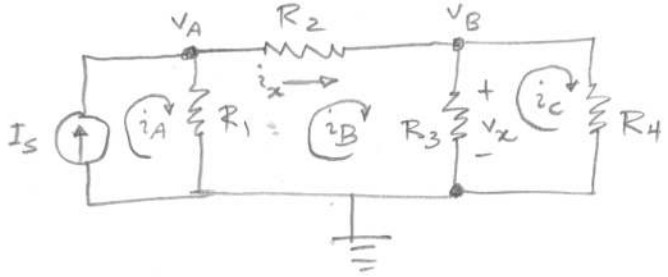
HW 4

3.2



Sum of voltages around mesh B is

$$R_1(i_B - i_A) + R_2 i_B + R_3(i_B - i_C) = 0 \qquad (*)$$

sum of voltage around mesh C is

$$R_3(i_C - i_B) + R_4 i_C = 0 \qquad (**)$$

and also $I_S = i_A$

$i_x = i_B$

we substitute $I_S = i_A$ in then we get

$$R_1(i_B - I_S) + R_2 i_B + R_3(i_B - i_C) = 0$$

$$R_3(i_C - i_B) + R_4 i_C = 0$$

$$\Rightarrow \begin{cases} (R_1 + R_2 + R_3) i_B + (-R_3) i_C = R_1 I_S \\ (-R_3) i_B + (R_3 + R_4) i_C = 0 \end{cases} \longrightarrow i_C = \frac{R_3}{R_3 + R_4} i_B$$

$$\left(R_1 + R_2 + R_3 - \frac{R_3^2}{R_3 + R_4}\right) i_B = R_1 I_S \longrightarrow i_B = \frac{(R_3 + R_4) R_1 I_S}{(R_1 + R_2 + R_3)(R_3 + R_4) - R_3^2}$$

3.2

$$i_L = \frac{R_3 R_1 I_S}{(R_1+R_2+R_3)(R_3+R_4) - R_3^2}$$

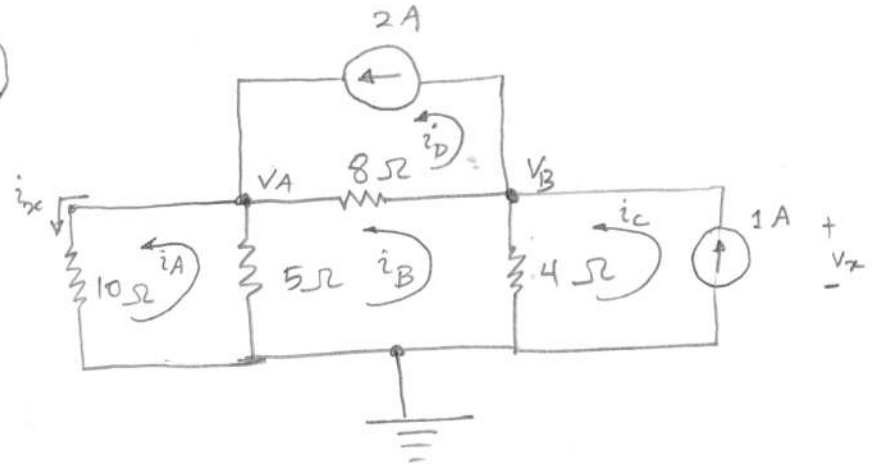
$$V_A = (i_A - i_B) R_1 = \left[I_S - \frac{(R_3+R_4) R_1 I_S}{(R_1+R_2+R_3)(R_3+R_4) - R_3^2} \right] R_1$$

$$V_x = V_B = (i_B - i_L) R_3 = \frac{R_4 R_1 I_S R_3}{(R_1+R_2+R_3)(R_3+R_4) - R_3^2}$$

$$i_B = \frac{\begin{vmatrix} R_1 I_S & -R_3 \\ 0 & R_3+R_4 \end{vmatrix}}{\begin{vmatrix} R_1+R_2+R_3 & -R_3 \\ -R_3 & R_3+R_4 \end{vmatrix}} = \frac{R_1 I_S (R_3+R_4)}{(R_3+R_4)(R_1+R_2+R_3) - R_3^2}$$

$$i_L = \frac{\begin{vmatrix} R_1+R_2+R_3 & R_1 I_S \\ -R_3 & 0 \end{vmatrix}}{\begin{vmatrix} (R_1+R_2+R_3 & -R_3 \\ -R_3 & R_3+R_4 \end{vmatrix}} = \frac{R_1 R_3 I_S}{(R_3+R_4)(R_1+R_2+R_3) - R_3^2}$$

3_4



A: $5(i_A - i_B) + 10 i_x = 0$ $\quad (i_x = i_A)$

B: $4(i_B - i_C) + 8(i_B - i_D) + 5(i_B - i_A) = 0$

C: $i_C = 1 \text{ A}$

D: $i_D = 2 \text{ A}$

now substitute i_C, i_D in above equations

$5(i_A - i_B) + 10 i_A = 0 \longrightarrow \boxed{15 i_A - 5 i_B = 0} \longrightarrow i_B = 3 i_A$

$4(i_B - 1) + 8(i_B - 2) + 5(i_B - i_A) = 0 \Longrightarrow$

$17 i_B - 5 i_A - 4 - 16 = 0 \longrightarrow \boxed{17 i_B - 5 i_A = 20}$

$\Longrightarrow 51 i_A - 5 i_A = 20 \longrightarrow \boxed{i_A = \frac{20}{46} = 0.43 \text{ A}}$ ✓

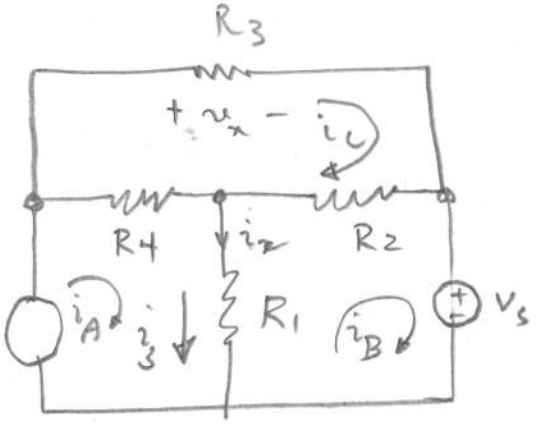
$i_A = i_x = 0.43$

$\boxed{i_B = 1.3 \text{ A}}$ ✓

$V_B = 4(-i_B + i_C) = 4 \times 0.3 = -1.2 \text{ V}$

$V_A = 10 i_x = 4.3 \text{ A}$

3.6



$R_1 = R_2 = R_4 = 5\ k\Omega$

$i_A = i_s = 2$

mesh A: $i_A = i_s$

mesh B: $i_B(R_1+R_2) - i_A R_1 - i_C R_2 + V_s = 0$

mesh C: $i_C(R_2+R_3+R_4) - i_A R_4 - i_B R_2 = 0$

$$\begin{bmatrix} 1 & 0 & 0 \\ -R_1 & R_1+R_2 & -R_2 \\ -R_4 & -R_2 & R_2+R_3+R_4 \end{bmatrix} \begin{bmatrix} i_A \\ i_B \\ i_C \end{bmatrix} = \begin{bmatrix} i_s \\ -V_s \\ 0 \end{bmatrix}$$

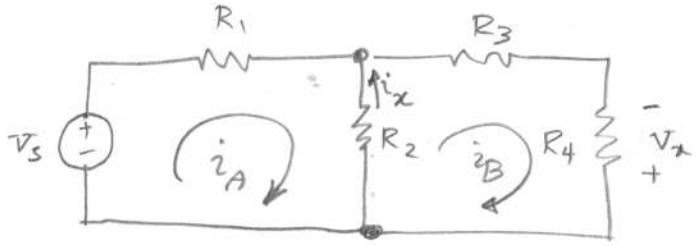
$$\begin{bmatrix} 1 & 0 & 0 \\ -5 & +10 & -5 \\ -5 & -5 & +15 \end{bmatrix} \begin{bmatrix} i_A \\ i_B \\ i_C \end{bmatrix} = \begin{bmatrix} 2 \\ -12 \\ 0 \end{bmatrix}$$

$i_A = 2\ mA \qquad i_B = 0.16\ mA \qquad i_C = 0.72\ mA$

$v_x = i_C R_3 = 0.72 \times 5 = 3.6\ V$

$i_2 = i_A - i_B = 2 - 0.16 = 1.84\ mA$

3.13



$$A: \quad -v_s + R_1 i_A + R_2 (i_A - i_B) = 0$$

$$B: \quad R_2 (i_B - i_A) + R_3 i_B + R_4 i_B = 0$$

$$i_x = i_B - i_A$$

$$\Rightarrow \quad (R_1 + R_2) i_A + (-R_2) i_B = +v_s$$

$$(-R_2) i_A + (R_2 + R_3 + R_4) i_B = 0 \longrightarrow i_A = \frac{R_2 + R_3 + R_4}{R_2} i_B$$

$$\left[\frac{(R_1 + R_2)(R_2 + R_3 + R_4)}{R_2} - R_2 \right] i_B = +v_s \longrightarrow$$

$$i_B = \frac{R_2 v_s}{-R_2^2 + (R_1 + R_2)(R_2 + R_3 + R_4)},$$

$$i_A = \frac{(R_2 + R_3 + R_4) v_s}{-R_2^2 + (R_1 + R_2)(R_2 + R_3 + R_4)} \Rightarrow i_x = i_B - i_A = \frac{(R_3 + R_4) v_s}{+(R_1 + R_2)(R_2 + R_3 + R_4) + R_2^2}$$

$$v_x = -i_B R_4 = \frac{R_4 R_2 v_s}{-(R_1 + R_2)(R_2 + R_3 + R_4) + R_2^2}$$

3.14

C: $10\,i_C + 10(i_C - i_B) + 5(i_C - i_A) = 0$

B: $10(i_B - i_C) + 22\,i_B + 10(i_B - i_A) = 0$

A: $5(i_A - i_C) + 10(i_A - i_B) + 100 = 0$

$\Rightarrow$

$$25\,i_C - 10\,i_B - 5\,i_A = 0$$

$$-10\,i_C + 42\,i_B - 10\,i_A = 0$$

$$-5\,i_C - 10\,i_B + 15\,i_A = -100$$

$$i_C = \frac{\begin{vmatrix} 0 & -10 & -5 \\ 0 & 42 & -10 \\ -100 & -10 & +15 \end{vmatrix}}{\begin{vmatrix} +25 & -10 & -5 \\ -10 & 42 & -10 \\ -5 & -10 & +15 \end{vmatrix}} = \frac{-31000}{9700} = -3.19 \text{ mA}$$

$$i_B = \frac{\begin{vmatrix} 25 & 0 & -5 \\ -10 & 0 & -10 \\ -5 & -100 & 15 \end{vmatrix}}{|A|} = \frac{-30000}{9700}$$

$$i_A = \frac{\begin{vmatrix} 25 & -10 & 0 \\ -10 & 42 & 0 \\ -5 & -10 & -100 \end{vmatrix}}{|A|} = \frac{-95000}{9700} = -9.79 \text{ mA}$$

$i_C = -3.19$ mA

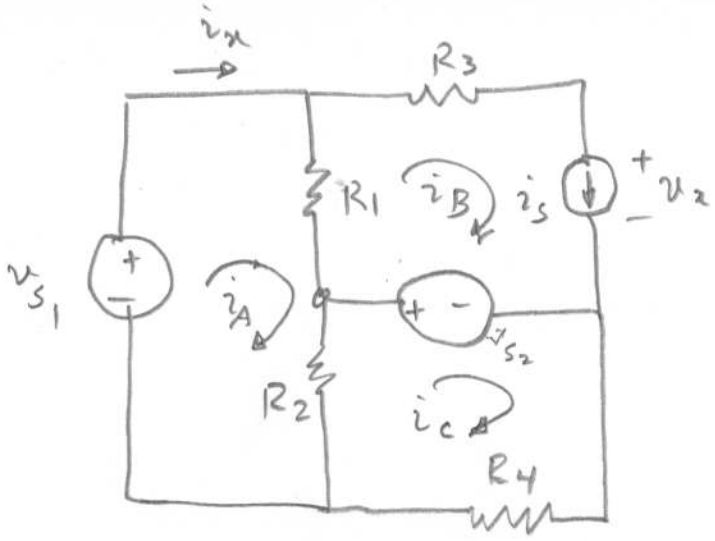
$i_B = -3.09$ mA

$i_A = -9.79$ mA

$v_x = 10(i_C - i_B) = -1.03$ V

$i_x = i_C = 3.19$ mA

3.20



$$A: \quad (R_1+R_2)\, i_A - R_1 i_B - R_2 i_C = v_{s_1}$$

$$C: \quad -R_2 i_A + (R_2+R_4)\, i_C = -v_{s_2}$$

$$i_B = 2.5$$

$$\begin{bmatrix} 16.4 & -8.2 & -8.2 \\ -8.2 & 0 & 11.5 \\ 0 & 1 & 6 \end{bmatrix} \begin{bmatrix} i_A \\ i_B \\ i_C \end{bmatrix} = \begin{bmatrix} 15 \\ -5 \\ 2.5 \end{bmatrix}$$

$i_A = 3.02$ mA $\qquad$ $i_B = 2.5$ mA $\qquad$ $i_C = 1.72$ mA

now we writ KVL for B

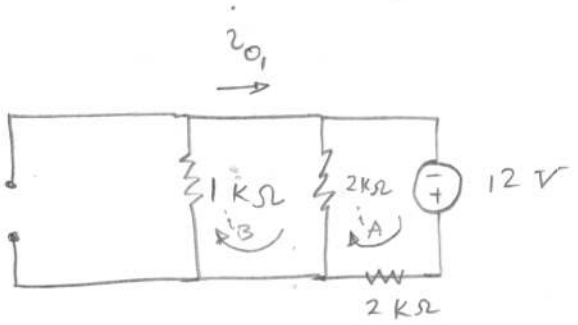
$$R_3 i_B + v_x - v_{s_2} + R_1 i_B - R_1 i_A = 0 \longrightarrow$$

$$2.2\,(2.5) + v_x - 5 + 8.2\,(2.5) - 8.2\,(3.02) = 0$$

$$v_x = 3.76 \text{ V}$$

$i_x = i_A = 3.02$ mA $\longrightarrow$ $P = v_{s_1} \cdot i_x = 15 \times 3.02 = 45.3$ mW

3.41



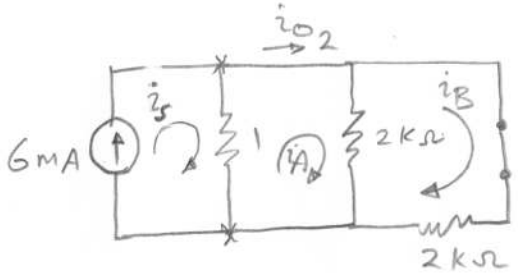
B: $i_B + 2(i_B - i_A) = 0 \longrightarrow 3i_B = 2i_A$

A: $2i_A + 2(i_A - i_B) - 12 = 0 \longrightarrow 4i_A - 2i_B = 12$

$6i_B - 2i_B = 12 \longrightarrow i_B = 3 \text{ mA} \longrightarrow i_A = 4.5 \text{ mA}$

$i_{o_1} = i_B = 3 \longrightarrow \boxed{i_{o_1} = 3 \text{ mA}}$

$\boxed{v_{o_1} = 1 \cdot i_B = 3 \text{ V}}$



A: $1(i_A - i_s) + 2(i_A - i_B) = 0$ $\qquad i_{o_2} = i_A \qquad i_s = 6 \text{ mA}$

B: $2(i_B - i_A) + 2i_B = 0$

$\Rightarrow \quad 3i_A - 2i_B = 6 \longrightarrow 4i_B = 6 \longrightarrow$

$4i_B - 2i_A = 0 \longrightarrow i_A = 2i_B$

3.41

$i_B = 1.5 \text{ mA}$ $\qquad$ $i_A = 3 \text{ mA}$

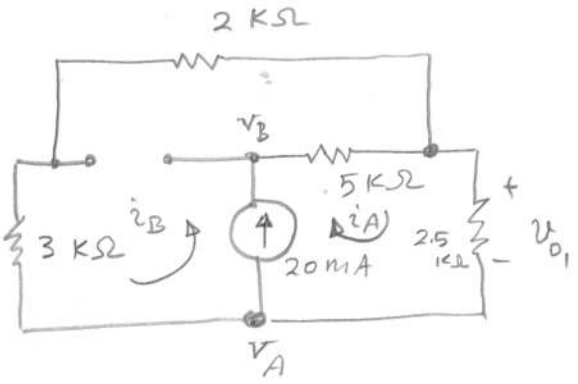
$$i_{o_2} = i_A = 3 \text{ mA}$$

$$v_{o_2} = 1(3-6) = -3$$

$$\Rightarrow \quad i_o = i_{o_2} + i_{o_1} = 3+3 = 6 \text{ mA}$$

$$v_o = v_{o_1} + v_{o_2} = 0$$

3.43



A: $(V_A - V_B) + 5(20) + 2.5\, i_A = 0$

B: $(V_A - V_B) + 5(20) + 2\, i_B + 3\, i_B = 0$

$\Rightarrow 2.5\, i_A = 5\, i_B$

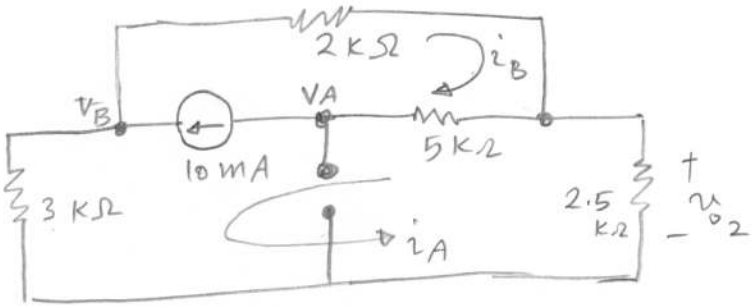
$\boxed{i_A = 2\, i_B}$

$$20 = i_A + i_B$$

$\rightarrow 20 = 3\, i_B \rightarrow \boxed{i_B = 6.67 \text{ mA}}$

$\boxed{i_A = 13.33 \text{ mA}} \rightarrow v_{o_1} = 2.5\, i_A$

$v_{o_1} = 33.32 \text{ V}$



B: $2(i_B) + 5(i_B + i_A) + V_A - V_B = 0$

$\rightarrow 2 i_B = 5.5\, i_A$

A: $5(i_B + i_A) + V_A - V_B + 3\, i_A + 2.5\, i_A = 0$

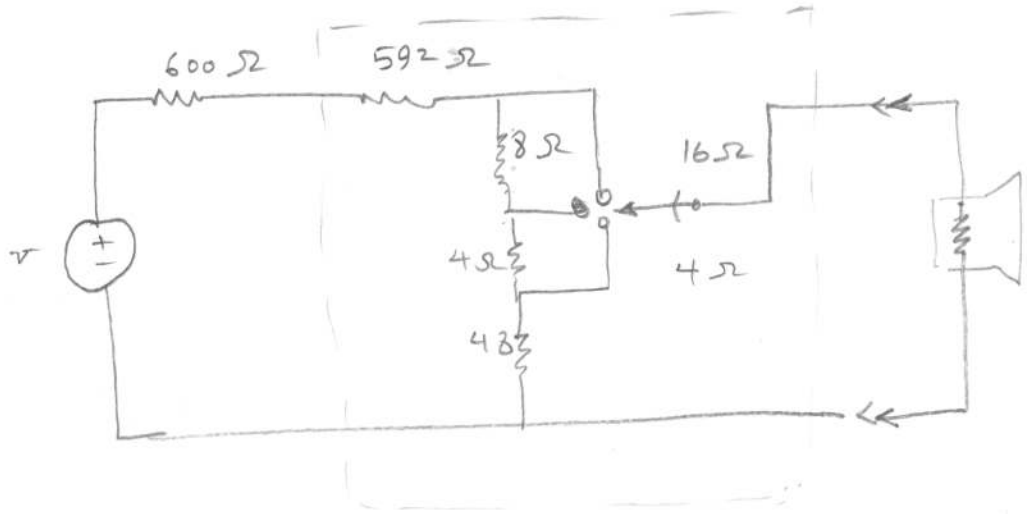
$\boxed{i_B = 2.75\, i_A}$

$i_A + i_B = 10 \text{ mA} \rightarrow i_A = 2.66 \text{ mA}$

$v_{o_2} = -6.65 \text{ V}$

$v_o = 33.32 - 6.65 = 26.67 \text{ V}$

3-97



(I) If switch 16 Ω is connected

$R_{in} = 592 + 16 \,||\, (8+4+4) = 600\ \Omega$

$R_{out} = (4+4+8) \,||\, (592+600) = 15.79\ \Omega$

(II) If switch 8 Ω is connected

$R_{in} = (592+8) + (4+4) \,||\, 8 = 604\ \Omega$

$R_{out} = (4+4) \,||\, (8+592+600) = 7.94$

III If switch 4 Ω is connected

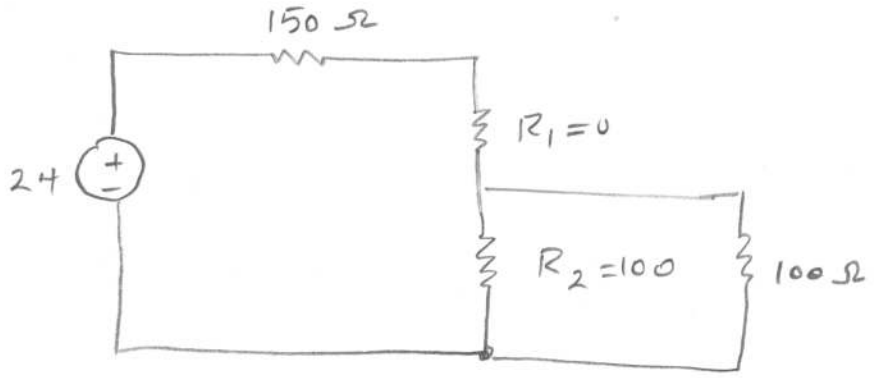
$R_{in} = (592+8+4) + (4 \,||\, 4) = 606\ \Omega$

$R_{out} = 4 \,||\, (4+8+592+600) = 3.99\ \Omega$

So, the claim is true

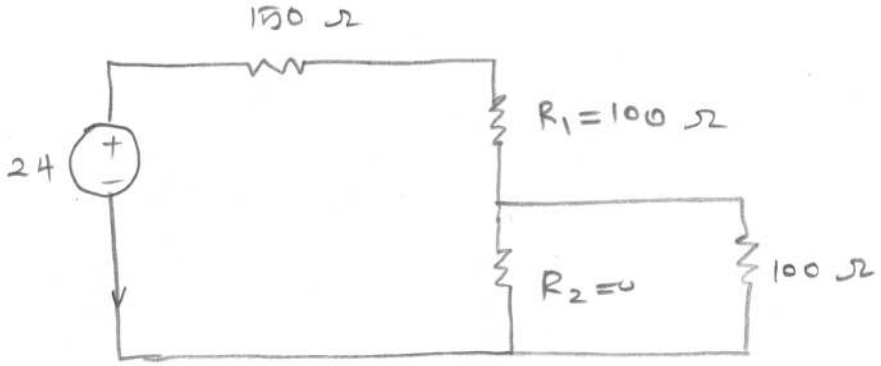
3-102

Let us try



$R_{eq} = 250 \longrightarrow i = \frac{24}{250} = 0.096$ A ✓

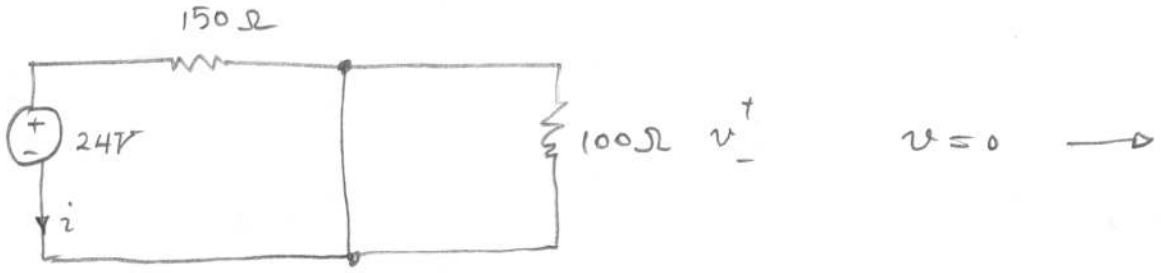
but now



$i = \frac{24}{350} = 0.06$

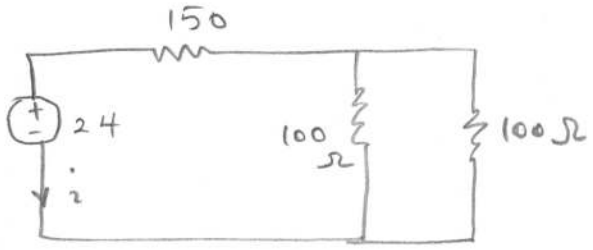
3-102

Let put R=0 then we have



$i = \frac{24}{150} = 0.16$ too high more than maximum!

but now



$R_{eq} = 150 + 50 = 200$

$i = \frac{24}{200} = 0.12$ A

acceptable!