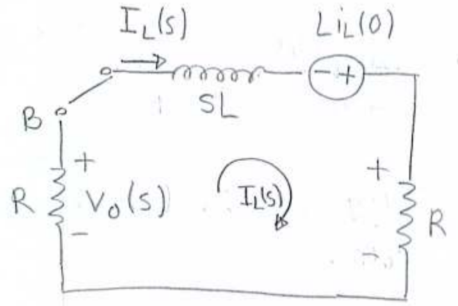
10.19 Solve for $I_L(s)$, $i_L(t)$, $V_o(s)$, $v_o(t)$; switch is moved to position B at time $t=0$



Initial condition

For $t<0$ the circuit is in a DC steady-state condition and the inductor acts like a short circuit ($\equiv$).
Consequently, $i_L(t) = \frac{V_A}{R}$

First, transform the circuit into the s domain for $t>0$



1. Using KVL we obtain

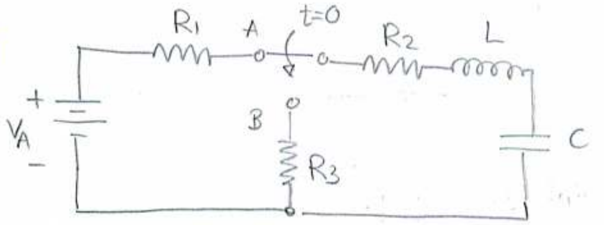
$$sLI_L(s) - Li_L(0) + RI_L(s) + RI_L(s) = 0$$
$$(sL+2R)I_L(s) = Li_L(0) = L\frac{V_A}{R}$$
$$I_L(s) = \frac{L\frac{V_A}{R}}{sL+2R} = \frac{\frac{1}{R}}{s+2\frac{R}{L}} V_A$$

$$\boxed{I_L(s) = \frac{\frac{1}{R}}{s+\frac{2R}{L}} V_A}$$

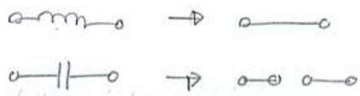
$$\boxed{i_L(t) = \left\{\frac{V_A}{R}\cdot e^{-\frac{2R}{L}t}\right\} u(t)}$$

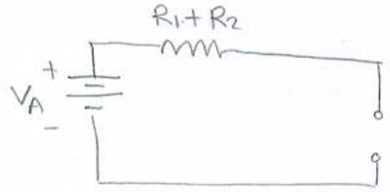
2.
$$\boxed{\begin{gathered} V_o(s) = -RI_L(s) = -\frac{V_A}{s+\frac{2R}{L}} \\ v_o(t) = \left\{-V_A e^{-\frac{2R}{L}t}\right\} u(t) \end{gathered}}$$

10.25



a) First we have to find the initial inductor current and capacitor voltage. The circuit is in a DC steady-state condition for $t<0$ and the
- inductor acts like a short circuit
- capacitor like an open circuit

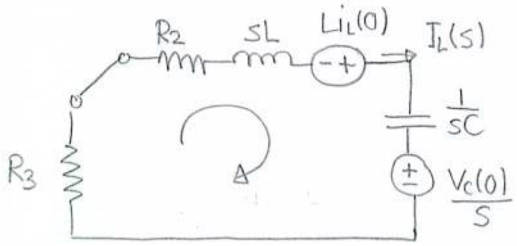




Thus $i_L(0)=0$

$V_C(0)=V_A$

Now transform the circuit into the s-domain for $t>0$



KVL:

$$\left(R_2+R_3+sL+\frac{1}{sC}\right) I_L(s) + \frac{\overset{V_A}{\cancel{V_S(0)}}}{s} = 0$$

$$I_L(s) = -\frac{V_A}{s^2L+(R_2+R_3)s+\frac{1}{C}}$$

$$\boxed{I_L(s) = -\frac{\frac{V_A}{L}}{s^2+\frac{R_2+R_3}{L}s+\frac{1}{LC}}}$$

b) $R_1=R_2=500\Omega$
$R_3=1k\Omega$
$L=0.5\ H$
$C=0.2\ \mu F$
$V_A=15\ V$

Have to find $\mathcal{L}^{-1}\{I_L(s)\}$ first

$$I_L(s) = -\frac{30V}{s^2+3000s+10\cdot 10^6}A = \frac{-30V}{(s^2+1500)^2+7\,750\,000}A$$

NOTE: $\frac{\beta}{(s+\alpha)^2+\beta^2} \xrightarrow{\mathcal{L}^{-1}} \{e^{-\alpha t}\sin\beta t\}u(t)$

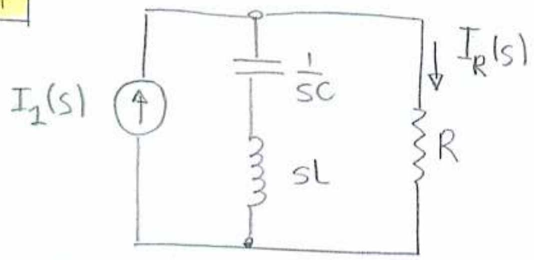
$$I_L(s) = \frac{-0.0108\cdot 2784}{(s^2+1500^2)^2+2784^2}A$$

$$\boxed{i_L(t) = \left[-0.0108\cdot e^{-1500t}\cdot\sin\{2784t\}\right]u(t)A}$$

Alternative way:

Poles at: $s_{1/2} = -1500 \pm j\sqrt{7{,}750{,}000} \rightarrow I_L(s) = \frac{k_1}{(s-s_1)}+\frac{k_2}{(s-s_2)}$ → should obtain the same result

10.27



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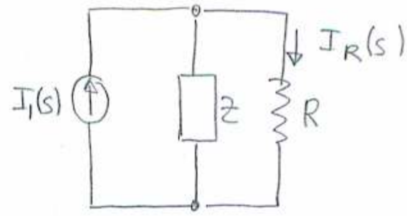


$$Z = \frac{1}{sC} + sL = \frac{1+s^2LC}{sC}$$

Current division

$$\frac{I_R(s)}{I_1(s)} = \frac{Z}{Z+R} = \frac{\frac{1+s^2LC}{sC}}{\frac{1+s^2LC}{sC} + R} = \boxed{\frac{1+s^2LC}{1+sCR+s^2LC}}$$

zeros: $1+s^2LC = 0$

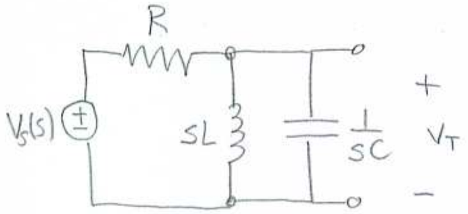
$$s = \pm j\sqrt{\frac{1}{LC}}$$

poles: $s^2 + \frac{R}{L}s + \frac{1}{LC} = 0$

$$s_{1/2} = -\frac{R}{2L} \pm \sqrt{\frac{R^2}{4L^2} - \frac{1}{LC}} = \frac{-RC \pm \sqrt{R^2C^2 - 4LC}}{2LC}$$

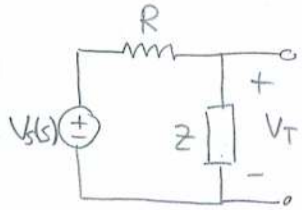
10.31

Thevenin equivalent



$\equiv$

1. Thévenin voltage



$$Z = sL \| \frac{1}{sC} = \frac{\frac{sL}{sC}}{sL+\frac{1}{sC}}$$

$$Z = \frac{sL}{1+s^2LC}$$

$$V_T = \frac{Z}{R+Z}V_s(s) = \frac{sL}{R+sL+s^2LCR}V_s(s)$$

$$\boxed{V_T = \frac{\frac{1}{RC}s}{\frac{1}{LC}+\frac{1}{CR}s+s^2}V_s(s)}$$

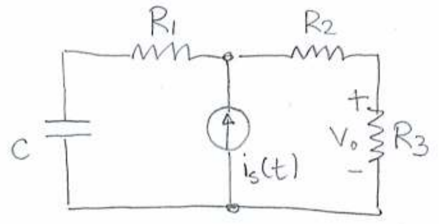
2. Equivalent resistance

set $V_s(s) = 0$

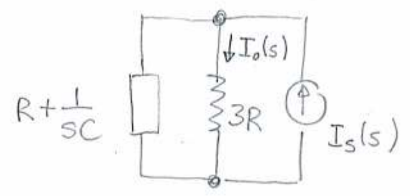
$$Z_{EQ} = Z \| R = \frac{sLR}{s^2LCR + sL + R}$$

$$\boxed{Z_{EQ} = \frac{\frac{1}{C}s}{s^2+\frac{1}{RC}s+\frac{1}{LC}}}$$

10.32 Compute $v_o(t)$ via current division (no initial conditions)



$R_1 = R_3 = R$
$R_2 = 2R$



$R_1 = R_3 = 5\,k\Omega$
$R_2 = 10\,k\Omega$
$C = 0.2\,\mu F = 0.2$

$i_s(t) = 5e^{-500t}u(t)\ mA$

$I_s(t) = \dfrac{0.005}{s+500}$ [A]

Using element voltage-current relation

$V_o(s) = R_3 \cdot I_o(s)$

Calculate $I_o(s)$ using current division

$$I_o(s) = \frac{R + \frac{1}{sC}}{\left(R+\frac{1}{sC}\right) + 3R} I_o(s) \qquad I_o(s) = \frac{1+sCR}{1+4sCR} I_o(s)$$

$$I_o(s) = \frac{1}{4} \cdot \frac{s + \frac{1}{CR}}{s + \frac{1}{4CR}} I_o(s)$$

$$V_o(s) = R_3 I_o(s) = \frac{R}{4} \quad \frac{s+\frac{1}{CR}}{s+\frac{1}{4CR}} \cdot \frac{0.005}{s+500}$$

NOTE: $\boxed{\dfrac{1}{CR} = \dfrac{1}{0.2\cdot 10^{-6}\cdot 5\cdot 10^{3}} = 10^3}$

$$V_o(s) = \frac{25}{4} \cdot \frac{(s+1000)}{(s+250)(s+500)} = \frac{k_1}{s+250} + \frac{k_2}{s+500}$$

$$k_1 = \lim_{s\to -250} \frac{25}{4} \cdot \frac{s+1000}{s+500} = \frac{25}{4} \cdot \frac{750}{250} = \frac{75}{4} = 18.75$$

$$k_2 = \lim_{s\to -500} \frac{25}{4} \cdot \frac{s+1000}{s+250} = \frac{25}{4} \cdot \frac{500}{-(250)} = -\frac{50}{4} = -12.5$$

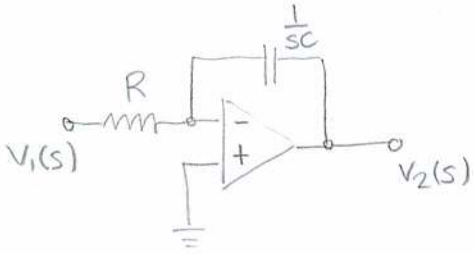
$$V_o(s) = \frac{18.75}{s+250} - \frac{12.5}{s+500}$$

$$\boxed{V_o(t) = \left\{18.75e^{-250t} - 12.5e^{-500t}\right\} u(t)}$$

NOTE TO READER:
students were supposed to solve 10.32; however, I scanned in problem 10.31;
Please give full credit if either 10.31 or 10.32 was solved correctly. Thanks!

10.61 $v_1(t) = \cos 1000t$ $\qquad V_1(s) = \dfrac{s}{s^2+1000^2}$

desired output $v_2(t) = -\sin 1000t$ $\qquad V_2(s) = -\dfrac{1000}{s^2+1000^2}$



This is an inverting amplifier and we know that

$$V_2(s) = K V_1(s) = -\frac{\frac{1}{sC}}{R} V_1(s) = -\frac{1}{sCR} V_1(s)$$

So $\quad V_2(s) = -\dfrac{1}{sCR} V_1(s)$

$$-\frac{1000}{s^2+1000^2} = -\frac{1}{\cancel{s}CR} \cdot \frac{\cancel{s}}{s^2+1000^2}$$

→ comparison shows that $\quad 1000 = \dfrac{1}{CR}$

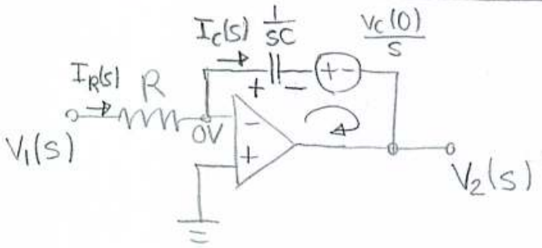
$$C = \frac{1}{1000R}$$

$$\boxed{C = 1\ \mu F}$$

select $\boxed{R = 1k\Omega}$

NOTE: Students can choose values for R or C randomly and the other value can be computed via $C = \frac{1}{1000R}$

What happens if $v_c(0) = 10V$?



We know that $i_N = i_P = 0$ so $I_c(s) = I_R(s) = I(s)$

$V^+ = V^- = 0$

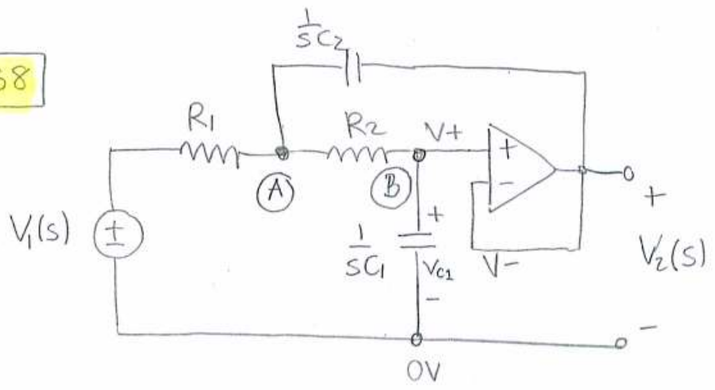
KVL: $\quad -V_2(s) = \dfrac{1}{sC} I(s) + \dfrac{V_c(0)}{s} \qquad I(s) = \dfrac{V_1(s)}{R}$

$$-V_2(s) = \frac{1}{sC} \cdot \frac{V_1(s)}{R} + \frac{V_c(0)}{s}$$

$$V_2(s) = -\frac{V_1(s)}{sCR} - \underbrace{\frac{V_c(0)}{s}}_{=\text{ step function in time domain}}$$

→ output is shifted by $V_c(0) = 10V$

10.68



Note: $i_N = i_p = 0$ and $V^+ = V^- = V_2(s)$

Now use KCL at node Ⓐ and Ⓑ to find $V_2(s)$

KCL @ Ⓐ
$$\frac{V_1(s) - V_A(s)}{R_1} + \frac{V_2(s) - V_A(s)}{\frac{1}{sC_2}} + \frac{0V - V_A(s)}{R_2 + \frac{1}{sC_1}} = 0$$

$$\frac{1}{R_1}V_1(s) + sC_2V_2(s) = \left(\frac{1}{R_1} + sC_2 + \frac{sC_1}{1 + sC_1R_2}\right)V_A(s) \qquad (1)$$

KCL @ Ⓑ
$$\frac{V^+(s) - V_A(s)}{R_2} + \frac{V^+(s) - 0V}{\frac{1}{sC_1}} = 0 \qquad V^+(s) = V^-(s) = V_2(s)$$

$$V_2(s) - V_A(s) + sC_1R_2V_2(s) = 0$$

$$V_A = (1 + sC_1R_2)V_2(s) \qquad (2)$$

Substitute (2) in (1)

$$\frac{1}{R_1}V_1(s) + sC_2V_2(s) = \left(\frac{1 + sC_2R_1}{R_1} + \frac{sC_1}{1 + sC_1R_2}\right)(1 + sC_1R_2)V_2(s)$$

$$\frac{1}{R_1}V_1(s) = \left\{\frac{1}{R_1}(1 + sC_2R_1)(1 + sC_1R_2) + sC_1 - sC_2\right\}V_2(s)$$

$$\frac{V_1(s)}{V_2(s)} = 1 + s(C_1R_2 + \cancel{C_2R_1}) + s^2C_1C_2R_1R_2 + sC_1R_1 - \cancel{sC_2R_1}$$

$$V_2(s) = \frac{1}{s^2C_1C_2R_1R_2 + s(C_1R_2 + C_1R_1) + 1}V_1(s)$$

$$\boxed{V_2(s) = \frac{\frac{1}{C_1C_2R_1R_2}}{s^2 + s\left(\frac{1}{C_2R_1} + \frac{1}{C_2R_2}\right) + \frac{1}{C_1C_2R_1R_2}}V_1(s)}$$