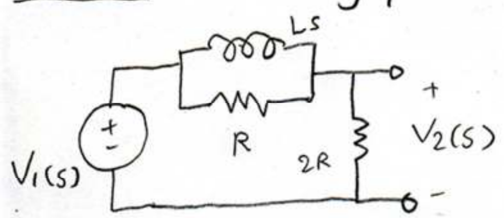
MAE 140 HW 8 Solution

11-2 Find driving point impedance and TF $T_V(s) = V_2(s) / V_1(s)$

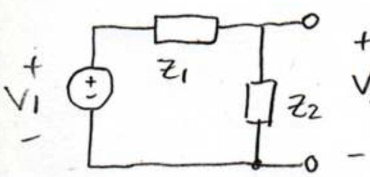


$$Z_1 = \left(\frac{1}{R} + \frac{1}{Ls}\right)^{-1} = \frac{RLs}{R+Ls}$$

$$Z_2 = 2R$$

Driving pt impedance $\rightarrow$ $Z_d = Z_1 + Z_2$

$$\boxed{Z_d = \frac{RLs}{R+Ls} + 2R}$$

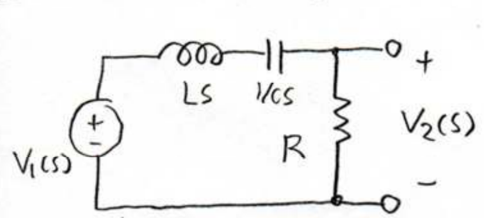


Volt Divider

$$V_2(s) = \frac{Z_2}{Z_1 + Z_2} V_1(s) = \frac{2R}{2R + \frac{RLs}{R+Ls}} V_1(s)$$

$$\boxed{T_V(s) = (R+Ls)\frac{2R}{2R^2 + 3RLs}}$$

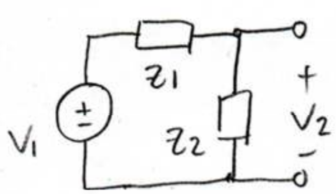
11-3 Find driving pt impedance and $T_V(s) = V_2(s)/V_1(s)$



$$Z_1 = Ls + \frac{1}{Cs} = \frac{1 + LCs^2}{Cs}$$

$$Z_2 = R$$

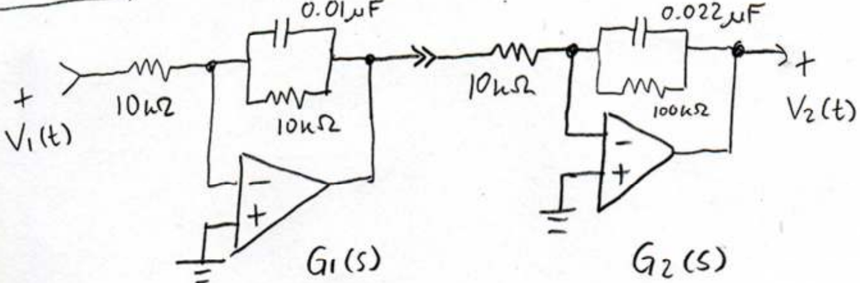
$$\boxed{Z_d = Z_1 + Z_2 = R + Ls + \frac{1}{Cs} = \frac{1 + RCs + LCs^2}{Cs}}$$



$$V_2(s) = \frac{Z_2}{Z_1 + Z_2} V_1(s) = \frac{R}{R + Ls + \frac{1}{Cs}} V_1(s)$$

$$\boxed{T_V(s) = \frac{RCs}{1 + RCs + LCs^2}}$$

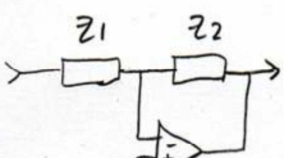
11-14 Find $T_V(s) = V_2(s)/V_1(s)$ and its poles and zeros.



$G_1(s)$ $G_2(s)$

Cascade of 2 inverting OP Amps.

$T_V(s) = G_1(s) \cdot G_2(s)$



$TF = -\frac{Z_2}{Z_1}$

$$G_1(s) = \frac{-\left(10k\Omega \,\|\, \frac{1}{0.01\mu F\, s}\right)}{10\,k\Omega} = -\frac{\left(\frac{1}{10^4\Omega} + 10^{-8}\mu F \cdot s\right)^{-1}}{10^4\,\Omega}$$

$$= -\frac{\frac{10^4}{1+10^{-4}s}}{10^4} = -\frac{1}{1+10^{-4}s} = -\frac{10000}{s+10000}$$

$$G_2(s) = -\frac{(100k\Omega \,\|\, 0.022\mu F)}{10\,k\Omega} = -\frac{\left(\frac{1}{10^5} + 22\cdot 10^{-9}s\right)^{-1}}{10^4}$$

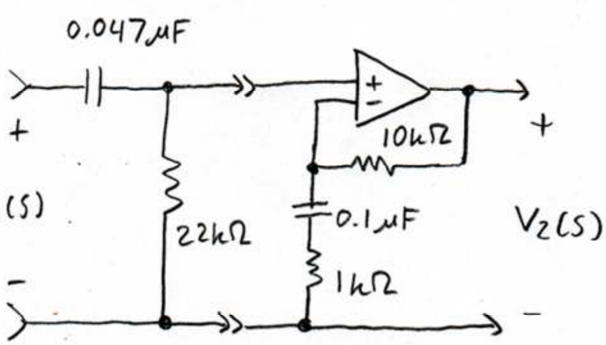
$$= -\frac{10^5}{10^4}\left(\frac{1}{1+22\cdot 10^{-4}s}\right) = -\frac{4545}{s+454.5}$$

$T_V(s) = G_1(s) \cdot G_2(s)$

$$\boxed{T_V(s) = 4.545\cdot 10^7\left(\frac{1}{s+10000}\right)\left(\frac{1}{s+454.5}\right)}$$

$$\boxed{\begin{array}{l} \text{zeros} = \{\} \\ \text{poles} = \{-10^4, -454.5\} \end{array}}$$

11-15 Find $T_V(s) = V_2(s)/V_1(s)$ and its poles and zeros.



$G_1(s)$ $G_2(s)$

Cascade of Volt. divider and non-inverting OP Amp.

It's non-loading, so

$T_V(s) = G_1(s) \cdot G_2(s)$

$$G_1(s) = \frac{22k\Omega}{22k\Omega + \frac{1}{47\cdot 10^{-9}F\cdot s}} = \frac{22k\Omega \cdot 47\cdot 10^{-9} F\cdot s}{1 + 22k\Omega \cdot 47\cdot 10^{-9} F\cdot s} = \frac{1.034\cdot 10^{-3} s}{1 + 1.034\cdot 10^{-3} s}$$

$$\boxed{G_1(s) = \frac{s}{s + 967.12}}$$

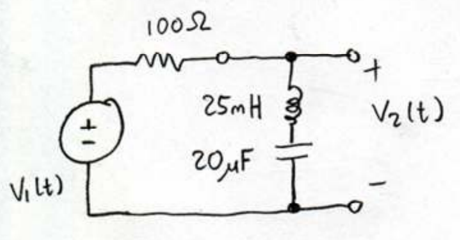
$$G_2(s) = 1 + \frac{10k\Omega}{1k\Omega + \frac{1}{0.1\cdot 10^{-6} F\cdot s}} = 1 + \frac{10^4 \cdot 0.1\cdot 10^{-6} s}{1 + 10^3 \cdot 0.1 \cdot 10^{-6} s} = 1 + \frac{10^{-3} s}{1 + 10^{-4} s}$$

$$G_2(s) = 1 + \frac{10s}{s + 10000} = \frac{11s + 10000}{s + 10000} \rightarrow \boxed{G_2(s) = 11\left(\frac{s + 909.1}{s + 10000}\right)}$$

$$T_V(s) = G_1(s) \cdot G_2(s) \rightarrow \boxed{T_V(s) = 11 \frac{s(s+909.1)}{(s+967.12)(s+10000)}}$$

$$\boxed{\text{zeros} = \{-909.1, 0\} \quad \text{poles} = \{-967.12, -10000\}}$$

11-34 Find $V_{2ss}(t)$ for $v_1(t) = 5\cos(1414.21t)\,V$, $v_1(t) = 5\cos(1kt)\,V$ and $V_1(t) = 5V$



Volt. Divider

Find the transfer function.

$$T_V(s) = \frac{V_2(s)}{V_1(s)} = \frac{\left(25mH\cdot s + \frac{1}{20\mu F\cdot s}\right)}{100\Omega + \left(25mH\cdot s + \frac{1}{20\mu F\cdot s}\right)}$$

$$= \frac{5\cdot 10^{-7} s^2 + 1}{5\cdot 10^{-7} s^2 + 0.002s + 1}$$

$$\boxed{T_V(s) = \frac{s^2 + 2\cdot 10^6}{s^2 + 4000s + 2\cdot 10^6}}$$

Find magnitude and phase of $T_V(j\omega)$ at the ω of the sinusoidal input.

$$T_V(j\cdot 1414.21) = \frac{(1414.21)^2 j^2 + 2\cdot 10^6}{(1414.21)^2 j^2 + 4000\cdot 1414.21j + 2\cdot 10^6} = \frac{10.0759}{10.0759 + j\,5.657\cdot 10^6}$$

$$T_V(j\,1414.21) = \frac{10.076}{10.076 + j\,5.657\cdot 10^6}\left(\frac{10.076 - j\,5.657\cdot 10^6}{10.076 - j\,5.657\cdot 10^6}\right)$$

$$= \frac{101.526 - j\,5.7\cdot 10^7}{3.2\cdot 10^{13}}$$

$$\left|T_V(j\,1414.21)\right| = \frac{\sqrt{101.526^2 + (5.7\cdot 10^7)^2}}{3.2\cdot 10^{13}} = \underline{\underline{1.7813\cdot 10^{-6}}}$$

$$\measuredangle T_V(j1414.21) = \tan^{-1}\left(\frac{-5.7\cdot 10^7}{101.526}\right) = \underline{-1.5708 \text{ rad} \quad \text{or} \quad -90°}$$

For $V_1(t) = 5\cdot\cos(1414.12t)$ V

$\hookrightarrow V_{2ss}(t) = (5\cdot 1.7813\cdot 10^{-6})\cos(1414.12t - 1.5708)$ V

$$\boxed{V_{2ss}(t) := 8.906\cdot 10^{-6}\cos(1414.21t - 1.5708)\ \text{V}}$$

Similarly for $V_1(t) = 5\cos(1kt)$ V, find magnitude and phase of $T_V(j1000)$.

$$T_V(j1000) = \frac{-10^6 + 2\cdot 10^6}{-10^6 + j4\cdot 10^6 + 2\cdot 10^6} = \frac{10^6}{10^6 + j\cdot 4\cdot 10^6} = \frac{10^{12}(1-4j)}{17\cdot 10^{12}} = \underline{\underline{\frac{1-4j}{17}}}$$

$$\left|T_V(j1000)\right| = \frac{\sqrt{1+16}}{17} = \underline{\underline{0.2425}} \qquad \measuredangle T_V(1000j) = \tan^{-1}\left(\frac{-4}{1}\right) = \underline{\underline{-1.3258}}$$

$\hookrightarrow V_{2ss}(t) = (5\cdot 0.2425)\cos(1kt - 1.3258)$ V

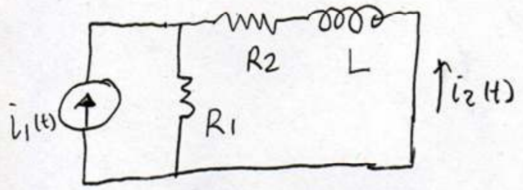
$\hookrightarrow \boxed{V_{2ss}(t) = 1.2620\cdot\cos(1kt - 1.3258)\ \text{V}}$

For $V_1(t) = 5$V, use $V_1(t) = 5\text{V}\cdot\cos(0\cdot t) \rightarrow \omega = 0$

$T_V(0) = 1 \rightarrow \boxed{V_{2ss}(t) = 5\text{V}}$

11-37

Find $i_{2ss}(t)$ w/ $i_1(t) = 10\cos(50kt)$ mA, $i_2(t) = 10\cos(5kt)$ mA.
Where is the pole? $R_1 = 100\Omega$, $R_2 = 400\Omega$, $L = 100$ mH.



Current Divider

Find the transfer function $G(s) = \frac{I_2(s)}{I_1(s)}$

$$G(s) = \frac{-R_1}{R_1+R_2+Ls} = \frac{-100}{500+0.1s}$$

$$\boxed{G(s) = \frac{-1000}{s+5000}} \longrightarrow \text{Pole} = -5000$$

↳ $i_1(t) = 10\cos(50kt)$ mA

$$G(j\cdot 50k) = \frac{-1000}{50000j+5000} = \frac{-1}{2525}(5-50j) = \frac{-1+10j}{505}$$

95.7° ↗

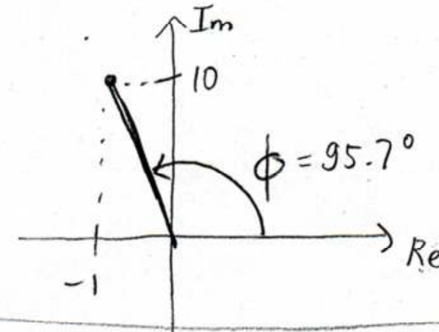
$$|G(j50k)| = \frac{\sqrt{100+1}}{505} = 0.02 \qquad \measuredangle G(j50k) = \tan^{-1}\left(\frac{10}{-1}\right) = 1.6705 \text{ rad}$$

!!

$\tan^{-1}(-10) = -84.3^\circ$, but we want 95.7°.

Always check the complex plane diagram.

Use atan2 or phase in Matlab.



$$i_{2ss}(t) = (10 \cdot 0.02)\cos(50kt + 1.6075)\text{ mA} = 0.2\cos(50kt+1.6075)\text{ mA}$$

↳ $i_1(t) = 10\cos(5kt)$ mA

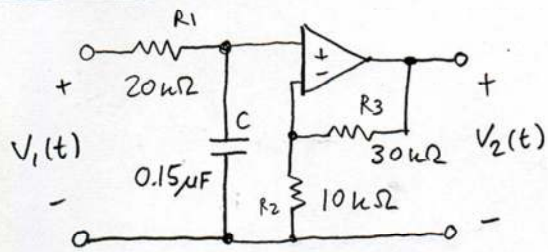
$$G(j5k) = \frac{-1000}{5000j+5000} = \frac{-1}{50}(5-5j) = \frac{-1+j}{10}$$

135° ↗

$$|G(j5k)| = \frac{\sqrt{2}}{10} = 0.1414 \qquad \measuredangle G(j5k) = \tan^{-1}\left(\frac{1}{-1}\right) = 2.3562 \text{ rad}$$

$$\boxed{i_{2ss}(t) = 1.414\cos(5kt + 2.3562)\text{ mA}}$$

12-6 | Find $T_V(s) = V_2(s) / V_1(s)$!



Volt Div → Non-inv- OP Amp

$$T_V(s) = G_1(s) \cdot G_2(s) = \left(\frac{1/cs}{R_1 + 1/cs}\right)\left(1 + \frac{R_3}{R_2}\right)$$

$$= \frac{1}{1+(20k\Omega)(0.15\mu F)s}\left(\frac{40k\Omega}{10k\Omega}\right) = \frac{4}{1+0.003s}$$

$$\boxed{T_V(s) = \frac{1333}{s + 333.3}}$$

a) DC Gain → $|T_V(0)| = \underline{4}$

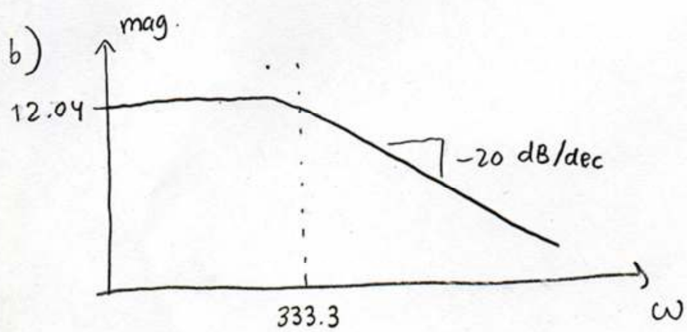
$\infty\omega$ gain → $|T_V(j\infty)| = \underline{0}$

cutoff ω → $\underline{\omega_c = 333.3 \text{ rad/s}}$

Response type : Low-pass filter

$20 \cdot \log_{10}(4) = 12.04$ dB

b)



d) on other page

c) $|T_V(0.1\omega_c)| = \text{DC Gain} = 4$

$|T_V(\omega_c)| = \text{DC Gain}/\sqrt{2} = 2.8284$

$|T_V(10\omega_c)| = \text{DC Gain}/10 = 0.4$

Actual values :

3.98

2.8284

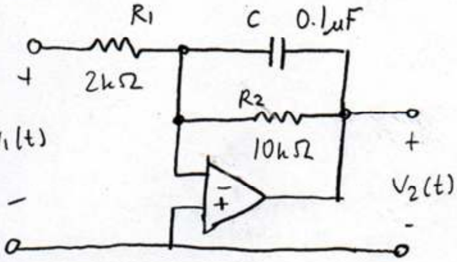
0.398

e) Should change the OP Amp's gain. Find R_3 such that $G_2(s) = 1 + \frac{R_3}{R_2} = 10$

↳ $R_3 = 9R_2 = \underline{90k\Omega}$

f) About $\underline{-20 \text{ dB}}$

12.7 Find $T_v(s) = V_2(s)/V_1(s)$



Inverting OP Amp

$$T_v(s) = -\frac{\left(\frac{1}{R_2} + Cs\right)^{-1}}{R_1} = -\frac{R_2}{R_1}\left(1 + R_2 C s\right)^{-1}$$

$$= -\frac{10k\Omega}{2k\Omega}\,\frac{1}{(1 + 10k\Omega \cdot 0.1\mu F \cdot s)} = \frac{-5}{1 + 0.001s}$$

$$\boxed{T_v(s) = \frac{-5000}{s + 1000}}$$

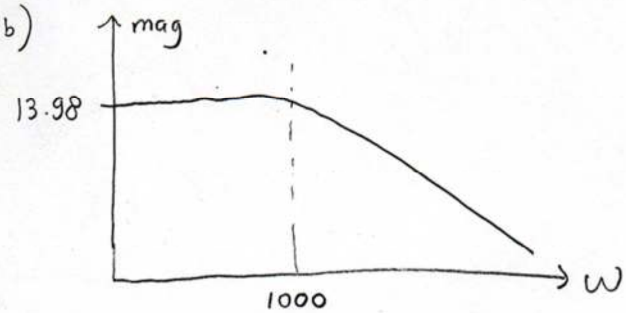
a) DC Gain $= |T_v(0)| = \underline{+5}$

$\infty\omega$ gain $= |T_v(j\infty)| = \underline{0}$

$\omega_c = \underline{1000 \text{ rad/s}}$

Type = low pass filter

b)



$20 \cdot \log^{10}(5) = 13.98$ dB

d) on other page.

c) Gain at $\omega = 0.1\omega_c \rightarrow |T_v(j0.1\omega_c)| \simeq$ DC Gain $= \underline{5}$

Gain at $\omega = \omega_c \rightarrow |T_v(j\omega_c)| =$ DC Gain$/\sqrt{2} = \underline{3.536}$

Gain at $\omega = 10\omega_c \rightarrow |T_v(j10\omega_c)| =$ DC Gain$/10 = \underline{0.5}$

Actual value :-

4.9752

3.5355

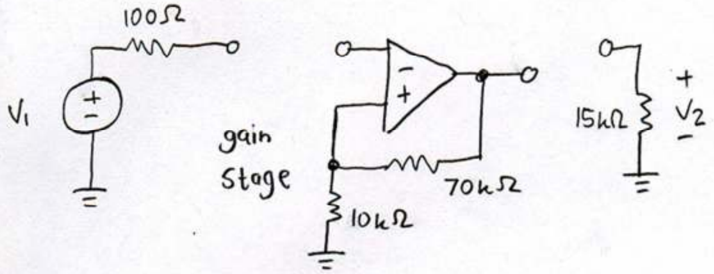
0.4975

e) Double the passband gain

↳ make $\frac{R_2}{R_1} = 10$ instead of 5 $\rightarrow \boxed{R_1 = 1k\Omega}$

Can't change R_2 because it affects the ω_c.

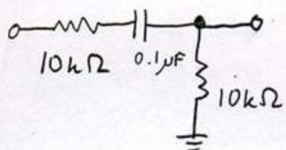
12-8 Connect to make a $T_V(s)$ w/ gain 4, $\omega_c = 500$ rad/s.
Repeat if $\omega_c = 625$ rad/s and gain doesn't matter.



3 options:

1. Just the filter stage
2. Filter stage → gain stage
3. Gain stage → filter stage.

filter stage



→ Only the filter stage affects the ω_c.
Op Amp always have the same gain.

$$\text{OP Amp gain} = 1 + \frac{70k\Omega}{10k\Omega} = 8$$

Just the filter:

$$T_V(s) = \frac{\overbrace{(10k\Omega \parallel 15k\Omega)}^{\to 6k\Omega}}{100\Omega + 10k\Omega + \frac{1}{0.1\mu F\, s} + (10k\Omega \parallel 15k\Omega)}$$

$$= \frac{6k\Omega \cdot 0.1\mu F \cdot s}{16.1k\Omega \cdot 0.1\mu F \cdot s + 1} = \frac{0.0006 \cdot s}{1 + 0.00161 s} = \frac{0.3727 \cdot s}{s + 621}$$

Filter → gain

$$T_V(s) = \left(\frac{10k\Omega \cdot \overset{\checkmark \text{ op Amp gain}}{8}}{100\Omega + 10k\Omega + \frac{1}{0.1\mu F s} + 10k\Omega} \right) = \frac{10k\Omega \cdot 0.1\mu F s \cdot 8}{20.1k\Omega \cdot 0.1\mu F s + 1}$$

$$= \frac{0.001 \cdot 8 \cdot s}{1 + 0.00201 s} = \frac{3.98 \cdot s}{s + 497.5}$$

gain → filter

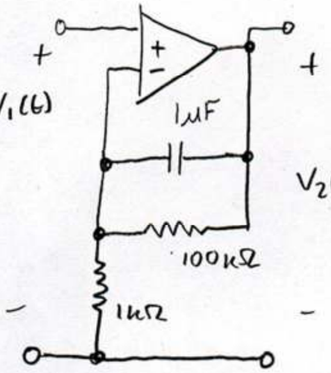
$$T_V(s) = 8 \left(\frac{(10k\Omega \parallel 15k\Omega)}{10k\Omega + \frac{1}{0.1\mu F s} + (10k\Omega \parallel 15k\Omega)} \right) = \frac{8 \cdot 6k\Omega \cdot 0.1\mu F \cdot s}{1 + 16k\Omega \cdot 0.1\mu F \cdot s}$$

$$= \frac{8 \cdot 0.0006 \cdot s}{1 + 0.0016 s} = \frac{3 \cdot s}{s + 625}$$

So, use filter stage → gain stage for T_V with gain 4 and $\omega_c = 500$ rad/s

use gain stage → filter stage for T_V w/ $\omega_c = 625$ rad/s

$\boxed{12\text{-}44}$ Find $T_V(s) = V_2(s)/V_1(s)$



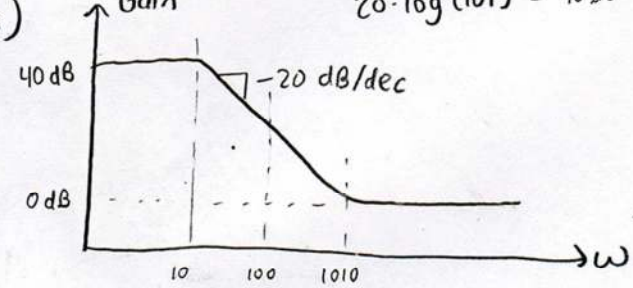
$$T_V(s) = 1 + \frac{\left(100k\Omega \parallel \frac{1}{1\mu F s}\right)}{1k\Omega} = 1 + \frac{\left(\frac{1}{100k\Omega} + 1\mu F s\right)^{-1}}{1k\Omega}$$

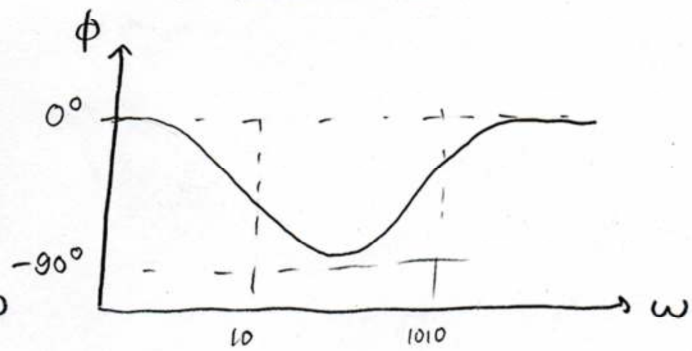
$$= 1 + \frac{\dfrac{100k\Omega}{1 + 100k\Omega\, 1\mu F s}}{1k\Omega} = 1 + \frac{100}{1 + 0.1 s}$$

$$= 1 + \frac{1000}{s+10} = \underline{\underline{\frac{s + 1010}{s+10}}}$$

Non-inverting amp.

a) $20 \cdot \log^{10}(101) = 40\,dB$





For $v_1(t) = 10 \sin(100t)$ V, gain ≈ 20 dB $= 10$, phase $\approx -90° = -\pi/2$

$\hookrightarrow$ $\boxed{V_2(t) \approx 100 \sin(100t - \pi/2)}$

b) Actual gain and phase

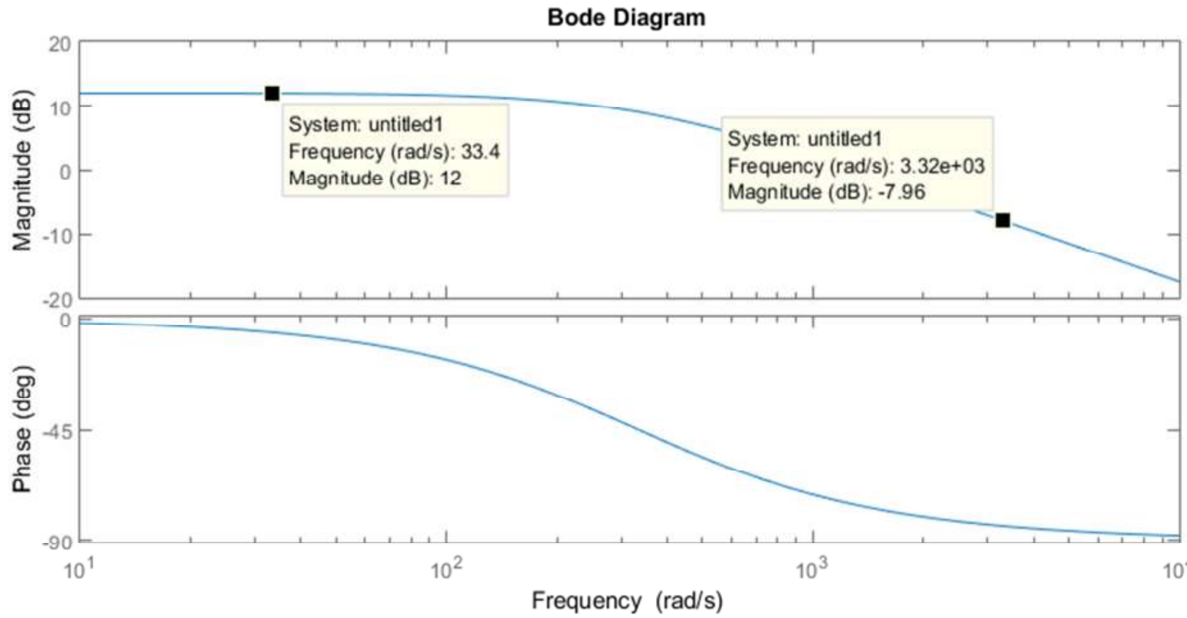
$$T_V(100j) = \frac{100j + 1010}{100j + 10} = \frac{(1010 + 100j)(10 - 100j)}{100^2 + 10^2}$$

$$= \frac{10100 + 100^2 + 1000j - 101000j}{10100} = \frac{201 - 1000j}{101}$$

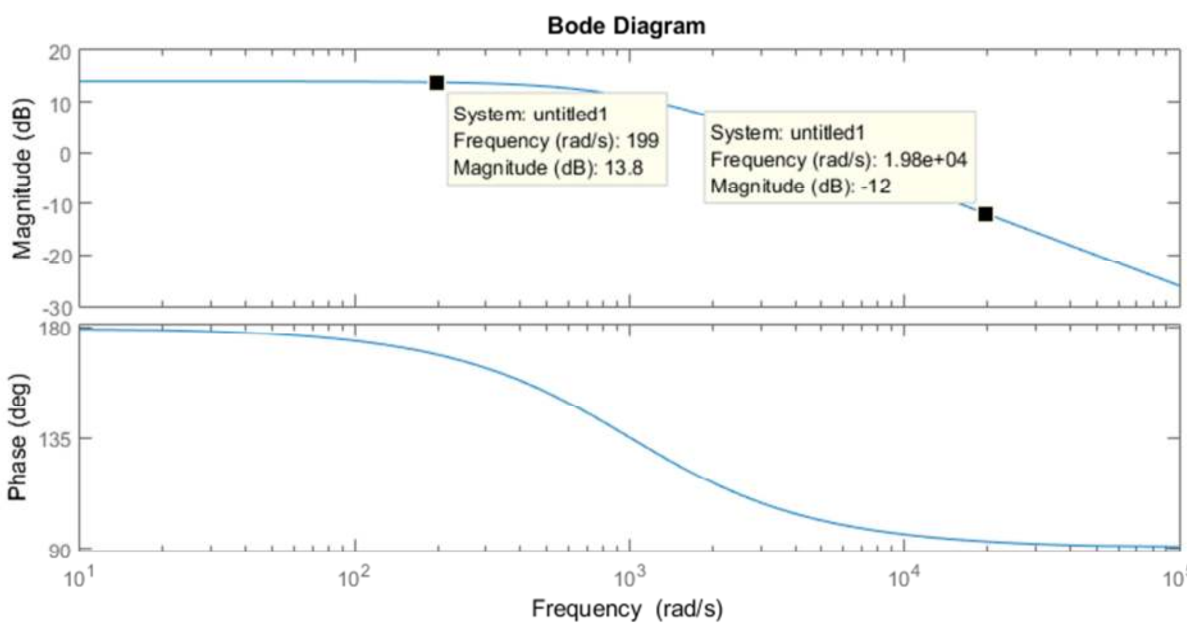
$$|T_V(100j)| = \frac{\sqrt{201^2 + 1000^2}}{101} = \underline{\underline{10.099}} \qquad \measuredangle T(100j) = \underline{\underline{-1.3724 \text{ rad}}}$$

c) on other page.

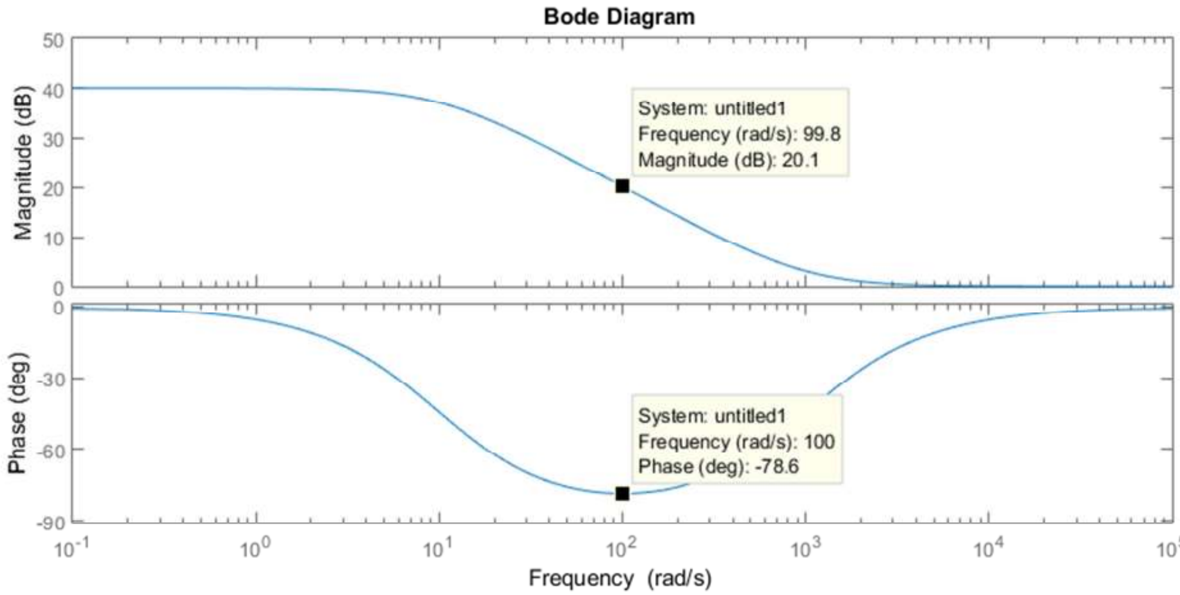
12.6) d


Bode Diagram
System: untitled1
Frequency (rad/s): 33.4
Magnitude (dB): 12
System: untitled1
Frequency (rad/s): 3.32e+03
Magnitude (dB): -7.96
Magnitude (dB)
Phase (deg)
Frequency (rad/s)
20
10
0
-10
-20
0
-45
-90
10^1
10^2
10^3
10^4

12.7) d


Bode Diagram
System: untitled1
Frequency (rad/s): 199
Magnitude (dB): 13.8
System: untitled1
Frequency (rad/s): 1.98e+04
Magnitude (dB): -12
Magnitude (dB)
Phase (deg)
Frequency (rad/s)
20
10
0
-10
-20
-30
180
135
90
10^1
10^2
10^3
10^4
10^5

12.44) c



Bode Diagram
Magnitude (dB)
Phase (deg)
System: untitled1
Frequency (rad/s): 99.8
Magnitude (dB): 20.1
System: untitled1
Frequency (rad/s): 100
Phase (deg): -78.6
Frequency (rad/s)