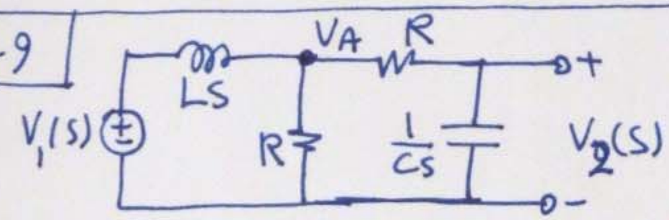
MAE 140: Linear Circuits – Fall 2012

Homework #8 – Solutions

11–9



Voltage division:

$$V_2(s) = \frac{\frac{1}{Cs}}{R+\frac{1}{Cs}} V_A = \frac{1}{1+RCs} V_A \quad \text{(I)}$$

Voltage division:

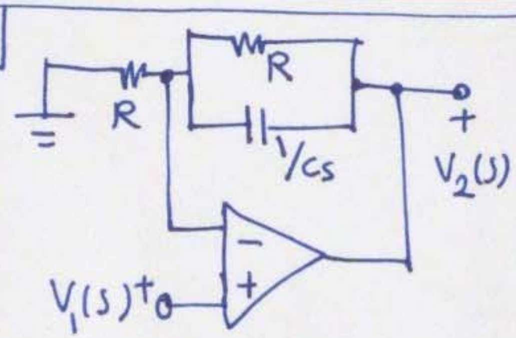
$$V_A = \frac{R \,\|\, (R+\frac{1}{Cs})}{LS + R\,\|\,(R+\frac{1}{Cs})} V_1(s) = \frac{\dfrac{R\,\frac{(1+RCs)}{Cs}}{R+\frac{1+RCs}{Cs}}}{LS + \bigcirc} V_1(s)$$

$$= \frac{R(1+RCs)}{2RCLS^2+(L+R^2C)s+R} V_1(s) \quad \text{(II)}$$

Subutitute (II) into (I):

$$T_V(s) = \frac{V_2(s)}{V_1(s)} = \boxed{\frac{R}{2RCLs^2+(L+R^2C)s+R}}$$

11–11



non-inverting module:

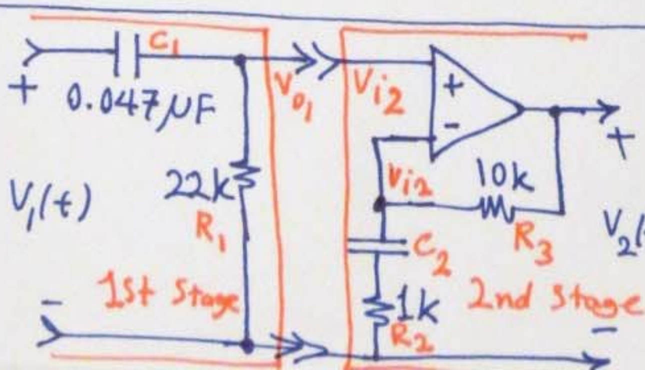
$$T_V(s) = \frac{V_2(s)}{V_1(s)} = 1 + \frac{R\,\|\,\frac{1}{Cs}}{R} = 1 + \frac{\frac{R/Cs}{R+1/Cs}}{R}$$

$$= 1 + \frac{\frac{R}{1+RCs}}{R} = \boxed{\frac{2+RCs}{1+RCs}}$$

pole: $s = -\frac{1}{RC} = -100 \text{ krad/s} \Rightarrow RC = \frac{1}{100k} = 10^{-5}$

one degree of freedom: $\boxed{\begin{array}{l} R = 10k \longrightarrow C = 1nF \\ R = 1k \longrightarrow C = 10nF \end{array}}$

11–15



1st stage:

Voltage division:

$$T_{V_1}(s) = \frac{V_{o_1}(s)}{V_1(s)} = \frac{R_1}{R_1+\frac{1}{C_1 s}} = \frac{R_1 C_1 s}{1+R_1 C_1 s}$$

2nd Stage:

Voltage division: (non-inverting) module

$$T_{V_2}(s) = \frac{V_2(s)}{V_{i_2}(s)} = 1 + \frac{R_3}{R_2 + \frac{1}{C_2 s}} = 1 + \frac{R_3 C_2 s}{1 + R_2 C_2 s} = \frac{1 + (R_2 + R_3) C_2 s}{1 + R_2 C_2 s}$$

We have a cascade connection of 1st and 2nd stages ($V_{i_2} = V_{o_1}$) with no loading issues. So, the overall transfer function is

$$T_V(s) = \frac{V_2(s)}{V_1(s)} = T_{V_1}(s)\, T_{V_2}(s) = \frac{R_1 C_1 s}{1 + R_1 C_1 s} \, \frac{1 + (R_2 + R_3) C_2 s}{1 + R_2 C_2 s}$$

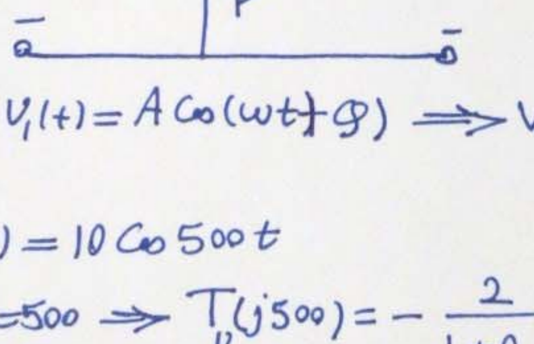
$$= \frac{s\left[1 + (R_2 + R_3) C_2 s\right]}{R_2 C_2 \left(s + \frac{1}{R_1 C_1}\right)\left(s + \frac{1}{R_2 C_2}\right)} = \frac{s\left[(R_2 + R_3)s + \frac{1}{C_2}\right]}{R_2 \left(s + \frac{1}{R_1 C_1}\right)\left(s + \frac{1}{R_2 C_2}\right)}$$

substitute the element values: $T_V(s) = \dfrac{s(11s + 10^4)}{(s + 967.1)(s + 10^4)}$

Zeros: $\boxed{s = 0}$, $s = -\frac{10^4}{11} \boxed{= -909.1}$

Poles: $\boxed{s = -967.1}$, $\boxed{s = -10^4}$

11-35



Inverting module:

$$T_V(s) = \frac{V_2(s)}{V_1(s)} = -\frac{R_2 \parallel \frac{1}{Cs}}{R_1} = -\frac{\frac{R_2/Cs}{R_2 + 1/Cs}}{R_1} = -\frac{R_2}{R_1(1 + R_2 C s)} = \boxed{-\frac{2}{1 + 0.001s}}$$

if $v_1(t) = A\cos(\omega t + \varphi) \Longrightarrow v_{2_{ss}}(t) = A \cdot |T_V(j\omega)| \cos(\omega t + \varphi + \measuredangle T_V(j\omega))$

- $v_1(t) = 10\cos 500t$

$\omega = 500 \Rightarrow T_V(j500) = -\dfrac{2}{1 + 0.001(j500)} = -\dfrac{2}{1 + j0.5} = -1.6 + j0.8$
$= 1.789 \measuredangle 153.43°$

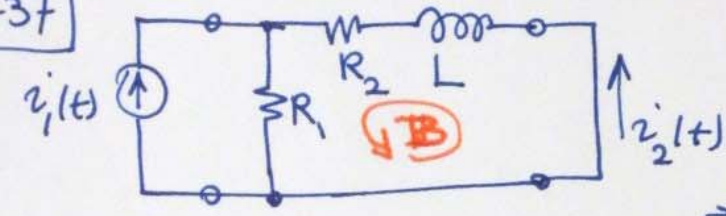
$\Rightarrow \boxed{v_{2_{ss}}(t) = 10 \times 1.789 \cos(500t + 153.43)}$

• $v_1(t) = 10\cos(10k\,t)$

$$\omega = 10k \Rightarrow T_v(j10k) = -\frac{2}{1+j10} = 0.199 \angle 95.71^\circ$$

$$\Rightarrow \boxed{v_{2ss}(t) = 1.99\cos(10k\,t + 95.71^\circ)} \qquad \boxed{\text{Pole: } s = -1000} \ \text{rad/s}$$

11-37



KVL in loop B:

$$(Ls + R_2)I_2 + R_1(I_2 + I_1) = 0$$

$$\Rightarrow T_I(s) = \frac{I_2(s)}{I_1(s)} = \frac{-R_1}{R_1 + R_2 + Ls}$$

$$= \frac{-100}{100 + 400 + 100\times 10^{-3}s} = \boxed{\frac{-100}{500 + 0.1s}}$$

$$\text{Pole} = -\frac{500}{0.1} = \boxed{-5000}$$

• $i_1(t) = 10\cos 50kt$ mA

$$\omega = 50k \Rightarrow T_I(j50k) = \frac{-100}{500 + j500} = \frac{-0.2}{1 + j10} = 0.0199 \angle 95.71^\circ$$

$$\Rightarrow i_{1ss}(t) = 0.199\cos(50kt + 95.71^\circ) \text{ mA}$$

$$= \boxed{199\cos(50kt + 95.71^\circ)\ \mu A}$$

• $i_2(t) = 10\cos 5kt$ mA

$$\omega = 5k \Rightarrow T_I(j5k) = \frac{-100}{500 + j50} = \frac{-0.2}{1 + j} = -0.1 + j0.1 = 0.1414 \angle 135^\circ$$

$$\Rightarrow i_{2ss}(t) = 10 \times 0.1414\cos(5kt + 135^\circ) = \boxed{1.414\cos(5kt + 135^\circ) \text{ mA}}$$

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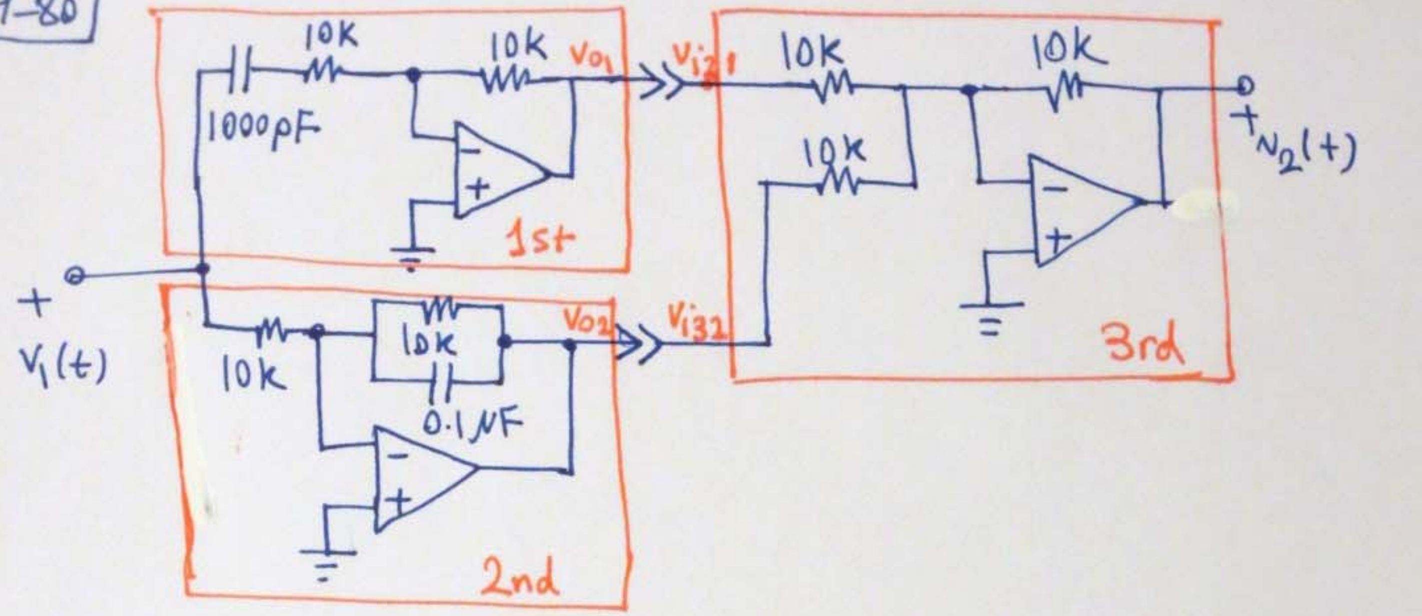
11-80



(a) This interconnection doesn't involve any loading because the two input channels of 3rd stage are directly driven by the OpAmps at 1st and 2nd stages. We know that an ideal OpAmp, without considering any practical limitations, can supply any amount of current to the load (here to the 3rd stage) without any voltage drop at its output.

(b) 1st and 2nd stages are inverting modules and 3rd stage is an inverting summation module with unity gain.

1st stage: $T_{V_1}(s) = \dfrac{V_{o_1}(s)}{V_1(s)} = -\dfrac{10\,k}{10\,k + \frac{1}{1000p\,s}} = -\dfrac{10^4}{10^4 + \frac{10^9}{s}} = -\dfrac{10^4 s}{10^4 s + 10^9} = -\dfrac{s}{s+10^5}$

2nd stage: $T_{V_2}(s) = \dfrac{V_{o2}(s)}{V_1(s)} = -\dfrac{10k \,\|\, \frac{1}{0.1\mu s}}{10k} = -\dfrac{\frac{10k}{1+10k\times 0.1\mu s}}{10k} = -\dfrac{1}{1+10^{-3}s} = -\dfrac{10^3}{s+10^3}$

3nd stage: $V_2(s) = -\left(V_{i31}(s) + V_{i32}(s)\right) = -\left(V_{o_1}(s) + V_{o2}(s)\right)$

$$= -\left(T_{V_1}(s)\,V_1(s) + T_{V_2}(s)\,V_1(s)\right) = -\left(T_{V_1}(s) + T_{V_2}(s)\right) V_2(s)$$

$\Rightarrow$ overall transfer function:

$$T_v(s) = \frac{V_2(s)}{V_1(s)} = -\left(T_{v_1}(s) + T_{v_2}(s)\right) = \frac{s}{s+10^5} + \frac{10^3}{s+10^3}$$

$$= \boxed{\frac{s^2 + 2\times10^3 s + 10^8}{(s+10^5)(s+10^3)}}$$

zeros: $s = -10^3 \pm j\,10^3\sqrt{99} = \boxed{-1000 \pm j994.99}$

poles: $\boxed{s=-10^3}$, $\boxed{s=-10^5}$

(c) • $v_1(t) = \cos 500t$

$$\omega = 500 \Rightarrow T(j500) = \frac{(j500)^2 + 2\times10^3(j500) + 10^8}{(j500)^2 + 101\times10^3(j500) + 10^8} = 0.8 - j0.395$$

$$= 0.892 \angle -26.27^\circ$$

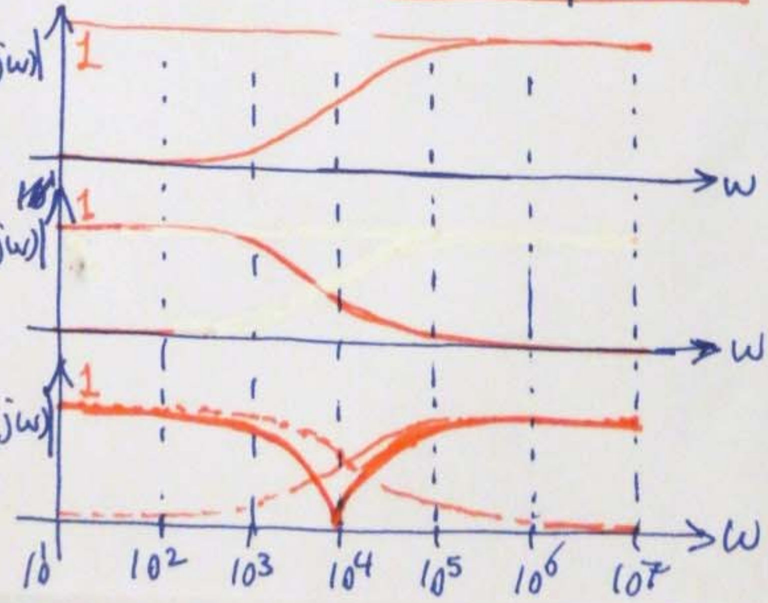
$$\boxed{\Rightarrow v_{2ss}(t) = 0.892\cos(500t - 26.27^\circ)}$$

• $v_1(t) = \cos 200kt$

$$\omega = 200k \Rightarrow T(j200k) = 0.8 + j0.395 = 0.892 \angle 26.27^\circ$$

$$\boxed{\Rightarrow v_{2ss}(t) = 0.892\cos(500t + 26.27^\circ)}$$

(d) this circuit is a band-stop filter with stop band $\omega \in (10^3, 10^5)$



Note 1: $|T_v(j\omega)| \neq |T_{v_1}(j\omega)| + |T_{v_2}(j\omega)|$

based on triangular inequality:

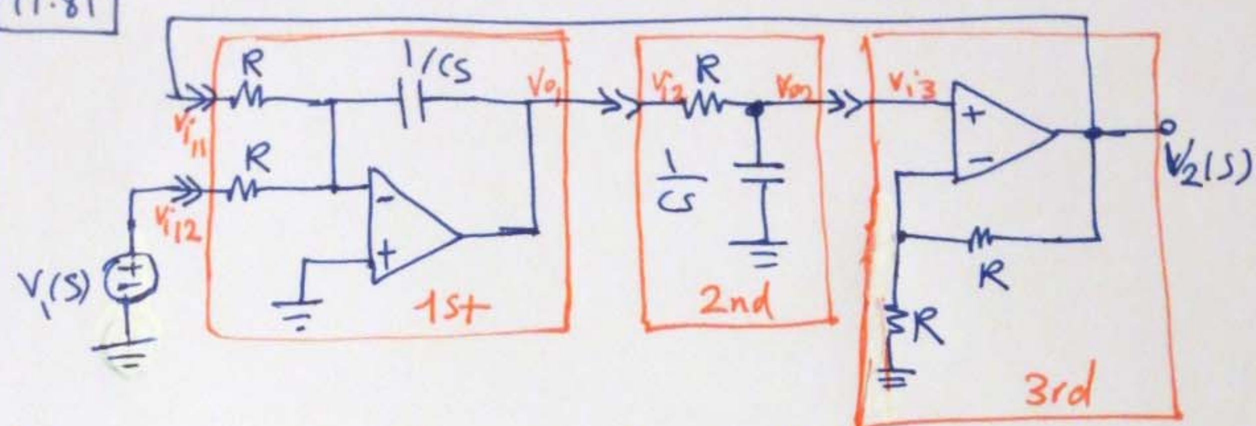
$$|T_v(j\omega)| \leq |T_{v_1}(j\omega)| + |T_{v_2}(j\omega)|$$

Note 2: It could be interesting if we were asked to find v_{2ss} when $v_1(t) = \cos 10^4 t$

$$T(j10^4) = 0.0198 \angle 0$$

$$\Rightarrow v_2(t) = 0.0198\cos 10^4 t \simeq 0$$

11.81



The interconnection doesn't involve any loading. The second stage is directly driven by 1st stage's OpAmp. So, it can receive any amount of current from the OpAmp without creating a voltage drop at output of 1st stage (no loading).

The 2^{nd} stage has been connected directly to the input terminal of 3rd stage's OpAmp. So, 3rd stage demands no current from 2nd stage (no loading).

1st stage is an inverting summation & integration module. 2^{nd} stage is an RC passive low-pass filter, and 3rd stage is a non-inverting gain module.

1st stage: Using the superposition principle we have:

$$V_{o_1}(s) = T_{V_{11}}(s)\,V_{i_{11}}(s) + T_{V_{12}}(s)\,V_{i_{12}}(s)$$

$$T_{V_{11}}(s) = \frac{V_{o_1}(s)}{V_{i_{11}}(s)} = -\frac{\frac{1}{Cs}}{R} = -\frac{1}{RC}$$

$$T_{V_{12}}(s) = \frac{V_{o_1}(s)}{V_{i_{12}}(s)} = -\frac{1}{RC}$$

$$\Rightarrow V_{o_1}(s) = -\frac{1}{RC}V_{i_{11}}(s) - \frac{1}{RC}V_{i_{12}}(s)$$

$$= -\frac{1}{RC}\left(V_{i_{11}}(s) + V_{i_{12}}(s)\right)$$

$$= T_{V_1}(s)\left(V_{i_{11}}(s) + V_{i_{12}}(s)\right)$$

$$, \boxed{T_{V_1}(s) = \frac{1}{RC}}$$

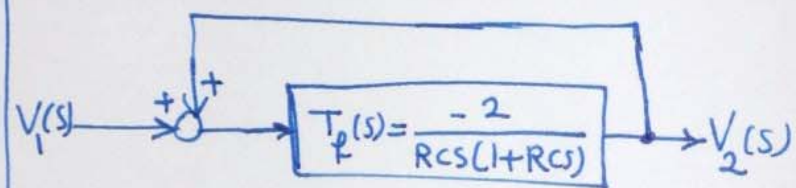
$\underline{2^{nd}\text{ stage:}}$ $T_{V_2}(s) = \dfrac{V_{o2}(s)}{V_{i2}(s)} = \dfrac{\frac{1}{Cs}}{R+\frac{1}{Cs}} = \boxed{\dfrac{1}{1+RCs}}$

voltage division

$\underline{3^{rd}\text{ stage:}}$ $V_{o3}(s) = 1+\frac{R}{R}V_{i3} \Rightarrow T_{V3}(s) = \dfrac{V_{o3}(s)}{V_{i3}(s)} = \boxed{2}$

We have a cascade connection of three stages, $V_{i2}=V_{o1}$ and $V_{i3}=V_{o2}$. Since the interconnection involves no loading, the transfer function of forward path is $T_f(s) = T_{V_1}(s)\,T_{V_2}(s)\,T_{V_3}(s) = \dfrac{-2}{RCs(1+RCs)}$

We can simplify the circuit to the following block diagram:



$V_2(s) = T_f(s)\left(V_1(s)+V_2(s)\right) \Rightarrow V_2(s)\left(1-T_f(s)\right) = T_f(s)V_1(s)$

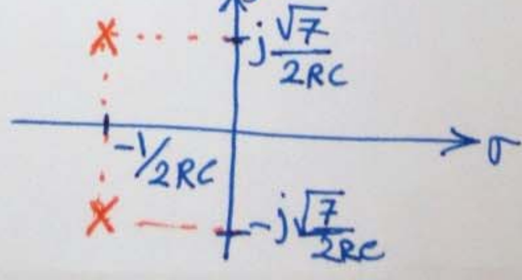
$\Rightarrow$ overal transfer function $T_V(s) = \dfrac{V_2(s)}{V_1(s)} = \dfrac{T_f(s)}{1-T_f(s)} = \dfrac{\frac{-2}{RCs(1+RCs)}}{1+\frac{2}{RCs(1+RCs)}}$

$$\Rightarrow \boxed{T_V(s) = \frac{-2}{R^2C^2s^2+RCs+2}}$$

Zeros: $\boxed{s=-\infty,\ -\infty}$

Poles: $s = \dfrac{-RC \pm \sqrt{R^2C^2-8R^2C^2}}{2R^2C^2} = \boxed{\dfrac{-1\pm j\sqrt{7}}{2RC}}$

the circuit is stable because both poles are located in right-half plane.



12-9

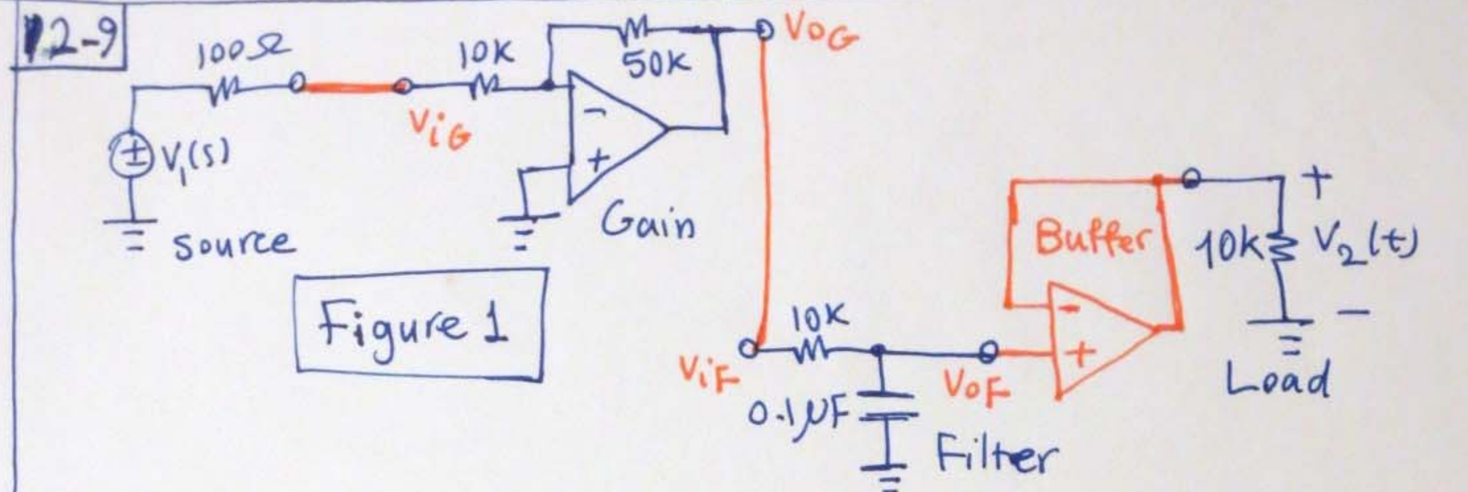


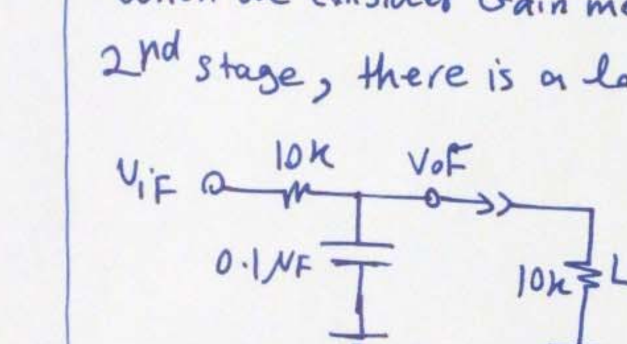
Figure 1

The transfer function of Gain and Filter stages when connections are open are:

$$T_{G_{open}}(s) = \frac{V_{oG}(s)}{V_{iG}(s)} = -\frac{50k}{10k} = -5$$

$$T_{F_{open}}(s) = \frac{V_{oF}(s)}{V_{iF}(s)} = \frac{\frac{1}{0.1\mu s}}{10k + \frac{1}{0.1\mu s}} = \frac{1000}{s+1000}$$ Low-pass filter cut-off freq. $w_c = 1000 \frac{rad}{s}$

when we consider Gain module as 1st stage and Filter module as 2nd stage, there is a loading issue at Load as follows:



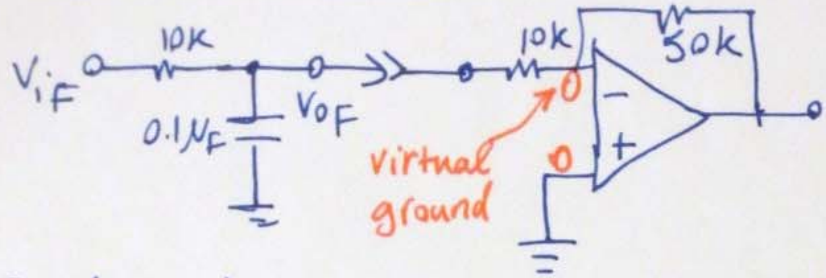
$$\Rightarrow T_{F_{connected}}(s) = \frac{V_{oF}(s)}{V_{iF}(s)} = \frac{10k \,||\, \frac{1}{0.1\mu s}}{10k + 10k \,||\, \frac{1}{0.1\mu s}} \neq T_{F_{open}}(s)$$

for example, at $s=0$

$$T_{F_{connected}}(0) = \frac{10k \,||\, \infty}{10k + 10k \,||\, \infty} = \frac{10k}{10k+10k} = \frac{1}{2}$$

$$T_{F_{open}}(0) = \frac{1000}{0+1000} = 1 \quad \neq$$

when we put the Filter at 1st stage and Gain module at 2nd stage, there is a loading issue at the interconnection of two stages, as follows



Considering the virtual short circuit at OpAmp input terminals, the negative (⊖) input of OpAmp is virtually grounded and again we have

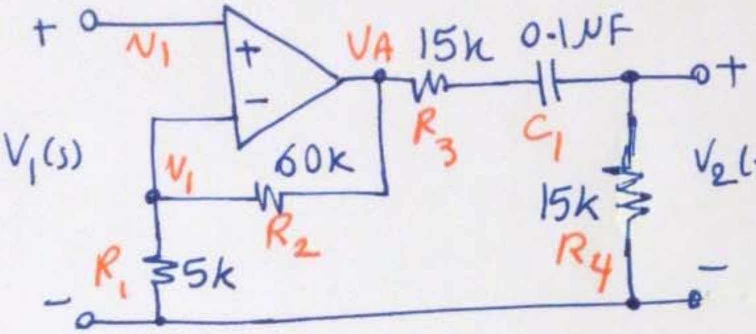
$$T_{F_{connected}}(s) = \frac{V_{oF}(s)}{V_{iF}(s)} = \frac{10k \,\|\, \frac{1}{0.1\mu s}}{10k + 10k \,\|\, \frac{1}{0.1\mu s}} \neq T_{F_{open}}(s)$$

Inorder to achieve the desired result we put Gain module at 1st stage, Filter at 2^{nd} stage, and then add a 3rd buffer stage before conneting to the load. This interconnection has been shown in Figure 1. Then

- There is no loading issue at source, because $(R_s = 100\,\Omega) \ll$ (input resistance of Gain stage $= 10k$)
- There is no loading at intercormection of Gain and Filter stages because the Filter is directly driven by OpAmp.
- There is no loading issue at interconnection of Filter and Buffer because buffer doesn't demand any current from filter.
- There is no loading issue at Load because the load is driven by buffer's OpAmp.

⇒ the circuit shown in Figure 1 provides the desired result.

12-14



Voltage division: $V_1(s) = \dfrac{5k}{5k+60k} V_A \Rightarrow V_A = 13\,V_1$

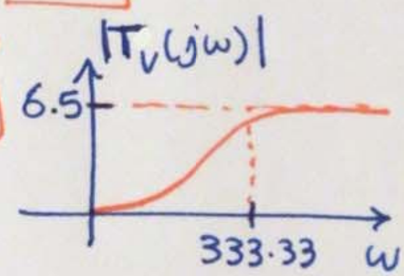
Voltage division: $V_2(s) = \dfrac{15k}{15k + 15k + \frac{1}{0.1\mu s}} V_A = \dfrac{15k \times 0.1\mu s}{1 + 30k \times 0.1 \mu s} \times 13 V_1$

$\Rightarrow T_V(s) = \dfrac{V_2(s)}{V_1(s)} = \dfrac{0.0195\,s}{1 + 0.003\,s} = \boxed{\dfrac{6.5\,s}{s + 333.33}}$

(a) DC-gain $= T_V(0) = \dfrac{6.5 \times 0}{0 + 333.33} = 0$

infinte freq. gain $= T_V(\infty) = \lim_{s\to\infty} \dfrac{6.5\,s}{s + 333.33} = 6.5$

cut-off frequency $\boxed{\omega_c = 333.33 \text{ rad/s}}$



filter type: high-pass filter

(c) the cut-off frequency is $\omega_c = \dfrac{1}{(R_3 + R_4) C_1}$

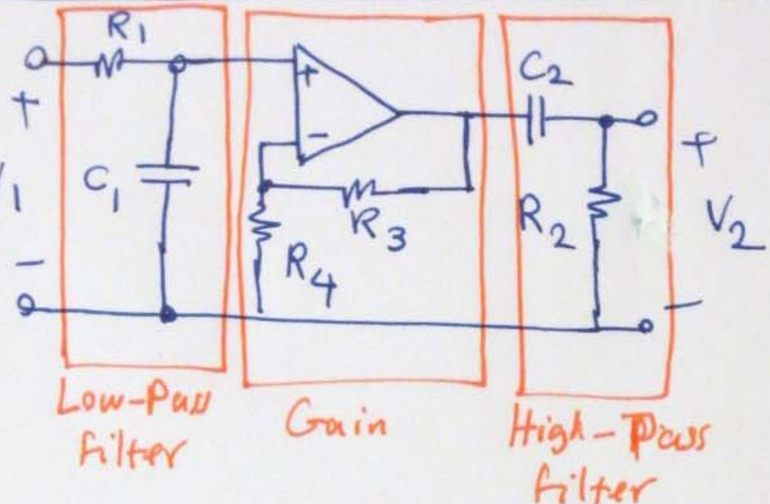
in order to increase the cut-off frequency by one decade (10 times) we can change C_1 to $0.1 C_1$ or $(R_3 + R_4)$ to $0.1 (R_3 + R_4)$

$\boxed{C_1 \rightarrow 0.01\,\mu F}$

or

$\boxed{R_3, R_4 \rightarrow 1.5\,k}$

12-21



The circuit is a cascade connection of a low-pass filter, a gain stage, and a high-pass filter.

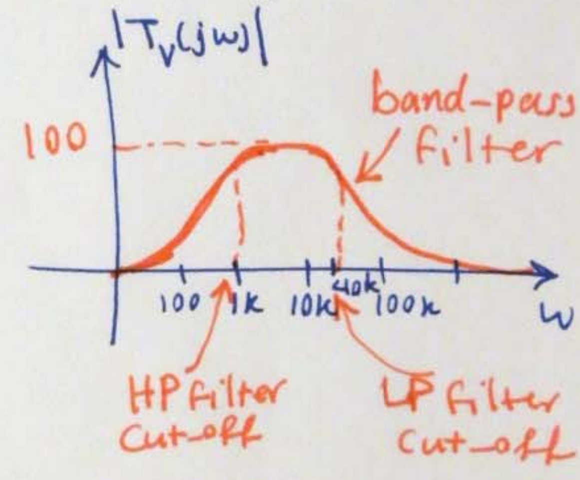
There is no loading issue in the interconnetion, so we have:

overal transfer function $T_V(s) = T_{LP}(s)\, T_G(s)\, T_{HP}(s)$

$$T_{LP}(s) = \frac{\frac{1}{C_1 s}}{R_1 + \frac{1}{C_1 s}} = \frac{1}{1 + R_1 C_1 s}$$

$$T_G(s) = 1 + \frac{R_3}{R_4}$$

$$T_{HP}(s) = \frac{R_2}{R_2 + \frac{1}{C_2 s}} = \frac{R_2 C_2 s}{1 + R_2 C_2 s}$$



$\omega_{C_{LP}} = \frac{1}{R_1 C_1} = 40\,k\,\frac{rad}{s}$ ⟶ we have one degree of freedom $\boxed{R_1 = 25k \rightarrow C_1 = 1\,nF \text{ or } R_1 = 10k \rightarrow C_1 = 2.5\,nF}$

$\omega_{C_{HP}} = \frac{1}{R_2 C_2} = 1000$ ⟶ $\boxed{R_2 = 10k \rightarrow C_2 = 0.1\,\mu F \text{ or } R_2 = 100k \rightarrow C_2 = 10\,nF}$

$T_G(s) = 1 + \frac{R_3}{R_4} = 100 \Rightarrow \frac{R_3}{R_4} = 99$ ⟶ $\boxed{R_4 = 1k \rightarrow R_3 = 99k \text{ or } R_4 = 10k \rightarrow R_3 = 990k}$