MAE140 HW#7 SOLUTIONS

9.5

Find the Laplace transform of $f(t) = [e^{-2t} - 2e^{t}]u(t)$,

Looking directly at the Laplace transform table and using the linearity property of the Laplace transform, we get that

$$F(s) = \mathcal{L}[e^{-2t} - 2e^{t}]$$
$$= \mathcal{L}[e^{-2t}] - 2\mathcal{L}[e^{t}]$$
$$= \frac{1}{s+2} - \frac{2}{s-1}$$

To determine the poles and zeros, we need to

$$= \frac{(s-1) - 2(s+2)}{(s+2)(s-1)}$$
$$F(s) = \frac{-(s+5)}{(s+2)(s-1)}$$

Which has a single zero at $s = -5$ and two poles at $s = -2$ and $s = +1$

9.8

$$f(t) = 5[4\cos(10t) - 3\sin(10t)]u(t)$$
$$F(s) = \frac{20s}{s^2 + 10^2} - \frac{5 \cdot 3 \cdot 10}{s^2 + 10^2}$$

$$F(s) = \frac{20s - 150}{s^2 + 10^2}$$

Let us compute the location of the poles and zeros

$$20s - 150 = 0$$
$$\Rightarrow \quad s = \frac{150}{20} \longrightarrow \text{location of zero}$$

$$s^2 + 100 = 0$$
$$s^2 = -100 \Rightarrow s = \pm 10j \longrightarrow \text{location of two imaginary poles.}$$

9.10

$f(t) = 2 - 5t - 2e^{-25t}$

$F(s) = \frac{2}{s} - \frac{5}{s^2} - \frac{2}{s+25}$.

$= \frac{45s - 125}{s^2(s+25)}$, zero at $s = \frac{125}{45}$, pole at $s = 0$ (twice), $s = -25$.

9.18

$f(t) = Au(t) - 2Au(t-T) + Au(t-2T)$

9.21

Find the inverse Laplace transforms of the following functions

(a) $F(s) = \frac{10}{s(s+50)}$

Using a partial fraction decomposition, we have that

$F(s) = \frac{A}{s} + \frac{B}{s+50} = \frac{10}{s(s+50)}$

Solve for A & B,

$\frac{A(s+50) + Bs}{s(s+50)} = \frac{10}{s(s+50)}$

$\Rightarrow As + 50A + Bs = 10$

$\Rightarrow 50A = 10, \quad A \Rightarrow \frac{1}{5}$

$B = \frac{-1}{5}$

so,

$F(s) = \frac{1}{5s} - \frac{1}{5(s+50)}$

So, now we can take the inverse Laplace transform of $F(s)$,

$$F(s) = \frac{1}{5s} - \frac{1}{5(s+50)}$$

$$= \frac{1}{5} - \frac{1}{5}e^{-50t}$$

$$= \underline{\frac{1}{5}(1 - e^{-50t})\,u(t)}$$

(b) $$F(s) = \frac{s+2}{(s+3)(s+4)} = \frac{A}{s+3} + \frac{B}{s+4} = \frac{A(s+4) + B(s+3)}{(s+3)(s+4)}$$

$$\frac{s+2}{(s+3)(s+4)} = \frac{As + 4A + Bs + 3B}{(s+3)(s+4)}$$

Comparing coefficients,

~~[illegible]~~ $\Rightarrow$ ~~$B = \frac{2}{3}$~~, ~~[illegible]~~

~~$2 = 3B$,~~

~~$A+$~~

$$A + B = 1 \qquad \Rightarrow \qquad 4A + 3(1 - A) = 2$$

$$4A + 3B = 2 \qquad\qquad 4A + 3 - 3A = 2$$

$$A = -1$$

$$\underline{B = 2}$$

So, now we can take the inverse Laplace transform,

$$\underline{f(t) = (-1e^{-3t} + 2e^{-4t})u(t)}$$

9.23

(a)

Find the inverse Laplace tranform of

$$F(s) = \frac{50(s+1000)(s+2000)}{(s+5000)(s+500)} = A + \frac{B}{s+500} + \frac{C}{s+5000}$$

$$50(s+1000)(s+2000) = A(s+500)(s+5000) + B(s+5000) + C(s+500)$$

$$50(s^2+3000s+2000000) = A(s^2+5500s+2500000) + Bs + 5000B + Cs + 500C$$

$$50s^2 + 150000s + 100e^6) = As^2 + 5500As + 2500000A + Bs + 5000B + Cs + 500C$$

Now, comparing coefficients,

(1) $50s^2 = As^2 \Rightarrow A = 50$

(2) $5500(50) + B + C = 150000$

$\Rightarrow B + C = -125000$

(3) $\Rightarrow 125 \times 10^6 + 5000B + 500C = 100 \times 10^6$

$5000B + 500C = -25 \times 10^6$

$10B + C = -50000$

$B = \frac{25000}{3}$, $C = \frac{-400000}{3}$

so, now we can take the inverse laplace transform

$$f(t) = 50\delta(t) + \left[\frac{25000}{3}e^{-500t} - \frac{400000}{3}e^{-5000t}\right]u(t)$$

9.23b)

$$F(s) = \frac{50s^2}{(s+100)(s+500)} = A + \frac{B}{s+100} + \frac{C}{s+500}$$

Following the same procedure as we did before, we get the following

$$f(t) = 50\delta(t) + [1250e^{-100t} - 31250e^{-500t}]u(t)$$

~~9.27~~

~~Find the inverse laplace transform of the following~~

$$\cancel{F(s) = \frac{\alpha^2}{s^2(s+\alpha)} \Rightarrow \frac{A}{s^2} + \frac{Bs}{s+\alpha}}$$

~~Comparing coefficients,~~

$$\cancel{A(s+\alpha) + Bs^2 = \alpha^2}$$

$$\cancel{As + \alpha A + Bs^2 = \alpha^2}$$

$$\cancel{F(s) = \frac{\alpha^2}{s^2(s+\alpha)} = \frac{A}{s+\alpha} + \frac{B}{s} + \frac{C}{s^2}}$$

~~Now, as we have done before, we can compare coefficients~~

$$\cancel{\alpha^2 = As^2 + Bs + C}$$

/

9.24

(a) Find the inverse Laplace transform of the following functions,

$$F(s) = \frac{\alpha^2}{s^2(s+\alpha)} = \frac{A}{s^2} + \frac{B}{s} + \frac{C}{s+\alpha}$$

We have the above partial fractions decomposition. Now we can compute the values of A, B & C.

$$\alpha^2 = A(s+\alpha) + Bs(s+\alpha) + Cs^2$$
$$\alpha^2 = As + \alpha A + Bs^2 + \alpha Bs + Cs^2$$
$$\alpha^2 = (B+C)s^2 + (A+\alpha B)s + \alpha A$$

Now comparing coefficients,

$$\alpha A = \alpha^2 \qquad B = -C$$
$$\underline{A = \alpha} \qquad A + \alpha B = 0 \Rightarrow \alpha + \alpha B = 0$$
$$\alpha = -\alpha B$$
$$B = -1$$
$$\underline{C = 1}$$

Which gives us the following function after consulting the Laplace transform table

$$\underline{f(t) = [\alpha t - 1 + e^{-\alpha t}]u(t)}$$

(9.27b)

Find the inverse Laplace transform of the following

$$F(s) = \frac{\alpha^2}{s(s+\alpha)^2} = \frac{A}{s} + \frac{B}{s+\alpha} + \frac{C}{(s+\alpha)^2}$$

$$\alpha^2 = A(s+\alpha)^2 + Bs(s+\alpha) + Cs$$
$$\alpha^2 = A(s^2+2\alpha s+\alpha^2) + Bs^2 + sB\alpha + Cs$$
$$\alpha^2 = As^2 + 2A\alpha s + A\alpha^2 + Bs^2 + B\alpha + Cs$$

$$\Rightarrow \left.\begin{aligned} A + B &= 0 \\ 2A\alpha + B\alpha + C &= 0 \\ A\alpha^2 &= \alpha^2 \end{aligned}\right\} \Rightarrow \begin{aligned} A &= 1 \\ B &= -1 \\ C &= -\alpha \end{aligned}$$

Again, by looking at the table of inverse laplace transforms, we get

$$f(t) = [1 - \alpha t e^{-\alpha t} - e^{-\alpha t}]\, u(t)$$

9.30

(a) Find the inverse Laplace transforms of the following

$$F(s) = \frac{16s}{(s+3)(s^2+21s+20)} = \frac{16s}{(s+3)(s+1)(s+20)}$$

$$= \frac{A}{s+1} + \frac{B}{s+3} + \frac{C}{s+20}$$

Comparing coefficients as we have previously,

$$16s = A(s+3)(s+20) + B(s+1)(s+20) + C(s+1)(s+3)$$
$$16s = A(s^2+23s+60) + B(s^2+21s+20) + C(s^2+4s+3)$$
$$16s = s^2(A+B+C) + s(23A+21B+4C) + 60A+20B+3C$$

$$\Rightarrow \quad \begin{aligned} A+B+C &= 0 \\ 60A+20B+3C &= 0 \\ 23A+21B+4C &= 16 \end{aligned} \quad \Rightarrow \begin{bmatrix} 1 & 1 & 1 \\ 60 & 20 & 3 \\ 23 & 21 & 4 \end{bmatrix} \begin{bmatrix} A \\ B \\ C \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 16 \end{bmatrix}$$

Using Matlab to solve the above matrix equation gives us

$$A = -8/19$$
$$B = 24/7$$
$$C = -320/323$$

so $f(t) = \left[-\frac{8}{19}e^{-t} + \frac{24}{7}e^{-3t} - \frac{320}{323}e^{-20t}\right] u(t)$

(b) $$F(s) = \frac{60(s^2+16)}{s(s^2+36)} = \frac{A}{s} + \frac{Bs+C}{s^2+36}$$

$$A(s^2+36) + (Bs+C)s = 60s^2 + (60)(16)$$
$$As^2 + 36A + Bs^2 + Cs = 60s^2 + 960$$

$$\Rightarrow \quad A+B=60, \quad 36A = 960, \quad C = 0 \qquad \Big| \qquad A = \frac{960}{36}, \quad B = \frac{100}{3}$$

$$F(s) = \frac{960}{36s} + \frac{\frac{100}{3}s}{s^2+36}, =$$

Now taking the inverse laplace

$$F(t) = \left[\frac{960}{36} + \frac{100}{3}\cos(6t)\right] u(t)$$

9.35

$$F(s) = K \cdot \frac{s+\gamma}{s+\alpha} = \frac{s+\gamma}{s+50}$$

Select the appropriate value for γ. To do this lets take the inverse ~~...~~ Laplace transform.

$$\frac{s+\gamma}{s+50} = A + \frac{B}{s+50}$$

$\Rightarrow$ $A(s+50) + B = s+\gamma$

$As + 50A + B = s + \gamma$

$\Rightarrow$ $A = 1$

$50A + B = \gamma$

$50 + B = \gamma$, $B = \gamma - 50$

(a) we want $B = -5$, $\Rightarrow \gamma = 45$

(b) $\gamma = 50$

(c) $\gamma = 55$

9.37(a)

$$F(s) = \frac{s^2+1}{s+5},$$

Here the order of the numerator is greater than the order of the numerator, so we need to divide first

$$s+5 \overline{\big) s^2+1} = s-5 + \frac{26}{s+5}$$

(using long division here),

and now we can directly find f(t) using the table of laplace transforms,

$$f(t) = \left[\delta'(t) - 5\delta(t) + 26e^{-5t}\right]u(t)$$

(b) $$F(s) = \frac{(s+1000)^2}{(s+10000)^2} = 1 - \frac{18000s + 9.9\times10^7}{(s+10000)^2}$$

Now we can take a partial fraction expansion to give us

$$F(s) = 1 - \left[\frac{1}{s+10000} - \frac{4500}{(s+10000)^2}\right]18000$$

$$f(t) = \left[\delta(t) - 18000[1 - 4500t]e^{-10000t}\right]u(t)$$