MAE140 - Linear Circuits - Fall 2015
Midterm, October 30th

Instructions

1. This exam is open book. You may use whatever written materials you choose, including your class notes and textbook. You may use a hand calculator with no communication capabilities.
2. You have 50 minutes.
3. Write your answers on your "Blue Book".
4. Do not forget to write your name, student number, and instructor.

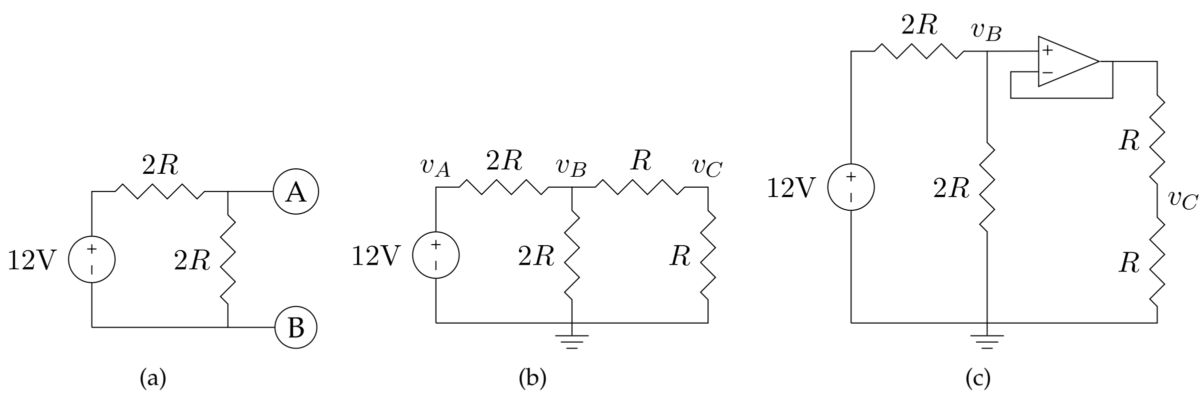
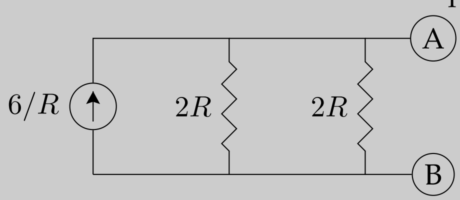


Figure 1: Circuits for all questions

1. **Equivalent Circuits**

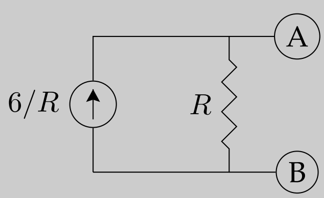
   (a) (5 points) For the circuit in Fig. 1(a), find the Thevenin and Norton equivalent circuits as seen from terminals A and B.

Solution:
We will use source transformations. First transform the voltage source in series with the resistor $2R$ into a current source in parallel with a resistor $2R$ to obtain the circuit:



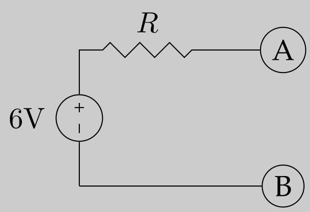
(+1 point)

Associate the two resistors in parallel to obtain the Norton equivalent:



(+2 points)

Apply another source transformation to obtain the Thevenin Equivalent:



(+2 points)

NOTE TO GRADERS: Other solutions are possible.

2. **Node Voltage Analysis**

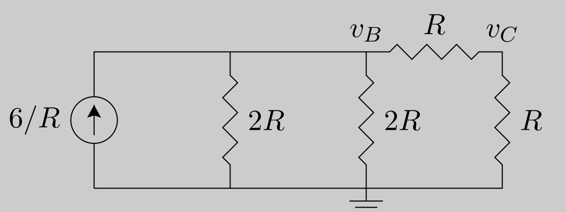
   (a) (3 points) Formulate node-voltage equations for the circuit in Fig. 1(b). Use the node-voltages and labels provided in the figure and clearly indicate the final equations and circuit variable unknowns. Make sure your final equations involve only node-voltages. You do not need to solve any equations at this point.

Solution:

We shall first convert the voltage source to a current source. We will use method #1, that is source transformation.

NOTE TO GRADERS: Method #2 is also fine!

Just as in problem 1:



(+1 point)

By inspection:

$$\begin{bmatrix} \frac{1}{2R} + \frac{1}{2R} + \frac{1}{R} & -\frac{1}{R} \\ -\frac{1}{R} & \frac{1}{R} + \frac{1}{R} \end{bmatrix} \begin{pmatrix} v_B \\ v_C \end{pmatrix} = \begin{pmatrix} \frac{6}{R} \\ 0 \end{pmatrix}$$ (+1 point)

These are the node-voltage equations that need to be solved for the node voltages v_B and v_C. v_A is obtained from the voltage source in the original circuit as $v_A = 12\text{V}$. (+1 point)

   (b) (2 points) Solve the node-voltage equations you obtained in (a) to calculate v_B and v_C.

Hint: $\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}^{-1} = \frac{1}{3}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$

Solution:

Multiplying by R on both sides

$$\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} \begin{pmatrix} v_B \\ v_C \end{pmatrix} = \begin{pmatrix} 6 \\ 0 \end{pmatrix},$$ (+1 point)

and using the hint:

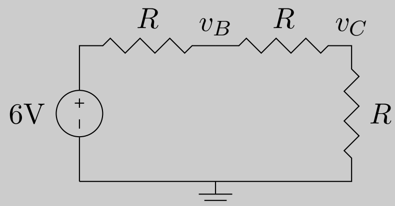
$$\begin{pmatrix} v_B \\ v_C \end{pmatrix} = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}^{-1} \begin{pmatrix} 6 \\ 0 \end{pmatrix} = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{pmatrix} 2 \\ 0 \end{pmatrix} = \begin{pmatrix} 4 \\ 2 \end{pmatrix}$$ (+1 point)

.

3. **Voltage Division and OpAmps**

(a) (2 points) Use the Thevenin equivalent you obtained in problem 1 and voltage division to calculate v_C for the circuit in Fig. 1(b).

Solution:
We shall replace the connection of the voltage source and the two $2R$ resistors in Fig. 1(b) by the Thevenin equivalent from problem 1 to obtain:



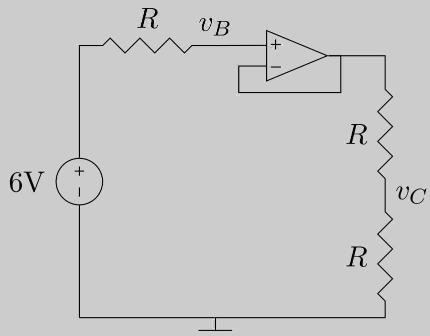
(+1 point)

The voltage v_C can now be computed using voltage division:

$$v_C = \frac{R}{3R} 6 = 2\text{V}. \qquad \text{(+1 point)}$$

(b) (3 points) Calculate v_B and v_C for the circuit in Fig. 1(c).

Solution:
We can once again make use of the Thevenin equivalent from problem 1:



(+1 point)

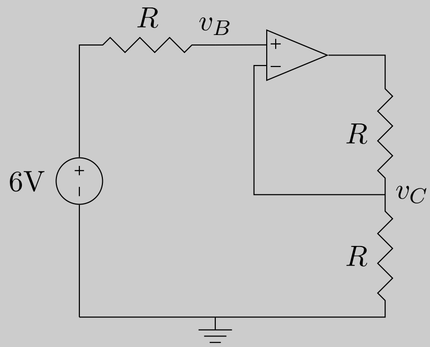
Because $i_P = i_N = 0$ we have that $v_B = 6$V. (+1 point)

Because $v_o = v_N = v_P = v_B = 6$V we have that

$$v_C = \frac{R}{R+R} v_o = \frac{1}{2} 6 = 3\text{V} \qquad \text{(+1 point)}$$

(c) (1 point (bonus)) What is the value of v_C if by mistake you connect the negative terminal of the opAmp to the node labeled v_C instead of the opAmp output?

Solution:
In that case the circuit becomes equivalent to:



from which $v_C = v_N = v_P = v_B = 6\text{V}$. (+1 bonus point)

(d) (1 point (bonus)) A friend does not seem to understand why the voltages v_B and v_C are not the same in the circuits of Fig. 1(b) and Fig. 1(c). Please explain.

Solution:
The circuits in Fig. 1(b) and Fig. 1(c) differ only by the way v_B connects to v_C. In Fig. 1(b) the voltage divider made by the R resistors *loads* the voltage divider made by the $2R$ resistors. The current drawn drops the voltage v_B from 6V to 4V. The opAmp in Fig. 1(c) decouple the two circuits and transfers the voltage v_B without loading the voltage divider made by the $2R$ resistors. (+1 bonus point)