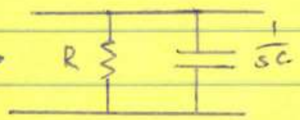
HW #7 SOLUTIONS
AMIT PANDEY

(10-2). For a parallel RC circuit, find $Z_{EQ}(s)$ and then select R & C so that there is a pole at -10 krad/s.

$Z_{eq}(s) \Rightarrow$ 

$$Z_{eq}(s) = \frac{R\left(\frac{1}{sC}\right)}{R + \frac{1}{sC}}$$

$$Z_{eq}(s) = \frac{R}{1 + sRC}$$

Pole is the root of the denominator:

$$1 + sRC = 0$$

$$1 = -sRC$$

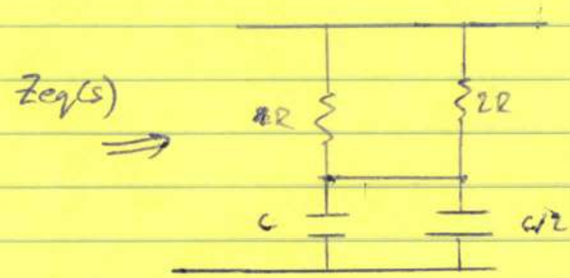
$$s = \frac{-1}{RC}$$

To achieve pole at -10 krad/s, select, $\underline{R = 10\text{k} \text{ and } C = 10\text{nF}}$

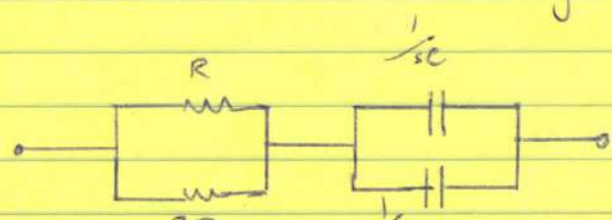
(or any other standard value of R, C which gives the correct location).

(10-4) (a) Find and express $Z_{eq}(s)$ as a rational transfer function and find its poles and zeros

(b) Select R & C to locate ~~pole~~ zero at -33 krad/s. What is the resulting ~~zero~~ pole?



This circuit is equivalent to the following:



$R_{eq} = \frac{2R}{3}$, $\qquad C_{eq} = \frac{2}{3Cs}$ (capacitance adds in parallel)

$$Z_{eq}(s) = \frac{2R}{3} + \frac{2}{3Cs} = \frac{2}{3}\left(R + \frac{1}{Cs}\right)$$

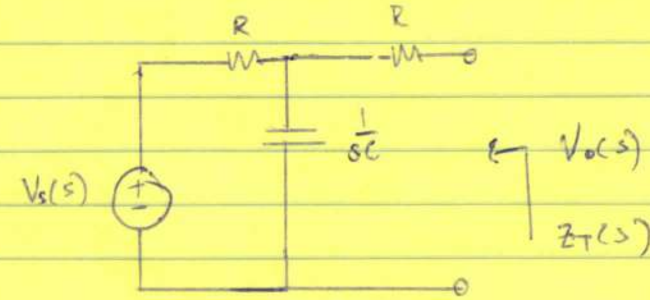
$$= \frac{2}{3}\left(\frac{RCs+1}{Cs}\right)$$

Location of pole $= 0$ rad/s

Location of zero $= \frac{-1}{RC}$ rad/s

(b) $\frac{1}{RC} = 33k$, so $R = 10\,k$, $C = 3\,nF$ (or otherwise)

(10.14)



(a) $$V_o(s) = \frac{V_s(s)}{\left(R + \frac{1}{sC}\right)} \cdot \frac{1}{sC}$$

$$= \frac{V_s(s)}{1 + RCs}$$

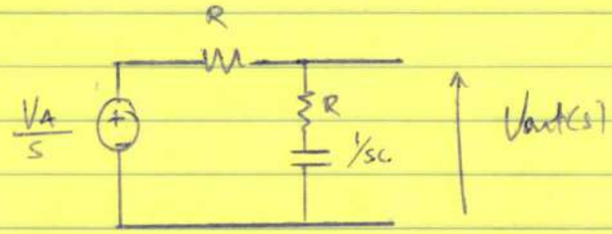
(b) Short out voltage source,

$$Z_T(s) = R \,//\, \frac{1}{sC} + R$$

$$= \frac{\frac{R}{sC}}{R + \frac{1}{sC}} + R \qquad = \frac{R(2 + RCs)}{1 + RCs}$$

(10-16)

$$V_o(s) = \frac{V_A(RCs+1)}{s(2RCs+1)}$$



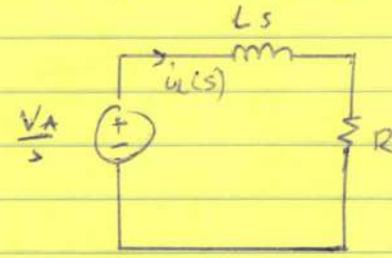
$$V_o(s) = \frac{V_A}{s} \cdot \frac{1}{2R + 1/sC}\left(R + \frac{1}{sC}\right)$$

$$= \frac{V_A}{s} \cdot \frac{1}{\frac{2RsC+1}{sC}} \cdot \frac{RsC+1}{sC}$$

$$= \frac{V_A}{s} \frac{(RsC+1)}{(2RsC+1)}$$

(10-20)

Switch in position (B) allows us to determine initial current through the inductor. There is no energy source, so $i_L(0) = 0$. Now, let us draw the s-domain representation of the switch in position - A.



$$\underline{I_L(s) = \frac{V_A}{s(Ls + R)}}$$

Now, we want to find $I_L(t)$. Need to take the inverse Laplace transform. We do this with a partial fraction expansion

$$I_L(s) = \frac{A}{s} + \frac{B}{s + R/L} = \frac{V_A/L}{s(s + R/L)}$$

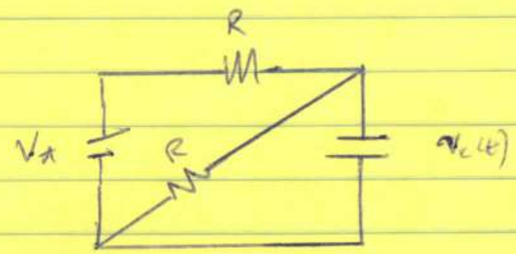
$$\Rightarrow \quad A\left(s + \frac{R}{L}\right) + Bs = \frac{V_A}{L}$$

evaluate @ $s = -R/L$ $\Rightarrow B = \frac{-V_A}{R}$, evaluate @ $s = 0$ $A = \frac{V_A}{R}$

$$\Rightarrow I_L(s) = \frac{V_A/R}{s} + \frac{\left(-\frac{V_A}{R}\right)}{s + R/L}$$

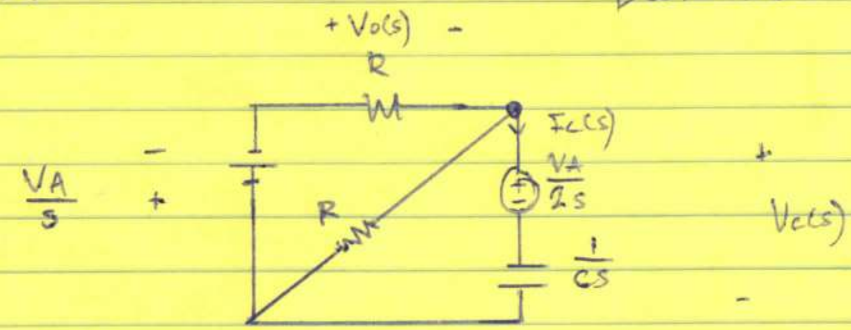
(10-21)

Determine initial condition by analysing circuit in position (A).



$$v_c(0) = \frac{V_A}{2}$$ (voltage divider, capacitor acts as open circuit as $t \to \infty$)

Now, we look at the s-domain circuit in position B.



Now, we find our unknown quantities:

$$\frac{V_c(s) + \frac{V_A}{s}}{R} + \frac{V_c(s)}{R} + \frac{V_c(s) - \frac{V_A}{2s}}{1/Cs} = 0$$

$$\underline{V_c(s) = \frac{V_A}{2s} \frac{(RCs - 2)}{2 + RCs}}$$

$$I_c(s) = \frac{V_c(s) - \frac{V_A}{2s}}{1/sC}$$

$$I_c(s) = 2V_A \cdot C \cdot \left[\frac{-1}{2 + RCs}\right]$$

Now, we can take the inverse Laplace transform.

$$I_L(t) = \mathcal{L}^{-1}\left(\frac{V_A}{R} s\right) + \mathcal{L}^{-1}\left(\frac{-V_A/R}{s + R/L}\right)$$

$$= \frac{V_A}{R} - \frac{V_A}{R} e^{-R/L\, t}$$

$$I_L(t) = \frac{V_A}{R}\left(1 - e^{-R/L\, t}\right) u(t)$$

Now, we want an expression for $V_c(t)$.

We can take a KCL expression about the outer loop,

$$+\frac{V_A}{s} + V_o(s) + V_c(s) = 0$$

$$\Rightarrow \quad V_o(s) = -V_c(s) - \frac{V_A}{s}$$

$$= -\frac{V_A}{2s}\,\frac{(RCs-2)}{(2+RCs)} - \frac{V_A}{s}$$

$$= -V_A\left(\frac{\frac{1}{RC}+\frac{3}{2}s}{s\left(s+\frac{2}{RC}\right)}\right)$$

~~Now doing partial fractions:~~

$$\cancel{-V_A\left(\frac{1}{RC}+\frac{3}{2}s\right) = A\left(s+\frac{2}{RC}\right)+Bs}$$

$$\cancel{\Rightarrow \quad A = -\frac{V_A}{2}, \quad B = -V_A}$$

$$\cancel{\frac{-V_A\left(\frac{1}{RC}+\frac{3}{2}s\right)}{s\left(s+\frac{2}{RC}\right)} = \frac{N_1}{s} + \frac{N_2}{s+\frac{2}{RC}}}$$

~~computing partial fractions~~ ; $\cancel{N_1 = \frac{-V_A}{2}}$

$\cancel{N_2 =}$

$$V_o(s) = -V_A\left(\frac{\frac{1}{RC} + \frac{3}{2}s}{s\left(s + \frac{2}{RC}\right)}\right)$$

Take partial fractions

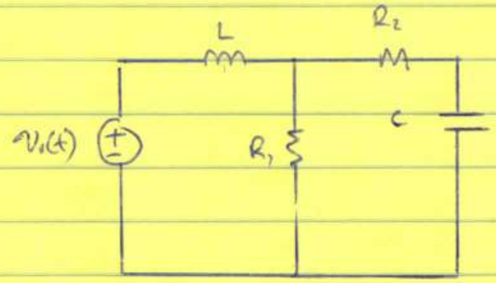
$$\frac{-V_A\left(\frac{1}{RC} + \frac{3}{2}s\right)}{s\left(s + \frac{2}{RC}\right)} = \frac{A}{s} + \frac{B}{s + \frac{2}{RC}}$$

$$\Rightarrow A = -\frac{V_A}{2}, \quad B = -V_A$$

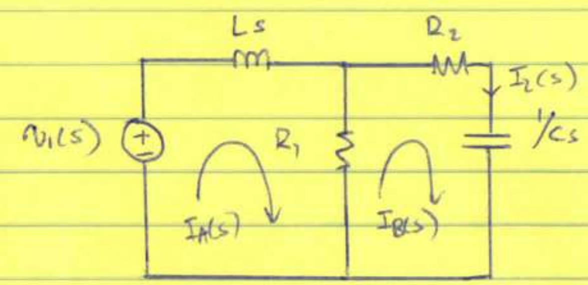
$$\Rightarrow V_o(s) = \frac{-V_A/2}{s} - \frac{V_A}{s + \frac{2}{RC}}$$

$$\Rightarrow V_o(t) = -\left[\frac{V_A}{2} + V_A e^{-\frac{2t}{RC}}\right] u(t)$$

10-41)



(a)



Express the circuit in the s-domain with no initial energy

Let us formulate the mesh current equations:

① $-V_1(s) + I_A(s)Ls + R_1(I_A(s) - I_B(s)) = 0$

$$V_1(s) = I_A(s)(Ls + R_1) - I_B(s)R_1$$

② $0 = (R_1 + R_2 + \frac{1}{Cs})I_B(s) - R_1 I_A(s) = 0$

(1*) $I_A(s) = \dfrac{V_1(s) + I_B(s)R_1}{Ls + R_1}$

$$I_B(s) = \frac{R_1 I_A(s)}{\frac{cs(R_1+R_2)+1}{cs}} = \frac{R_1 cs}{cs(R_1+R_2)+1}\left[\frac{V_1(s) + I_B(s)R_1}{Ls + R_1}\right]$$

$$I_B(s)\left[1 - \frac{R_1^2 cs}{(cs(R_1+R_2)+1)(Ls+R_1)}\right] = \frac{V_1(s)\cdot R_1 cs}{(cs(R_1+R_2)+1)(Ls+R_1)}$$

$$I_B(s)\left[(cs(R_1+R_2)+1)(Ls+R_1) - R_1^2 cs\right] = V_1(s)\cdot R_1 cs$$

$$\Rightarrow I_2(s) = \frac{R_1 Cs V_1(s)}{(R_1+R_2)LCs^2 + (R_1R_2C + L)s + R_1}$$

(c) Single zero of system located at $s = 0$.

Poles of system are roots of :

$$s^2(R_1+R_2)LC + (R_1R_2C + L)s + R_1 = 0$$

By the quadratic formula

$$p = \frac{-(R_1R_2C + L) \pm \sqrt{(R_1R_2C + L)^2 - 4(R_1+R_2)LCR_1}}{2(R_1+R_2)LC}$$

(d) $v_i(t) = 10u(t)$, $R_1 = 1K$, $R_2 = 2K$, $L = 2$, $C = 0.5\mu F$

$$I_2(s) = \frac{1K \times 0.5\mu\ s \cdot 10/s}{[3K]1\mu\ s^2 + (2K \cdot 1K \cdot 0.5\mu + 2)s + 1K}$$

$$= \frac{5\times10^{-3}}{(3\times10^{-3})s^2 + 3s + 1000}$$

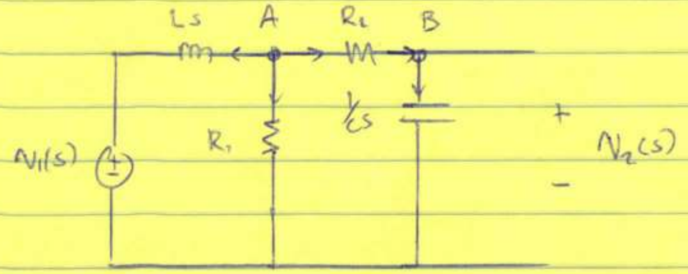
$$I_2(s) = \frac{5/3}{s^2 + 1000s + (\frac{1}{3})\times10^6} = \frac{5/3}{(s+500)^2 + (\frac{500}{\sqrt{3}})^2}$$

~~Now we take partial fractions~~ Taking the inverse laplace transform,

$$\frac{\sqrt{3}}{300} e^{-500t} \cdot \sin\left(\frac{500}{\sqrt{3}}t\right)u(t)$$

(10-42)

(a) Transform circuit into s domain and formulate node-voltage equations



Node voltage equations:

$$\cdot(1) \qquad \frac{v_A(s) - v_1(s)}{sL} + \frac{v_A(s)}{R_1} + \frac{v_A(s) - v_B(s)}{R_2} = 0$$

$$\cdot(2) \qquad v_B(s) \cdot sC + \frac{v_B(s) - v_A(s)}{R_2} = 0$$

(b) Solving for $v_B(s)$ gives

$$v_2(s) = v_B(s) = \frac{R_1 v_1(s)}{(R_1+R_2)LCs^2 + (R_1R_2C + L)s + R_1}$$

(c) Natural ~~resp~~ poles are the poles caused by circuit elements, not in response to external stimulus:

$$p = \frac{-(R_1R_2C + L) \pm \sqrt{(R_1R_2C + L)^2 - 4(R_1+R_2)LC \cdot R_1}}{2(R_1+R_2)LC}$$

Forced poles are unknown without knowing what $v_1(s)$ is.

(d) $V_1(t) = 25u(t)$, $R_1 = R_2 = 500$, $L = 0.5$, $C = 2\mu F$.

$$V_2(s) = \frac{500 \times \frac{25}{s}}{(500+500)0.5\times 2\mu F\, s^2 + (500\times 500\times 2\mu + 0.5)s + 500}$$

$$= \frac{1.25\times 10^7}{s\left(s^2 + 1000s + 0.5\times 10^6\right)} = \frac{12.5\times 10^6}{s\left((s+500)^2 + 500^2\right)}$$

$$= \frac{N_1}{s} + \frac{N_2 s + N_3}{(s+500)^2 + 500^2}$$

$$\Rightarrow \quad 1.25\times 10^7 = N_1\left((s+500)^2 + 500^2\right) + (N_2 s + N_3)s$$

@ $s=0$

$$N_1 = \underline{25}$$

Now for N_1 & N_2:

$N_1 s^2 + N_2 s^2 = 0$ (compare coefficients of s^2)

$\Rightarrow N_1 + N_2 = 0$

$\Rightarrow \underline{N_2 = -25}$

Compare coefficients of s

$$1000 N_1 + N_3 = 0$$

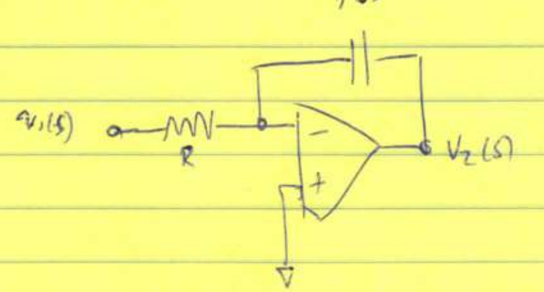
$$\Rightarrow \underline{N_3 = -25000}$$

$$V_2(s) = \frac{25}{s} + \frac{(-25)s - 25000}{(s+500)^2 + 500^2}$$

$$= \frac{25}{s} - 25\left(\frac{s+500}{(s+500)^2+500} + \frac{500}{(s+500)^2+500}\right)$$

$$\Rightarrow \quad v_2(t) = \left[25 - 25e^{-500t}\cos(500t) - 25e^{-500t}\sin(500t)\right]u(t)$$

10-61



$v_1(t) = \cos(1000t)$
$v_2(t) = -\sin(1000t)$

This is an inverting amplifier with a gain of $\frac{-R_2}{R_1} = \frac{-1}{CRs}$

So, $$\frac{V_2(s)}{V_1(s)} = -\frac{1}{CRs}.$$

$$V_2(s) = \frac{-1}{CRs} \cdot V_1(s)$$

$$V_1(s) = \mathcal{L}\left(\cos(1000t)\right) = \frac{s}{s^2+1000^2}$$

$$V_2(s) = \frac{-1}{CRs} \cdot \frac{s}{s^2+1000^2}$$

$$V_2(s). = \frac{-1}{RC} \cdot \frac{1}{s^2+1000^2}$$

The laplace transform of ~~-1000~~ $-\sin(1000t)$ is $\frac{-1000}{s^2+1000^2}$.

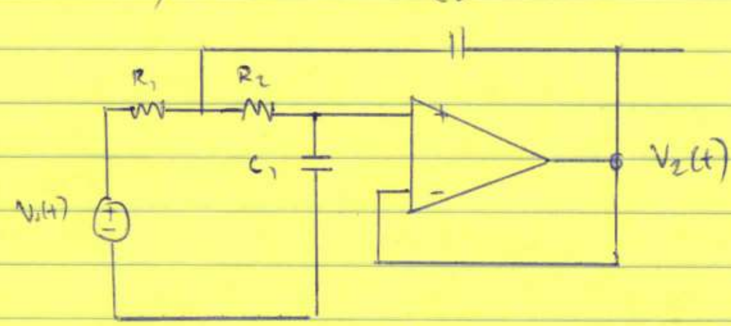
so we require $\frac{1}{RC} = 1000. \Rightarrow \underline{R = 1k,\ C = 1\mu F}$

or similar.

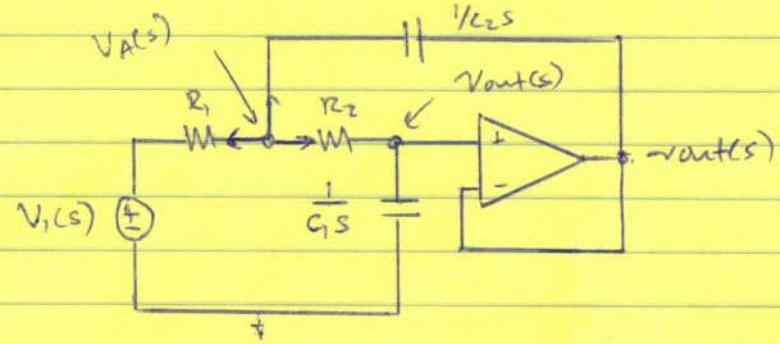
What if capacitor had 10V across it initially?

By superposition we can conclude that the output voltage will decrease by 10V.

10-68/



First, we find the s - domain representation:



We have : $V_-(s) = V_+(s) = V_{out}(s)$.

so,

$$\frac{V_A(s) - V_1(s)}{R_1} + \frac{V_A(s) - V_{out}(s)}{R_2} + \left(V_A(s) - V_{out}(s)\right)(C_2 s) = 0$$

(1) $$V_A(s)\left[\frac{1}{R_1} + \frac{1}{R_2} + C_2 s\right] = \frac{V_1(s)}{R_1} + \frac{V_{out}(s)}{R_2} + V_{out}(s)\, C_2 s$$

(2) $$V_{out}(s) = \frac{V_A(s)}{R_2 + \frac{1}{C_1 s}} \cdot \frac{1}{C_1 s} = \frac{V_A(s)}{1 + R_2 C_1 s}$$

$$\Rightarrow V_A(s) = V_{out}(s)\left(1 + R_2 C_1 s\right)$$

$$\cancel{\Rightarrow V_{out}(s)(1 + R_2 C_1 s)\left[\frac{1}{R_1} + \frac{1}{R_2} + C_2 s\right] = \frac{V_1(s)}{R_1} + V_{out}(s)\left[\frac{1}{R_2} + C_2 s\right]}$$

From (1)

$$V_A(s)\left[R_1 + R_2 + R_1R_2C_2s\right] = V_1(s)R_2 + V_{out}(s)(R_1 + R_1R_2C_2s)$$

$$\Rightarrow V_{out}(s)\left[(1+R_2C_1s)(R_1+R_2+R_1R_2C_2s) - R_1 - R_1R_2C_2s\right] = V_1(s)R_2$$

$$\Rightarrow \frac{V_{out}(s)}{V_1(s)} = \frac{R_2}{(1+R_2C_1s)(R_1+R_2+R_1R_2C_2s) - R_1 - R_1R_2C_2s}$$

$$= \frac{R_2}{R_1R_2^2C_1C_2s^2 + (R_1R_2C_1 + R_2^2C_1)s + R_2}$$