MAE140 – Linear Circuits – Fall – 2016
Final, December 6th

Instructions

1. This exam is open book. You may use whatever written materials you choose, including your class notes and textbook. You may use a handheld calculator with no communication capabilities.
2. You have 170 minutes.
3. Do not forget to write your name and student number.
4. This exam has 6 questions, for a total of 60 points and 2 bonus points.

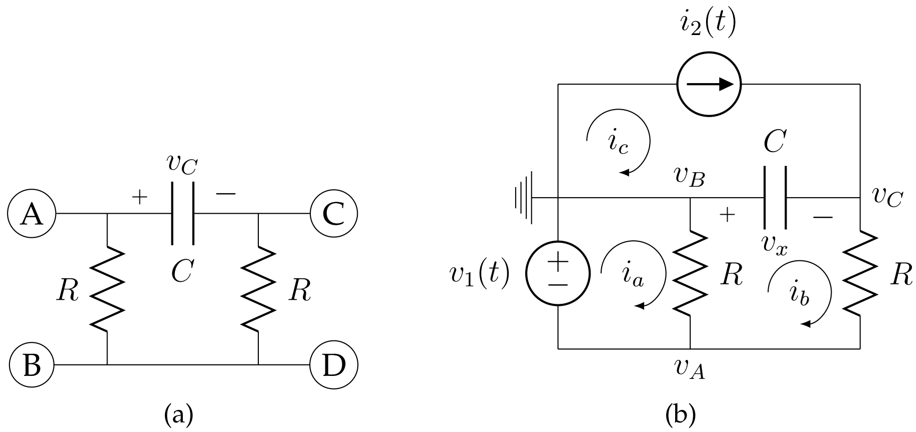

Figure 1: Circuits for Questions 1 and 2

1. **Circuit Equivalence**

   IMPORTANT: All impedances should be given as a ratio of two polynomials!

   (a) (3 points) Assume zero-initial conditions and find the s-domain equivalent circuit as seen from terminals (A) and (B) in Fig. 1a.

   (b) (3 points) Assume zero-initial conditions and find the s-domain equivalent circuit as seen from terminals (C) and (B) in Fig. 1a.

   (c) (4 points) Assume a non-zero initial condition $v_C(0) = 1$V as indicated and repeat parts (a) and (b).

2. **Circuit Analysis**

   (a) (4 points) Convert the circuit in Fig. 1b to the s-domain and formulate its node-voltage equations. Use the node-voltage and labels provided in the figure and clearly indicate the final equations and circuit variable unknowns. Do not assume zero initial conditions. Make sure your final equations only involve node-voltages.

   (b) (4 points) Convert the circuit in Fig. 1b to the s-domain and formulate its mesh-current equations. Use the mesh-currents and labels provided in the figure and clearly indicate the final equations and circuit variable unknowns. Do not assume zero initial conditions. Make sure your final equations only involve mesh-currents.

   (c) (2 points) Express the s-domain voltage $V_x(s)$ in terms of the node voltages and mesh currents from parts (a) and (b).

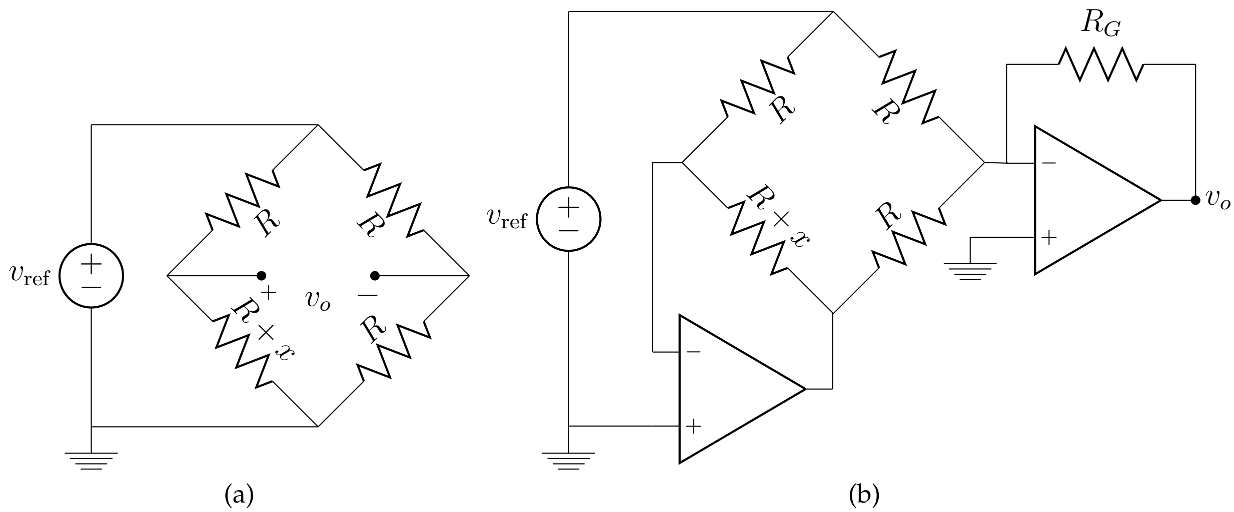

Figure 2: Circuits for Question 3

3. **Wheatstone Bridge**

   The circuits in Fig. 2 are used with strain-gauges to measure small displacements in structures. The displacement causes a change in resistance x indicated in the figure.

   (a) (5 points) For the circuit in Fig. 2a, show that

   $$\frac{v_0}{v_{\text{ref}}} \approx \frac{x}{4R}$$

   when x is small.

   Hint: Expand in series about $x = 0$ and discard higher order terms.

   (b) (5 points) For the circuit in Fig. 2b, show that

   $$\frac{v_0}{v_{\text{ref}}} = \frac{xR_G}{R^2}$$

   Hint: Don't get fooled by the drawing; this is simpler than what it seems!

   (c) (1 point (bonus)) Name one advantage and one disadvantage of each circuit.

4. (10 points) **OpAmp Design**

   Design an OpAmp circuit that can be used to amplify the voltage v_o shown in the circuit shown in Fig. 2a by a factor $K > 0$ to produce a voltage $v_o' = Kv_0$. Explain how your circuit works.

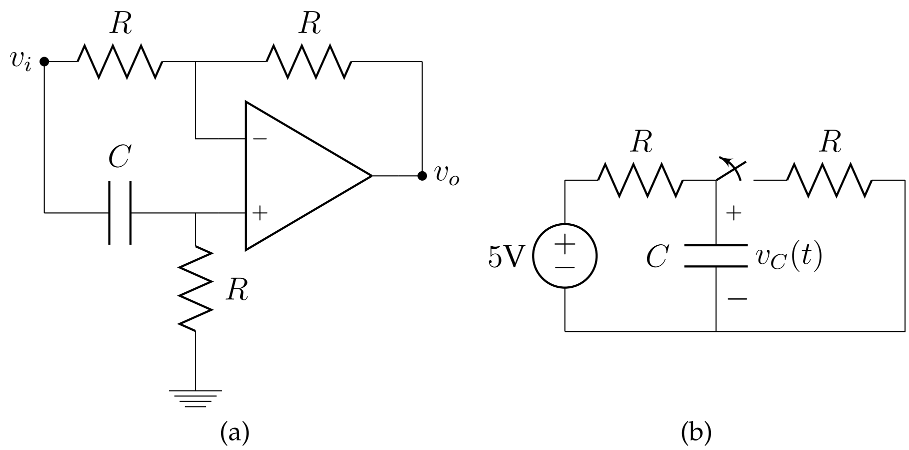

Figure 3: Circuits for Questions 5 and 6

5. **Frequency Response**

   (a) (3 points) Show that the transfer function $T(s) = V_o(s)/V_i(s)$ for the circuit in Fig. 3a is:

   $$T(s) = \frac{s - \omega_a}{s + \omega_a}, \quad \omega_a = \frac{1}{RC}$$

   IMPORTANT: For the rest of this question set $\omega_a = 1\text{rad/s}$.

   (b) (3 points) Sketch the magnitude and phase of the frequency response $T(j\omega)$.

   (c) (1 point) Explain what this circuit does based on its frequency response.

   (d) (3 points) Use the frequency response method or a direct calculation to compute the steady-state response of the circuit to an input of the form $v_i(t) = 2\cos(t)$V.

6. **Circuit with Switch**

   The switch in the circuit in Fig. 3b is kept closed for a long time. At $t = 0$ the switch is opened.

   (a) (3 points) Determine the initial condition at the capacitor at $t = 0$ and convert the circuit to the s-domain for $t \geq 0$.

   (b) (5 points) Show that

   $$v_c(t) = 5 - 2.5\, e^{-t/(RC)}\text{V}, \quad t \geq 0$$

   (c) (2 points) Calculate the power absorbed by the resistor for $t \geq 0$.

   (d) (1 point (bonus)) Calculate the total energy delivered by the voltage source after the switch is opened.