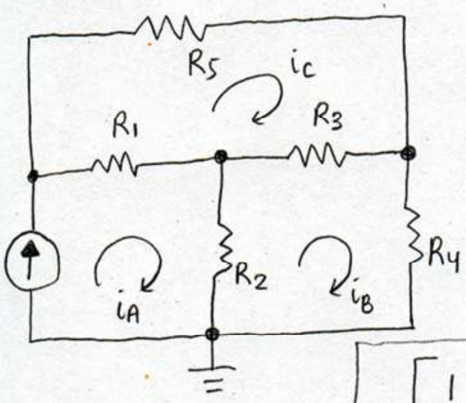
Homework 4 Solution

3.1

Formulate MESH CURRENT equations and arrange them into $A\vec{x} = \vec{b}$!



Draw the meshes and label them as you like.

There are 3 meshes → 3 equations.

Mesh A → $i_s = i_A$

mesh B → $i_B(R_2+R_3+R_4) - i_A R_2 - i_c R_3 = 0$

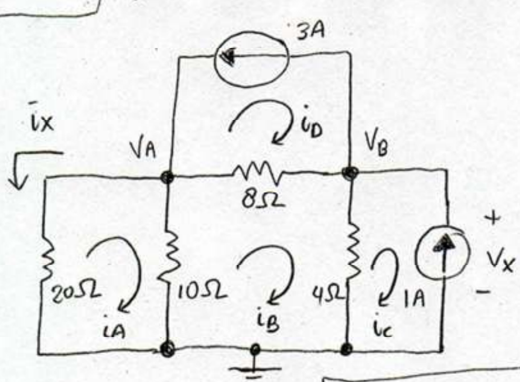
mesh C → $i_c(R_1+R_3+R_5) - i_A R_1 - i_B R_3 = 0$

Then we have →

$$\boxed{\begin{bmatrix} 1 & 0 & 0 \\ -R_2 & R_2+R_3+R_4 & -R_3 \\ -R_1 & -R_3 & R_1+R_3+R_5 \end{bmatrix}\begin{bmatrix} i_A \\ i_B \\ i_c \end{bmatrix} = \begin{bmatrix} i_s \\ 0 \\ 0 \end{bmatrix}}$$

3.4

a) Formulate MESH Current, b) Solve for V_A, V_B, c) Solve for V_x, i_x.



a) Draw and label the meshes.

mesh A → $i_A(20\Omega+10\Omega) - i_B(10\Omega) = 0$

Mesh B → $i_B(10\Omega+8\Omega+4\Omega) - i_A(10\Omega) - i_c(4\Omega) - i_D(8\Omega) = 0$

Mesh C → $i_c = -1A$

mesh D → $i_D = -3A$

So, we have →

$$\begin{bmatrix} 30\Omega & -10\Omega & 0 & 0 \\ -10\Omega & 22\Omega & -4\Omega & -8\Omega \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} i_A \\ i_B \\ i_c \\ i_D \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -1A \\ -3A \end{bmatrix} \rightarrow A\vec{x} = \vec{b}$$

Solve using Matlab $\vec{x} = A^{-1} \cdot \vec{b}$

↳ $i_A = -0.5A$ $\quad i_c = -1A$

$i_B = -1.5A$ $\quad i_D = -3A$

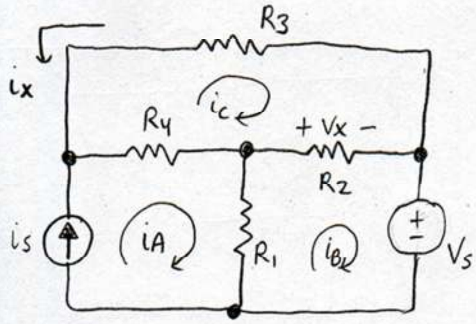
$V_A = 10\Omega(i_A - i_B) = 10V$

$V_B = 4\Omega(i_B - i_c) = -2V$

$V_x = V_B = -2V$

$i_x = -i_A = 0.5A$

3.6 a) Formulate MESH Current ; b) Solve for V_x, i_x using $R_1 = R_2 = R_3 = R_4 = 10k\Omega$, $V_s = 25V$ and $i_s = 1mA$.



a) We have 3 meshes → 3 equations.

Mesh A → $i_A = i_s$

Mesh B → $i_B (R_1 + R_2) - i_A R_1 - i_C R_2 + V_s = 0$

Mesh C → $i_C (R_2 + R_3 + R_4) - i_A R_4 - i_B R_2 = 0$

$$\hookrightarrow \boxed{\begin{bmatrix} 1 & 0 & 0 \\ -R_1 & R_1 + R_2 & -R_2 \\ -R_4 & -R_2 & R_2 + R_3 + R_4 \end{bmatrix} \begin{bmatrix} i_A \\ i_B \\ i_C \end{bmatrix} = \begin{bmatrix} i_s \\ -V_s \\ 0 \end{bmatrix}}$$

b) Plug in the values

$$\hookrightarrow \begin{bmatrix} 1 & 0 & 0 \\ -10k\Omega & 20k\Omega & -10k\Omega \\ -10k\Omega & -10k\Omega & 30k\Omega \end{bmatrix} \begin{bmatrix} i_A \\ i_B \\ i_C \end{bmatrix} = \begin{bmatrix} 1mA \\ -25V \\ 0 \end{bmatrix} \rightarrow$$

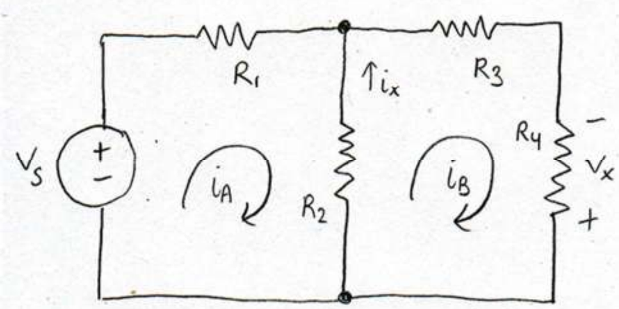
Solve using Matlab

$$\boxed{\begin{aligned} i_A &= 1mA \\ i_B &= -0.7mA \\ i_C &= 0.1\,mA \end{aligned}}$$

$$V_x = R_2 (i_B - i_C) = 10k\Omega (-0.7mA - 0.1\,mA) = \underline{\underline{-8V}}$$

$$i_x = -i_C = \underline{\underline{-0.1\,mA}}$$

3.16 a) Formulate MESH equations $A\vec{x} = \vec{b}$, b) Solve for i_A, i_B, c) Solve V_x, i_x.



Mesh A → $-V_s + i_A (R_1 + R_2) - i_B R_2 = 0$

Mesh B → $i_B (R_2 + R_3 + R_4) - i_A R_2 = 0$

$$\boxed{\begin{bmatrix} R_1 + R_2 & -R_2 \\ -R_2 & R_2 + R_3 + R_4 \end{bmatrix} \begin{pmatrix} i_A \\ i_B \end{pmatrix} = \begin{pmatrix} V_s \\ 0 \end{pmatrix}}$$

b) Solve for i_A, i_B

$\hookrightarrow$ $(R_1+R_2)\, i_A - R_2 i_B = V_S$

$-R_2 i_A = -(R_2+R_3+R_4)\, i_B$

$$\left.\begin{array}{l}(R_1+R_2)\, i_A - R_2 i_B = V_S \\ -R_2 i_A = -(R_2+R_3+R_4)\, i_B\end{array}\right\} \left[(R_1+R_2)\frac{(R_2+R_3+R_4)}{R_2} - R_2\right] i_B = V_S$$

$$\hookrightarrow \left(\frac{(R_1+R_2)(R_2+R_3+R_4) - R_2^2}{R_2}\right) i_B = V_S \rightarrow \boxed{i_B = \frac{R_2 \cdot V_S}{(R_1+R_2)(R_2+R_3+R_4) - R_2^2}}$$

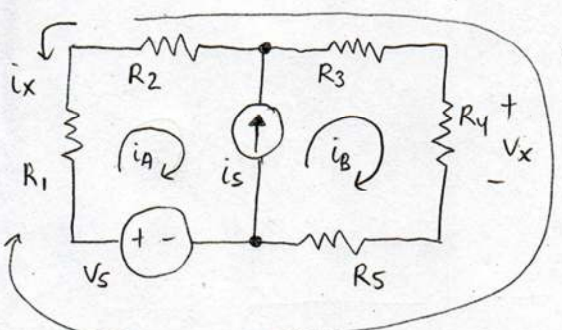
$$i_A = \left(\frac{R_2+R_3+R_4}{\cancel{R_2}}\right)\left(\frac{\cancel{R_2}}{(R_1+R_2)(R_2+R_3+R_4) - R_2^2}\right) V_S \Rightarrow \boxed{i_A = \left(\frac{R_2+R_3+R_4}{(R_1+R_2)(R_2+R_3+R_4) - R_2^2}\right) V_S}$$

c) Solve for V_x, i_x

$$i_x = i_B - i_A = \left(\frac{R_2 - (R_2+R_3+R_4)}{(R_1+R_2)(R_2+R_3+R_4) - R_2^2}\right) V_S = \underline{\underline{\frac{-R_3-R_4}{(R_1+R_2)(R_2+R_3+R_4) - R_2^2} V_S}}$$

$$V_x = i_B \cdot R_4 = \underline{\underline{\frac{R_2 R_4 V_S}{(R_1+R_2)(R_2+R_3+R_4) - R_2^2}}}$$

$\boxed{3.18}$ a) Formulate mesh current, b) Solve for V_x, i_x using $R_1 = 2k\Omega$, $R_2 = 3k\Omega$, $R_3 = 500\Omega$, $R_4 = 2.5\,k\Omega$, $R_5 = 2k\Omega$, $i_s = 10mA$, $V_s = 24V$, c) Find total power dissipated in the circuit!



Supermesh

a) There is a current source in the middle junction, so we need to use supermesh method. Formulate 2 equations 2 unknowns i_A, i_B.

$\hookrightarrow$ $i_s = i_B - i_A$

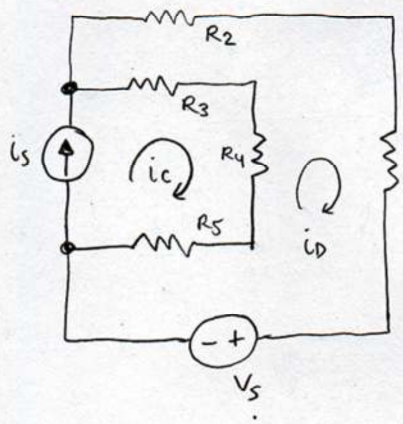
Supermesh $\rightarrow$ $-V_s + i_A(R_1+R_2) + i_B(R_3+R_4+R_5) = 0$

Then we have $\rightarrow$ $$\boxed{\begin{bmatrix} -1 & 1 \\ R_1+R_2 & R_3+R_4+R_5 \end{bmatrix}\begin{pmatrix} i_A \\ i_B \end{pmatrix} = \begin{pmatrix} i_s \\ V_s \end{pmatrix}}$$

Then $\begin{pmatrix} i_A \\ i_B \end{pmatrix} = \dfrac{1}{-(R_1+R_2+R_3+R_4+R_5)} \begin{pmatrix} R_3+R_4+R_5 & -1 \\ -R_1-R_2 & -1 \end{pmatrix} \begin{pmatrix} i_s \\ V_s \end{pmatrix}$

$$\begin{pmatrix} i_A \\ i_B \end{pmatrix} = \frac{1}{R_1+\cdots+R_5} \cdot \begin{pmatrix} -i_s\,(R_3+R_4+R_5) + V_s \\ i_s\,(R_1+R_2) + V_s \end{pmatrix}$$

↳ Can also do



→ $i_C = i_s$

→ $(R_1+R_2+R_3+R_4+R_5)\, i_D - (R_3+R_4+R_5)\, i_C + V_s = 0$

$$\boxed{\begin{bmatrix} 1 & 0 \\ -R_3-R_4-R_5 & R_1+R_2+R_3+R_4+R_5 \end{bmatrix} \begin{bmatrix} i_C \\ i_D \end{bmatrix} = \begin{bmatrix} i_s \\ -V_s \end{bmatrix}}$$

Note that $i_A = -i_D$, $\quad i_B = i_C - i_D$

b) Plug in the variables then we have :

$$\begin{pmatrix} i_A \\ i_B \end{pmatrix} = \begin{bmatrix} -1 & 1 \\ 2k\Omega+3k\Omega & 500\Omega + 2.5k\Omega + 2k\Omega \end{bmatrix}^{-1} \begin{pmatrix} 10mA \\ 24V \end{pmatrix} = \begin{pmatrix} -2.6\,mA \\ 7.4\,mA \end{pmatrix}$$

Then $\boxed{\begin{aligned} i_x &= -i_A = 2.6\,mA \\ V_x &= i_B \cdot R_4 = (7.4mA)(2.5k\Omega) = 18.5V \end{aligned}}$

c) Total power dissipated in the circuit : → power loss across the resistors.

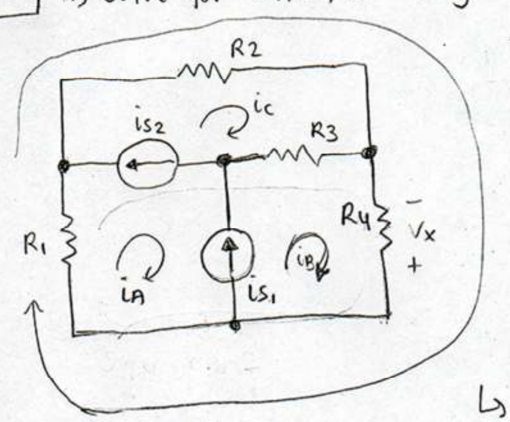
$$P = i_A^2 \cdot (R_1+R_2) + i_B^2 (R_3+R_4+R_5) = (-2.6\,mA)^2 (5k\Omega) + (7.4mA)^2 (5k\Omega)$$

$$= 0.0338\,W + 0.2738\,W$$

↳ $\boxed{P = 0.3076\ W}$

↳ Same as HW 3

3.19 a) Solve for i_A, i_B, i_C using 2 supermesh equations, b) Find V_x!



Supermesh 1

There is only 1 supermesh with no current source; Not sure why it wants 2 supermesh.

$$i_{s1} = i_B - i_A$$

$$i_{s2} = i_C - i_A$$

Supermesh $\rightarrow$ $R_1(i_A) + R_2(i_C) + R_4(i_B) = 0$

$$\hookrightarrow \boxed{\begin{bmatrix} -1 & 1 & 0 \\ -1 & 0 & 1 \\ R_1 & R_4 & R_2 \end{bmatrix} \begin{bmatrix} i_A \\ i_B \\ i_C \end{bmatrix} = \begin{bmatrix} i_{s1} \\ i_{s2} \\ 0 \end{bmatrix}}$$

Try subtitution to find i_A, i_B, i_C

$\hookrightarrow$ $i_A = i_B - i_{s1}$ $\rightarrow$ $i_{s2} = i_C - (i_B - i_{s1}) = i_C - i_B + i_{s1}$

$$i_C = -i_{s1} + i_{s2} + i_B$$

$$\hookrightarrow R_1(i_B - i_{s1}) + R_4 i_B + R_2(-i_{s1} + i_{s2} + i_B) = 0$$

$$i_B(R_1 + R_4 + R_2) - i_{s1}(R_1 + R_2) + i_{s2} R_2 = 0$$

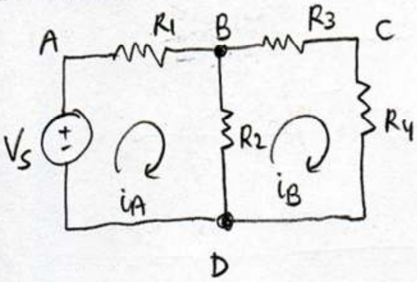
$$\boxed{\begin{aligned} i_B &= \frac{i_{s1}(R_1+R_2) - i_{s2}R_2}{R_1+R_2+R_4} \\ i_A &= \frac{-i_{s1}R_4 - i_{s2}R_2}{R_1+R_2+R_4} \\ i_C &= \frac{-i_{s1}R_4 + i_{s2}(R_1+R_4)}{R_1+R_2+R_4} \end{aligned}}$$

$$i_A = \frac{i_{s1}(R_1+R_2) - i_{s2}R_2}{R_1+R_2+R_4} - i_{s1}$$

$$i_C = -i_{s1} + i_{s2} + \frac{i_{s1}(R_1+R_2) - i_{s2}R_2}{R_1+R_2+R_4}$$

b) $$\boxed{V_x = i_B R_4 = R_4\left(\frac{i_s(R_1+R_2) - i_{s2}R_2}{R_1+R_2+R_4}\right)}$$

3.24 $V_s = 10V$, All resistors are $1k\Omega$. $V_C = -2V$ when $V_B = 0V$. Find i_A, i_B!



mesh $i_A \rightarrow -V_s + i_A(R_1+R_2) - i_B R_2 = 0$

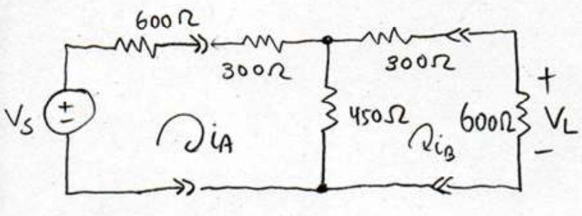
mesh $i_B \rightarrow i_B(R_2+R_3+R_4) - i_A R_2 = 0$

$$\boxed{\hookrightarrow \begin{bmatrix} R_1+R_2 & -R_2 \\ -R_2 & R_2+R_3+R_4 \end{bmatrix} \begin{pmatrix} i_A \\ i_B \end{pmatrix} = \begin{pmatrix} V_s \\ 0 \end{pmatrix}}$$

Plug in the values

$$\hookrightarrow \begin{bmatrix} 2k\Omega & -1k\Omega \\ -1k\Omega & 3k\Omega \end{bmatrix} \begin{pmatrix} i_A \\ i_B \end{pmatrix} = \begin{pmatrix} 10V \\ 0 \end{pmatrix} \rightarrow \boxed{\begin{pmatrix} i_A \\ i_B \end{pmatrix} = \begin{pmatrix} 6mA \\ 2mA \end{pmatrix}}$$

3.92 Find power delivered to the load in terms of V_s. Remove attenuator and find power again. Find attenuators power reduction in dB.



Solve with mesh current.

$$\begin{bmatrix} 1350\Omega & -450\Omega \\ -450\Omega & 1350\Omega \end{bmatrix} \begin{pmatrix} i_A \\ i_B \end{pmatrix} = \begin{pmatrix} V_s \\ 0 \end{pmatrix}$$

$\hookrightarrow i_A = (8.333 \cdot 10^{-4}\Omega^{-1})V_s, \quad i_B = (2.778 \cdot 10^{-4}\Omega^{-1})V_s$

Load voltage $V_L = (600\Omega) \cdot i_B$

$$\hookrightarrow V_L = (600\Omega)(2.778 \cdot 10^{-4}\Omega^{-1})V_s \Rightarrow \boxed{V_{L_1} = 0.1667\, V_s}$$

No attenuator $\rightarrow$ standard voltage divider

$$\hookrightarrow V_L = \frac{600\Omega}{600\Omega + 600\Omega} V_s \Rightarrow \boxed{V_{L_2} = 0.5\, V_s}$$

Power $\rightarrow$

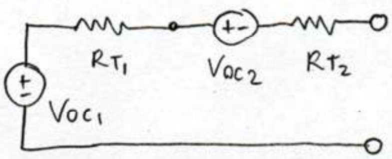
$$\boxed{\begin{aligned} P_1 &= \frac{V_{L_1}^2}{R} = \left(\frac{0.1667^2}{600\Omega}\right) V_s^2 = (4.6315 \cdot 10^{-5})V_s^2 \\ P_2 &= \frac{V_{L_2}^2}{R} = \left(\frac{0.5^2}{600\Omega}\right) V_s^2 = (4.1667 \cdot 10^{-4})V_s^2 \end{aligned}}$$

$$\text{Ratio} \rightarrow \boxed{\frac{P_1}{P_2} = 0.1112} \rightarrow \log^{10}(0.1112) \cdot 20\text{ dB} = \underline{\underline{-19.08\text{ dB}}}$$

3.94 | $V_{open\ circuit} = V_{oc} = 36V$. Thenevin resistance $R_T \leq 10\Omega$. ← Goal.

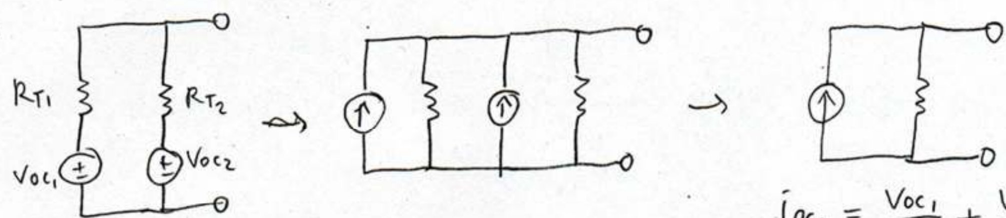
Select series or parallel combinations of 2 batteries : $V_{oc} = 9V$, $R_T = 4\Omega$, 30 grams or $V_{oc} = 4V$, $R_T = 2\Omega$, 18 grams.

Series →



$$\boxed{\begin{aligned} V_{oc\ total} &= V_{oc_1} + V_{oc_2} \\ R_{T\ total} &= R_{T_1} + R_{T_2} \end{aligned}}$$

Parallel →



$$i_{oc_t} = \frac{V_{oc_1}}{R_{T_1}} + \frac{V_{oc_2}}{R_{T_2}}$$

$$R_{T_t} = \frac{R_{T_1} R_{T_2}}{R_{T_1} + R_{T_2}}$$

$$\boxed{\begin{aligned} V_{oc_{total}} &= \frac{(V_{oc_1} R_{T_2} + V_{oc_2} R_{T_1})}{R_{T_1} + R_{T_2}} \\ R_{T_{total}} &= \frac{R_{T_1} R_{T_2}}{R_{T_1} + R_{T_2}} \end{aligned}}$$

→ If same battery → same voltage, half resistance.

To get 36V, stacking $9V \times 4$ batteries in series or $4V \times 9$

$4 \times 9V$ batteries → $V_{oc} : 36V$, $R_T = 16\Omega$, weight = 120 g ← best.

$9 \times 4V$ batteries → $V_{oc} = 36V$, $R_T = 18\Omega$, weight = 162 g

So, set $4 \times 9V$ series parallel with another $4 \times 9V$ series

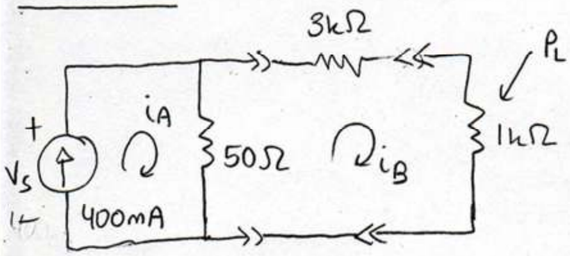
↳ $\boxed{V_{oc} = 36V,\ R_T = 8\Omega,\ \text{weight} = 240g}$

3.96 Compare 2 attenuators. Want $P_L = 25\,mW \pm 10\%$ with $R_L = 1k\Omega$.
Compare again for $P_L = 25\,mW \pm 5\%$.

$P_L = 25\,mW \pm 10\% \rightarrow P_L \in [22.5\,mW,\ 27.5\,mW]$

$\pm 5\% \rightarrow P_L \in [23.75\,mW,\ 26.25\,mW]$

Design 1



Solve with mesh current

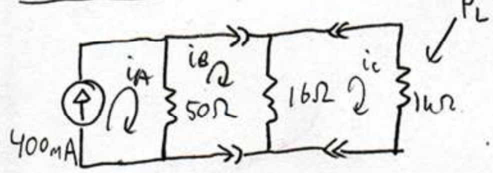
$$\begin{pmatrix} 1 & 0 \\ -50\Omega & 4050\Omega \end{pmatrix} \begin{pmatrix} i_A \\ i_B \end{pmatrix} = \begin{pmatrix} 400mA \\ 0 \end{pmatrix}$$

$\hookrightarrow i_A = 400\,mA, \quad i_B = 4.9\,mA$

$$P_L = i_B^2 R_L = (4.9mA)^2 (1k\Omega) = \underline{\underline{24\,mW}}$$

$$P_S = V_S \cdot i_S = (i_A - i_B)\,50\Omega \cdot i_S = (395.1\,mA)(50\Omega)(400\,mA) = \underline{\underline{7.902\,W}}$$

Design 2



$$\begin{pmatrix} 1 & 0 & 0 \\ -50\Omega & 66\Omega & -16\Omega \\ 0 & -16\Omega & 1016\Omega \end{pmatrix} \begin{pmatrix} i_A \\ i_B \\ i_c \end{pmatrix} = \begin{pmatrix} 400mA \\ 0 \\ 0 \end{pmatrix}$$

$\hookrightarrow i_A = 400\,mA, \quad i_B = 304.2\,mA, \quad i_c = 4.8\,mA$

$$P_L = (4.8\,mA)^2 (1k\Omega) = \underline{\underline{23mW}}$$

$$P_S = (400mA - 304.2\,mA)(50\Omega)(400mA) = \underline{\underline{1.9160\,W}}$$

For $P_L = 25mW \pm 10\%$, both design are within range, but Source power is much less on the 2nd design. Even though 16Ω resistor is less common, the source power requirement is worth it compored to the 1st design.

$\hookrightarrow$ $\boxed{\text{use design 2}}$

For $P_L = 25mW \pm 5\%$, only the 1st design meet the requirement

$\hookrightarrow$ $\boxed{\text{Use Design 1}}$