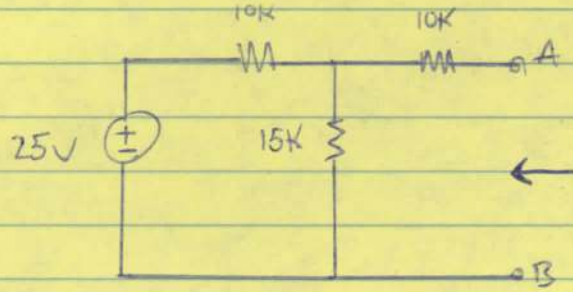
MAE140 HW#3 SOLUTIONS

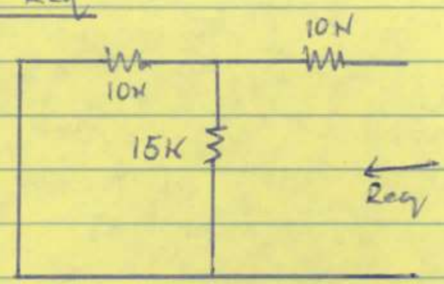
3.51



Let us start by finding V_{th}

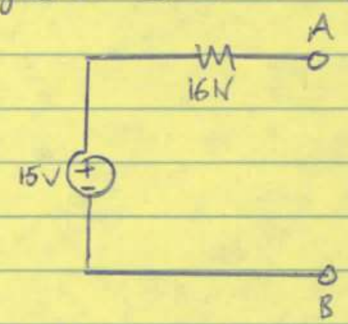
$$V_{th} = \frac{25}{10K + 15K} \cdot 15K = 15V$$

Now we can find R_{eq}



$$R_{eq} = 10K + 10K \parallel 15K$$
$$= \cancel{16K} \; \underline{16K}$$

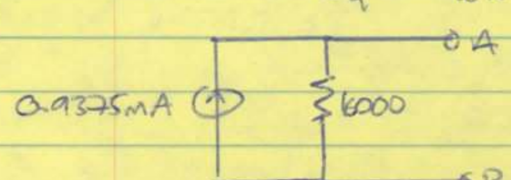
So, this gives us the Thevenin eq circuit



+5

To get the Norton circuit, we can do a source transformation

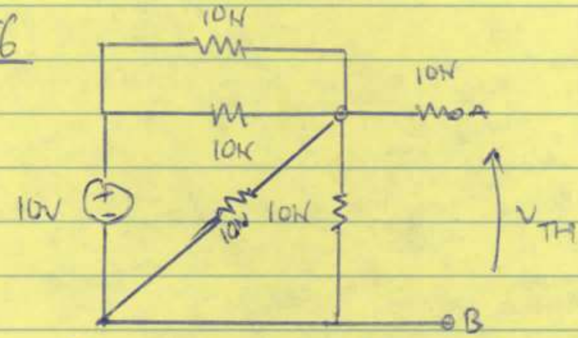
$$I_N = \frac{V_{TH}}{R_{eq}} = \frac{15}{16K} = 0.9375mA,$$



+5

(Award full points if they have both circuits correct. There are other ways to compute).

3-56



We will find the Thevenin eq circuit between terminals A & B. Note that when we do this, we do not consider R_L.

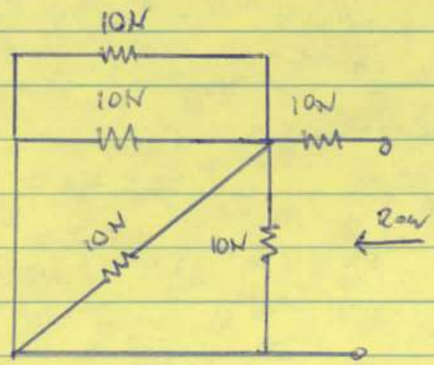
V_{TH}

$$V_{TH} = \left(\frac{10V}{10\Omega \| 10\Omega + 10\Omega \| 10\Omega} \right) 10\Omega \| 10\Omega = \underline{\underline{5V}}$$

+3

(We see that we have a voltage divider between two sets of $10\Omega \| 10\Omega$ resistances)

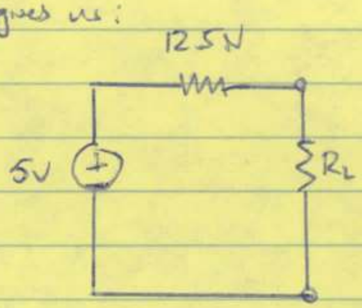
R_{eq}



$$R_{eq} = 10\Omega + 10\Omega \| 10\Omega \| 10\Omega \| 10\Omega$$
$$= \underline{\underline{12.5\Omega}}$$

+3

That gives us:



Now, we can compute the voltage across R_L

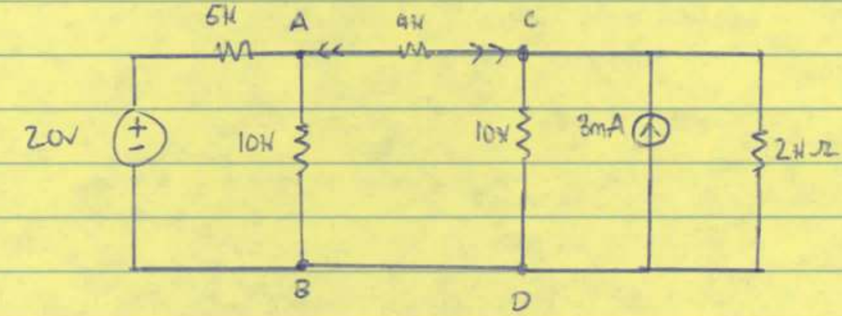
$$V_{R_L} = \frac{5V}{12.5\Omega + R_L} \cdot R_L$$

R_L	V_{R_L}
5Ω	1.429
10Ω	2.22
20Ω	3.077

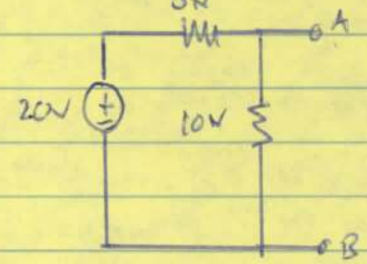
+4

NOTE: I ACCIDENTLY THOUGHT THE RESISTANCES WERE IN kΩ, THE ANSWERS ARE ALL THE SAME THOUGH.

3d



Let's find the circuit between nodes A & B.



$V_{TH} = \frac{20V}{15K} \cdot 10K$

$= \underline{13.3V}$

+2

$R_{eq} = 10K \parallel 5K = \underline{3333.3}$

And now the eq circuit between C & D.

$V_{TH} = 3m \times (10K \parallel 2K)$

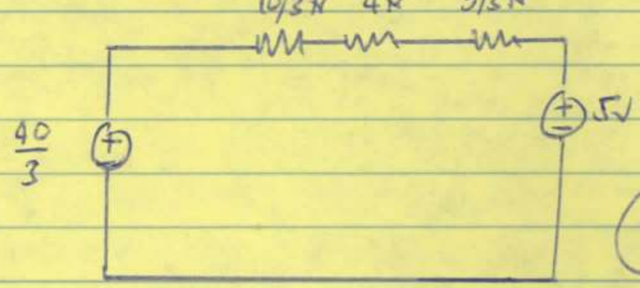
$= \underline{5V}$

+2

$R_{eq} = 10K \parallel 2K$

$= \underline{1666.67}$

Now we can combine both circuits together and add the load resistance



+6

$I_L = \dfrac{\frac{40}{3} - 5}{\frac{10}{3}K + 4K + \frac{5}{3}K}$

$= \underline{\frac{25}{27} mA}$ (0.926 mA)

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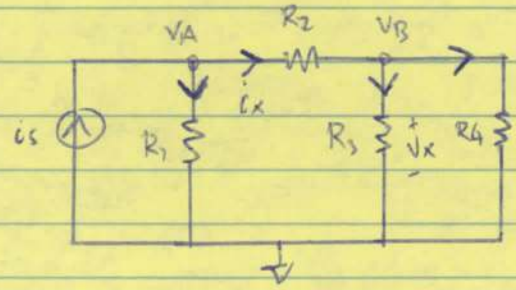


(1) Assign current directions
(2) Formulate node-voltage equations at V_A & V_B.

A $\quad i_s = \frac{V_A}{R_1} + \frac{V_A - V_B}{R_2}, \quad i_s = \frac{V_A}{R_1} + \frac{V_A}{R_2} - \frac{V_B}{R_2}$

B $\quad \frac{V_A - V_B}{R_2} = \frac{V_B}{R_3} + \frac{V_B}{R4} \qquad \frac{V_B}{R_2} + \frac{V_B}{R4} + \frac{V_B}{R_2} - \frac{V_A}{R_2} = 0$

(3) Write these equations in the form $Ax = b$.

$$\begin{bmatrix} \frac{1}{R_1} + \frac{1}{R_2} & -\frac{1}{R_2} \\ -\frac{1}{R_2} & \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R4} \end{bmatrix} \begin{bmatrix} V_A \\ V_B \end{bmatrix} = \begin{bmatrix} i_s \\ 0 \end{bmatrix} \qquad (+6)$$

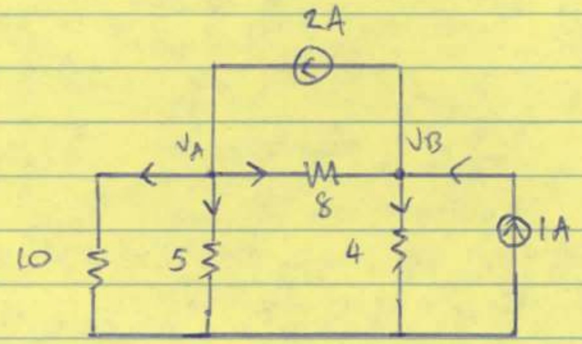
$$A = \begin{bmatrix} \frac{1}{R_1} + \frac{1}{R_2} & -\frac{1}{R_2} \\ \frac{-1}{R_2} & \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R4} \end{bmatrix}$$

$$A^{-1} = \frac{1}{\left(\frac{1}{R_1} + \frac{1}{R_2}\right)\left(\frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R4}\right) - \left(\frac{-1}{R_2}\right)^2} \begin{bmatrix} \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4} & +\frac{1}{R_2} \\ \frac{1}{R_2} & \frac{1}{R_2} + \frac{1}{R_1} \end{bmatrix}$$

$$\begin{bmatrix} V_A \\ V_B \end{bmatrix} = A^{-1} \cdot \begin{bmatrix} i_s \\ 0 \end{bmatrix}$$

$i_x = \frac{V_A - V_B}{R_2}, \quad V_x = V_B \quad (+4)$

3.4



① Assign current directions

② Formulate node-voltage equations at V_A & V_B

$\boxed{A}$

$$\frac{V_A}{10}+\frac{V_A}{5}+\frac{V_A-V_B}{8}=2, \quad \frac{V_A}{10}+\frac{V_A}{5}+\frac{V_A}{8}-\frac{V_B}{8}=2$$

$\boxed{B}$

$$\frac{V_B}{4}+2=\frac{V_A-V_B}{8}+1, \quad \frac{V_B}{4}+\frac{V_B}{8}-\frac{V_A}{8}=-1$$

③ Formulate equations in form $Ax=b$

$$\begin{bmatrix} \frac{1}{10}+\frac{1}{5}+\frac{1}{8} & -\frac{1}{8} \\ -\frac{1}{8} & \frac{1}{4}+\frac{1}{8} \end{bmatrix}\begin{bmatrix} V_A \\ V_B \end{bmatrix}=\begin{bmatrix} 2 \\ -1 \end{bmatrix}$$

+4

$$\Rightarrow \begin{bmatrix} V_A \\ V_B \end{bmatrix}=\begin{bmatrix} \frac{1}{10}+\frac{1}{5}+\frac{1}{8} & -\frac{1}{8} \\ -\frac{1}{8} & \frac{1}{4}+\frac{1}{8} \end{bmatrix}^{-1}\begin{bmatrix} 2 \\ -1 \end{bmatrix}$$

$$\begin{bmatrix} V_A \\ V_B \end{bmatrix}=\begin{bmatrix} 4.3478 \\ -1.2174 \end{bmatrix} V$$

+4

$V_x = V_b$

$$I_x=\frac{V_A}{10}=0.43478\ \underline{A}$$

+2

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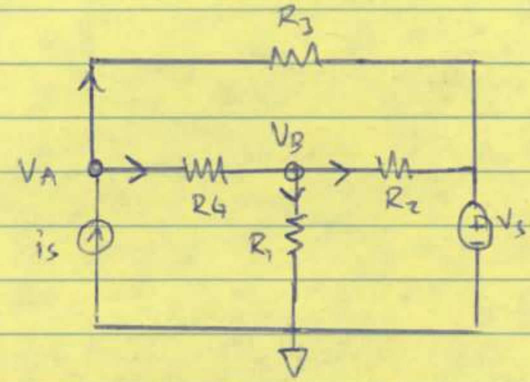

(1) Choose ground
(2) Assign current directions
(3) Formulate node-voltage equations at V_A & V_B.

A

$$i_s = \frac{V_A - V_c}{R_3} + \frac{V_A - V_B}{R_4}, \quad i_s + \frac{V_c}{R_3} = \frac{V_A}{R_3} + \frac{V_A}{R_4} - \frac{V_B}{R_4}$$

B

$$\frac{V_B}{R_1} + \frac{V_B - V_s}{R_2} = \frac{V_A - V_B}{R_4}, \quad \frac{V_B}{R_1} + \frac{V_B}{R_2} + \frac{V_B}{R_4} - \frac{V_A}{R_4} = \frac{V_s}{R_2}$$

(4) Formulate matrix

$$\begin{bmatrix} \frac{1}{R_3} + \frac{1}{R_4} & -\frac{1}{R_4} \\ -\frac{1}{R_4} & \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_4} \end{bmatrix} \begin{bmatrix} V_A \\ V_B \end{bmatrix} = \begin{bmatrix} i_s + V_s/R_3 \\ V_s/R_2 \end{bmatrix}$$ +4

(5) Solve when $R_1 = R_2 = R_3 = R_4 = 5k$, $V_s = 12V$, $i_s = 2m$

$$\begin{bmatrix} \frac{1}{5k} + \frac{1}{5k} & -\frac{1}{5k} \\ -\frac{1}{5k} & \frac{1}{5k} + \frac{1}{5k} + \frac{1}{5k} \end{bmatrix} \begin{bmatrix} V_A \\ V_B \end{bmatrix} = \begin{bmatrix} 2m + 12/5k \\ 12/5k \end{bmatrix}$$

$$\begin{bmatrix} V_A \\ V_B \end{bmatrix} = \begin{bmatrix} 15.6 \\ 9.2 \end{bmatrix} V$$ +3

$$V_x = 15.6 - 12 = \underline{3.6V} \qquad i_x = \frac{9.2}{5k} = \underline{\underline{1.84mA}}$$ +3

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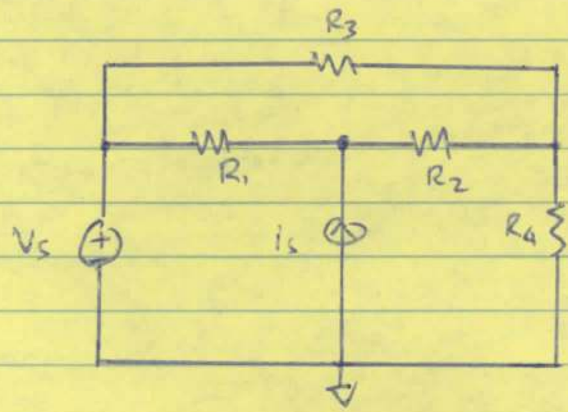
$V_A = V_S$

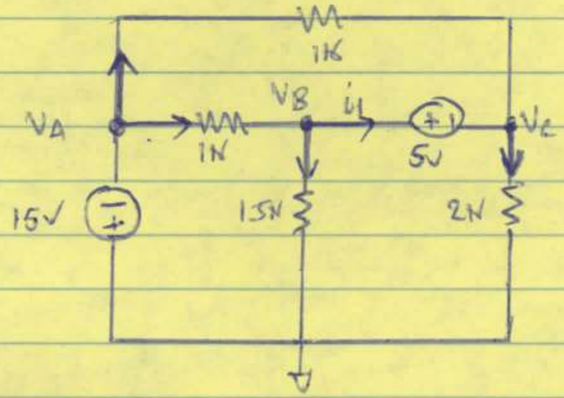
$$\frac{V_B - V_A}{R_1} + \frac{V_B - V_C}{R_2} - i_S = 0$$

$$\frac{V_C - V_A}{R_3} + \frac{V_C - V_B}{R_2} + \frac{V_C}{R_4} = 0$$

$V_D = 0$



3R



$V_A = -15V$

Formulate node voltage equation at B.

(1) $$\frac{V_B}{1.5K} + i_1 = \frac{V_A - V_B}{1K}$$

Formulate node voltage equation at C.

(2) $$\frac{V_C}{2K} = i_1 + \frac{V_A - V_C}{1K}$$

(1) - (2)

$$\frac{V_B}{1.5K} + \cancel{i_1 - i_1} - \frac{V_A - V_C}{1K} = \frac{V_A - V_B}{1K} - \frac{V_C}{2K}$$

Combine equations at B & C to eliminate i_1.

$$\Rightarrow \frac{V_B}{1.5K} - \frac{V_A - V_C}{1K} = \frac{V_A - V_B}{1K} - \frac{V_C}{2K}$$

$$\Rightarrow \frac{V_B}{1.5K} + \frac{V_C}{1K} + \frac{V_B}{1K} + \frac{V_C}{2K} = \frac{V_A}{1K} + \frac{V_A}{1K}$$

$$V_B - 5 = V_C$$

Super node equation

$$V_B\left(\frac{1}{1.5k}+\frac{1}{1k}\right)+V_c\left(\frac{1}{1k}+\frac{1}{2k}\right)=\frac{2V_A}{1k}$$

$$V_B\left(\frac{1}{1.5k}+\frac{1}{1k}\right)+(V_B-5)\left(\frac{1}{1k}+\frac{1}{2k}\right)=\frac{-30}{1k}$$

$$V_B\left(\frac{1}{1.5k}+\frac{1}{1k}\right)+V_B\left(\frac{1}{1k}+\frac{1}{2k}\right)=\frac{-30}{1k}+5\left(\frac{1}{1k}+\frac{1}{2k}\right)$$

$$V_B\left(\frac{1}{1.5k}+\frac{1}{1k}+\frac{1}{1k}+\frac{1}{2k}\right)=-0.0225$$

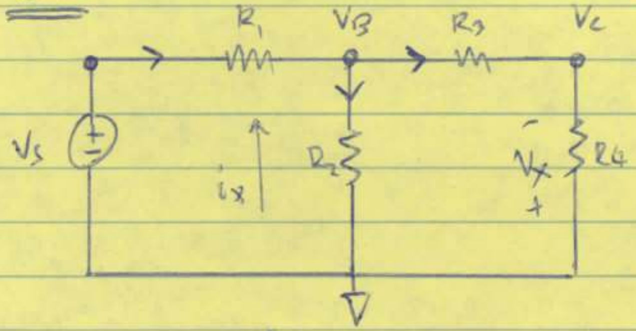
$$V_B = \underline{\underline{-7.105V}} \quad +4$$

$$V_C = \underline{\underline{-12.105V}} \quad +4$$

$$V_x = V_c$$

$$I_x = \frac{V_c+15}{1k} = \underline{\underline{2.8947mA}} \quad +2$$

3.15



(1) Assign circuit ground
(2) Formulate nodal equation at V_B,

$$\frac{V_S - V_B}{R_1} = \frac{V_B}{R_2} + \frac{V_B}{R_3 + R_4}$$

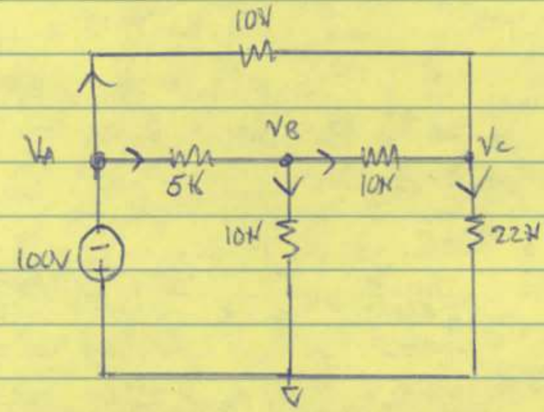
$$\frac{V_S}{R_1} = V_B\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3+R_4}\right), \quad V_B = \frac{V_S}{R_1\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3+R_4}\right)} \quad +5$$

$$i_x = -\frac{V_B}{R_2} \quad +2 \qquad V_x = -\frac{V_B}{R_3 + R_4} \cdot R4 \quad +3$$

(Negative sign because directions of voltage/current opposite to what we have assigned)

- Other solutions possible.

3.4



(1) Assign Ground and current directions
(2) Formulate node-voltage equations

(B)

$$\frac{V_A - V_B}{5K} = \frac{V_B}{10K} + \frac{V_B - V_C}{10K}$$

(C) $$\frac{V_A - V_C}{10K} + \frac{V_B - V_C}{10K} = \frac{V_C}{22K}$$

(3) Combine into matrix form

$$\begin{bmatrix} \frac{1}{5K} + \frac{1}{10K} + \frac{1}{10K} & -\frac{1}{10K} \\ -\frac{1}{10K} & \frac{1}{10K} + \frac{1}{10K} + \frac{1}{22K} \end{bmatrix} \begin{bmatrix} V_B \\ V_C \end{bmatrix} = \begin{bmatrix} \frac{-100}{5K} \\ \frac{-100}{10K} \end{bmatrix}$$

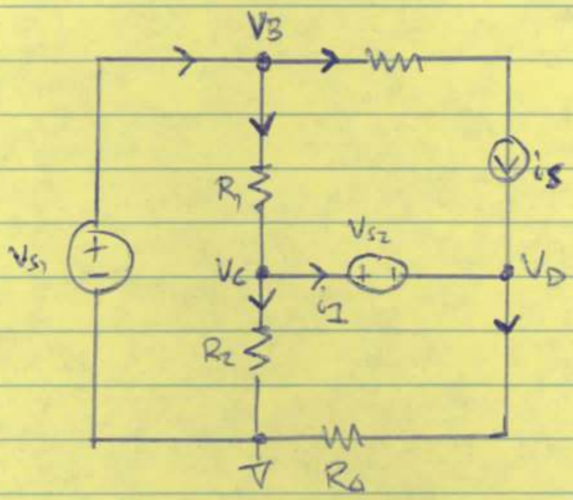
(4) Solve for V_B & V_C

$$\begin{bmatrix} V_B \\ V_C \end{bmatrix} = \begin{bmatrix} -67.0103 \\ -68.0412 \end{bmatrix} V$$

$V_x = V_C - V_B = -1.0309V$ +5

$i_x = \frac{-100 - V_C}{10K} = 3.2mA$ +5

3.20



(1) Assign a ground and current directions
(2) Formulate nodal equations

B/ $V_B = V_{S1}$.

C/ $$\frac{V_C}{R_2} + i_1 = \frac{V_{S1} - V_C}{R_1} \qquad (1)$$

D/ $$i_s + i_1 = \frac{V_D}{R4} \qquad (2)$$

(1) − (2) To eliminate i_1 term from our expression

$$\frac{V_C}{R_2} + \cancel{i_1} - i_s - \cancel{i_1} = \frac{V_{S1} - V_C}{R_1} - \frac{V_D}{R4}$$

$$\Rightarrow \frac{V_C}{R_2} + \frac{V_C}{R_1} + \frac{V_D}{R4} = \frac{V_{S1}}{R_1} + i_s$$

$$V_C - V_{S2} = V_D$$

These are our nodal equations

(c) $R_1 = R_2 = 8.2k$, $R_3 = 2.2k$, $R4 = 3.3k$, $i_s = 2.5mA$,
$V_{s1} = 15V$, $V_{s2} = 5V$

$$V_c\left(\frac{1}{R_1} + \frac{1}{R_2}\right) + (V_c - V_{s2})\frac{1}{R4} = \frac{V_{s1}}{R_1} + i_s$$

$$V_c\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R4}\right) = \frac{V_{s1}}{R_1} + i_s + \frac{V_{s2}}{R4}$$

$$V_c\left(\frac{1}{8.2k} + \frac{1}{8.2k} + \frac{1}{3.3k}\right) = \frac{15}{8.2k} + 2.5m + \frac{5}{3.3k}$$

$$V_c = 10.6858V$$
$$V_D = 5.6858V$$

Now we want to find i_x and v_x

$$i_x = \frac{V_B - V_c}{8.2k} + i_s$$

$$= \underline{\underline{3.026mA}}$$ +5

$$v_x = V_B - R_3 \cdot i_s - V_D$$
$$= 15 - 2.2k \times 2.5m$$
$$= 9.5 - 5.6858$$
$$= \underline{3.8142V}$$ +5