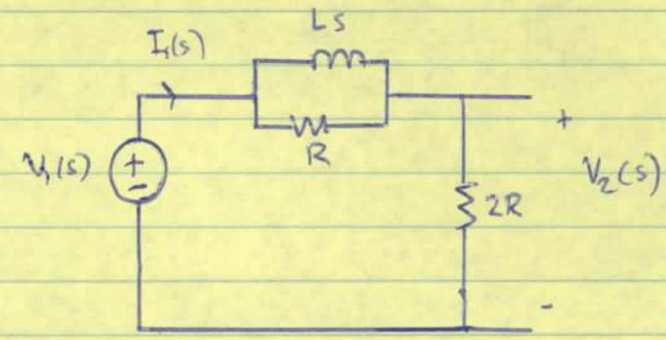
MAE140 HW#9 SOLUTIONS

11.2

(a)



We want to find the voltage transfer function. Let us first find the parallel resistance of the inductor and resistance

$$Ls \| R = \frac{1}{\frac{1}{R}+\frac{1}{Ls}} = \frac{1}{\frac{Ls+R}{RLs}} = \frac{sLR}{Ls+R}$$

Now,

$$I_1(s) = \frac{V_1(s)}{\frac{sLR}{Ls+R}+2R} = \frac{V_1(s)\cdot(Ls+R)}{sLR+2R(Ls+R)}$$

$$V_2(s) = I_1(s)\cdot 2R$$

$$= \frac{V_1(s)\cdot(Ls+R)\cdot 2R}{sLR+2R(Ls+R)} = V_1(s)\cdot\frac{(2RLs+2R^2)}{s(LR+2RL)+2R^2}$$

so,

$$\underline{\underline{T(s) = \frac{2RLs+2R^2}{3RLs+2R^2}}}$$

The driving point impedance is $Z(s) = \frac{sLR}{Ls+R} + 2R$

$$= \frac{sLR+2R(Ls+R)}{Ls+R}$$

(b) Select value of R & L such that transfer function has a zero at $s = -120$ rad/s and a pole at $s = -80$ rad/s.

The value of the zero is

$$2RLs + 2R^2 = 0$$

$$2RLs = -2R^2$$

$$s = \frac{-2R^2}{2RL} = -\frac{2R}{2L} = -\frac{R}{L}$$

and the value of the pole:

$$3RLs + 2R^2 = 0$$

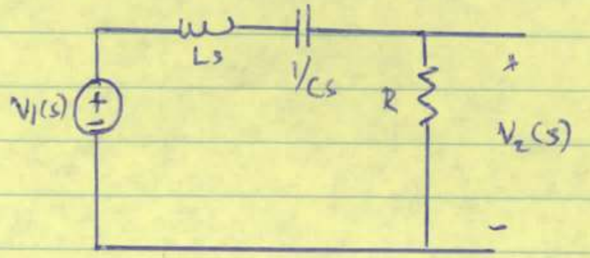
$$3RLs = -2R^2$$

$$s = \frac{-2R^2}{3RL}$$

$$= -\frac{2}{3} \cdot \frac{R}{L}$$

so, we select ~~[illegible]~~ $\boxed{R = 12,\ L = 0.1\,H}$

11.3



Let's first compute the driving point impedance $Z(s) = Ls + \frac{1}{Cs} + R$

$$= \frac{s^2 LC + 1 + sCR}{Cs}$$

$$= \frac{s^2 LC + sCR + 1}{Cs}$$

Now let us compute the transfer function.

$$I_1(s) = \frac{V_1(s)}{Z} = \frac{V_1(s) \cdot Cs}{s^2 LC + sCR + 1}$$

$$V_2(s) = I_1(s) \cdot R$$

$$= V_1(s) \cdot \frac{sCR}{s^2 LC + sCR + 1}$$

$$T(s) = \frac{sCR}{s^2 LC + sCR + 1} = \frac{\frac{R}{L} s}{s^2 + \frac{R}{L} s + \frac{1}{LC}} = \frac{R}{L} \cdot \frac{s}{s^2 + \frac{R}{L} s + \frac{1}{LC}}$$

There is a single zero at $s = 0$. For the poles we require

$$(s + 1000)(s + 1000) = s^2 + \frac{R}{L} s + \frac{1}{LC}$$

$$s^2 + 2000s + 1000000 = s^2 + \frac{R}{L} s + \frac{1}{LC}$$

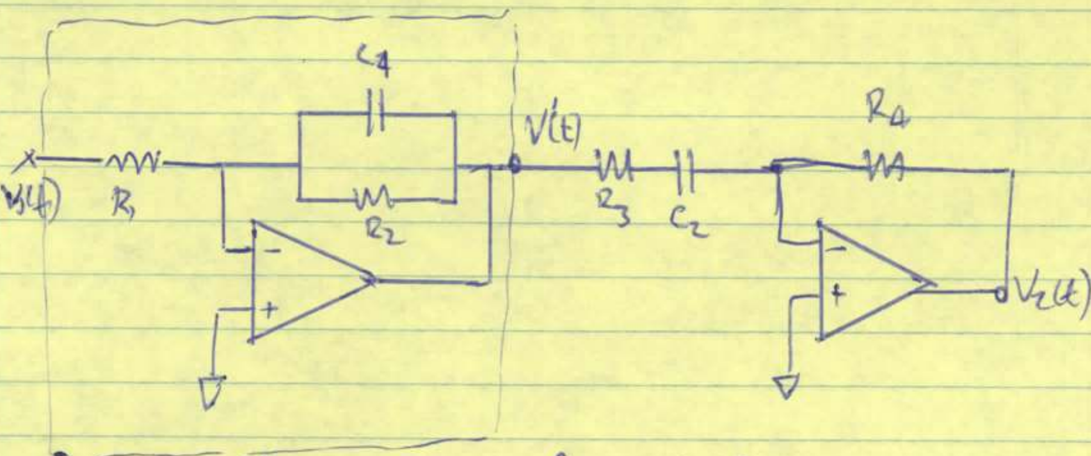
$\frac{R}{L} = 2000$,

$R = 2k$

$L = 1H$

$C = 1 \times 10^{-6}\ F$

12.32



Low pass filter (we know this from before)

The transfer function of the first stage circuit is

$$T(s) = \frac{-R_2}{R_1} \cdot \frac{1}{1 + R_2 C_1 s}$$

We can also find the transfer function of the second stage which is also a Inverting amplifier.

$$T(s) = \frac{-R_4}{R_3 + \frac{1}{sC_2}} = \frac{-R_4}{\frac{sR_3C_2 + 1}{sC_2}} = \frac{-sC_2R_4}{1 + sR_3C_2}$$

Which we see is a high pass response.

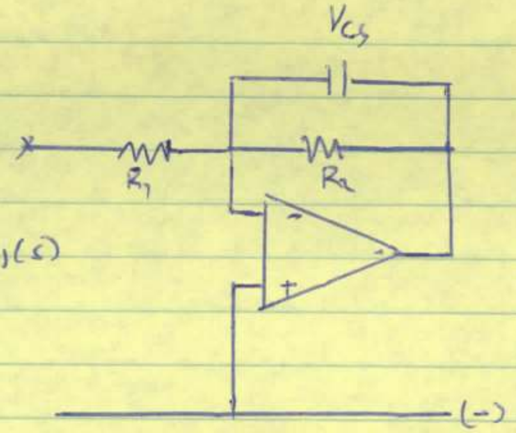
Let us select the low pass filter cutoff frequency

$$\omega_{c_2} = \frac{1}{R_2 C_1} = 40000, \quad R_2 = 10k, \quad C_1 = 2.5nF$$

$$\omega_{c_1} = \frac{1}{R_3 C_2} = 1000, \quad R_3 = 10k, \quad C_2 = 0.1\mu F$$

Back

11.12



This is an inverting amplifier with $H(s) = -\frac{Z_2}{Z_1}$ where $Z_1 = R_1$ and

$$Z_2 = \frac{1}{\frac{1}{R_2} + Cs} = \frac{1}{\frac{1+R_2Cs}{R_2}} = \frac{R_2}{1+R_2Cs}$$

so, $$H(s) = T(s) = -\frac{R_2}{1+R_2Cs} \cdot \frac{1}{R_1} = -\frac{R_2}{R_1} \cdot \frac{1}{1+R_2Cs}$$

We can see immediately that the input impedance is equal to R_1, so select $R_1 = 1.5\Omega$.

We can then select $R_2 = 150k$.

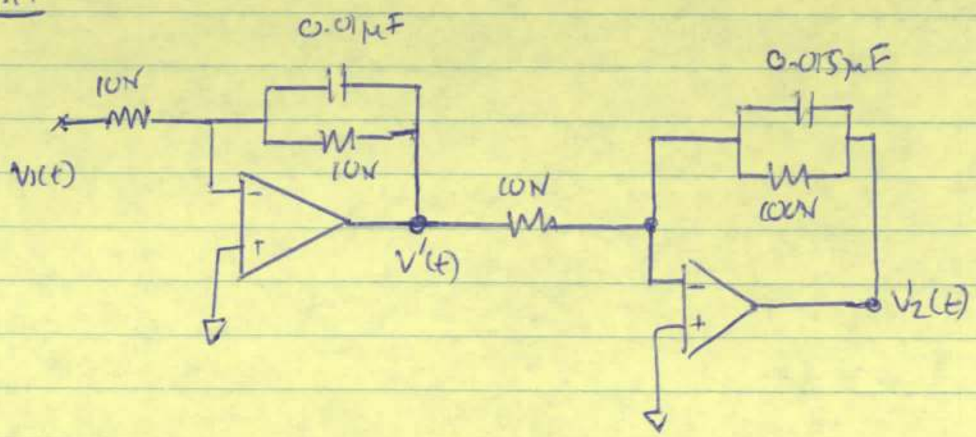
Finally, we need to determine the location of the pole

$$R_1 + R_1R_2cs = 0$$

$$s = \frac{-R_1}{R_1R_2C} = -500$$

$$c = \frac{+R_1}{(500)R_1R_2} = \frac{1}{500R_2} = \underline{\underline{13.3\,pF}}$$

11.14



This is two inverting amplifiers cascaded together, so

$$T(s) = T_1(s) \cdot T_2(s)$$

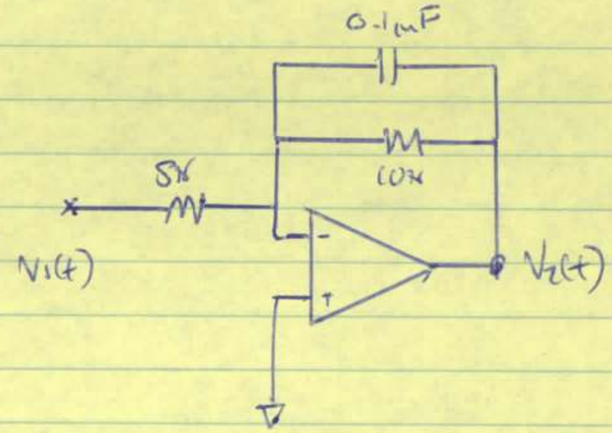
$$= \frac{-10K}{10K} \cdot \frac{1}{1+10K \cdot 0.01\mu \cdot s} \cdot \frac{-100K}{10K} \cdot \frac{1}{1+100K \cdot 0.015\mu\, s}$$

$$= \frac{-10}{(1+1\times10^{-4}s)(1+1.5\times10^{-3}s)}$$

$$s = \frac{-1}{10^{-4}} = -10^4, \quad \frac{1}{1.5\times10^{-3}} = -666.67$$

There are no zeros.

11.36



We have another inverting amplifier, so the transfer function is

$$T(s) = \frac{-R_2}{R_1} \cdot \frac{1}{1+R_2CS} = \frac{-10K}{5K} \cdot \frac{1}{1+10K \cdot 0.1\mu\, s}$$

$$= \frac{-2}{1+1\times10^{-3}\, s}$$

The pole is located at $s = \frac{-1}{1\times10^{-3}} = \underline{\underline{-1000}}$

$T(s) = -\frac{2000}{s+1000}$, $\quad T(j\omega) = \frac{-2000}{j\omega+1000}$

$v_1(t) = 5\cos(500t)\,V$, $\quad \omega = 500$

$T(j500) = -1.6 + 0.8j$

$|T(500j)| = 1.78885$

$\angle T(500j) = 153.435$

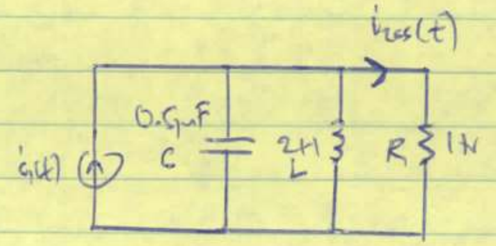
so, $V_2(t) = 5 \cdot 1.78885\cos(500t + 153.4°)$

$= 8.9442719\cos(500t + 153.4°)$

$v_1(t) = 5\cos(1000t)$, $\quad V_2(t) = 7.07\cos(1000t + 135°)$

$V_1(t) = 5\cos(10Kt)$, $\quad V_2(t) = 0.995\cos(10000t + 95.7°)$

11.40



$$I_1(s) = VsC + \frac{V}{sL} + \frac{V}{R}$$

$$I_1(s) = V\left(sC + \frac{1}{sL} + \frac{1}{R}\right)$$

$$I_1(s) = V\left(\frac{s^2RLC + R + sL}{sLR}\right)$$

$$V(s) = \frac{I_1 \cdot sLR}{s^2RLC + sL + R}$$

$$I_2(s) = \frac{V(s)}{R}$$

$$= \frac{I_1(s)\, sL}{s^2RLC + sL + R}$$

$$T(s) = \frac{I_2(s)}{I_1(s)} = \frac{sL}{s^2RLC + sL + R}$$

If $i_1(t) = 10\cos(100t)$ mA, we have $T(100j) = 0.1980 \angle 78.5788$

so,

$\underline{i_2(t)} = 10 \cdot 0.1980 \cos(100t + 78.58°)$ mA

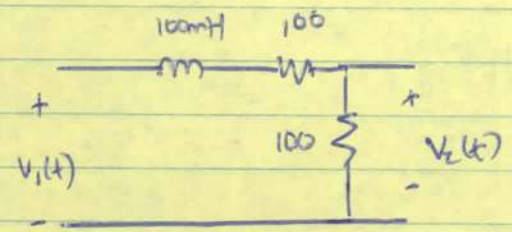
If $i_1(t) = 5\cos(1000t)$ mA, $T(1000j) = 1 \angle 0$, so

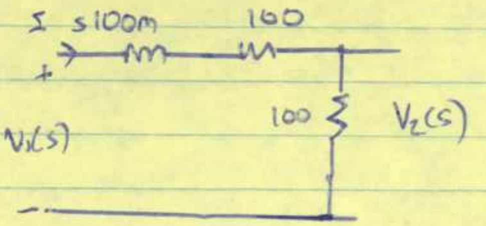
$\underline{i_{2ss}(t) = i_1(t)}$

If $i_1(t) = 5\cos(10000t)$ mA $T(10000j) = 0.198 \angle -78.5788$

$\underline{i_{2ss}(t)} = 0.99\cos(10000t - 78.58°)$ $\underline{\text{mA}}$

12.4





$$I(s) = \frac{V_1(s)}{200 + 0.1s}$$

$$V_2(s) = \frac{V_1(s) \cdot 100}{200 + 0.1s}$$

$$T(s) = \frac{100}{200 + 0.1s} = \frac{1000}{s + 2000} = \frac{0.5\,(2000)}{s + 2000}$$

We see immediately that this is a first order low pass response,

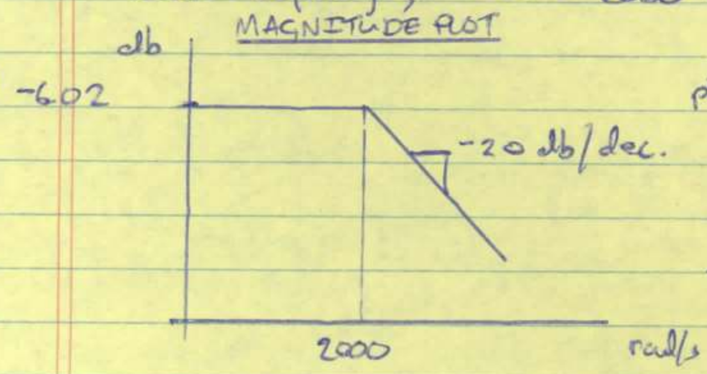
DC GAIN, $s = 0 \Rightarrow T(s=0) = \underline{0.5}$

Infinite frequency gain $T(s \to \infty) = 0$

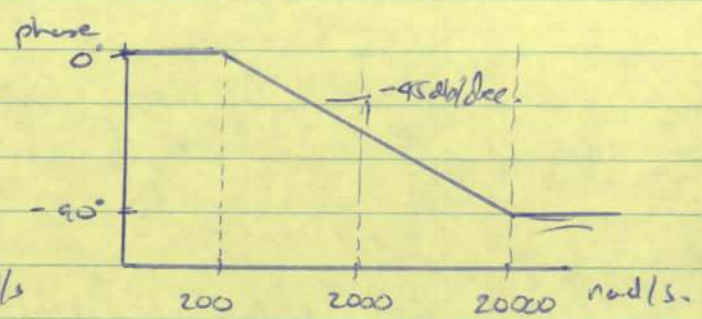
Cutoff frequency, $\omega_c = 2000$ rad/s.

MAGNITUDE PLOT

(b)



PHASE PLOT



(c) Calculate the gain at $\omega = 0.1\omega_c$, $\omega = 10\omega_c$,

$$T(j\omega) = \frac{1000}{j\omega + 2000}$$

$$T(200) = \frac{1000}{200j + 2000} = 0.4975\angle{-5.7106}$$

So gain is 0.4975 or -6.0638 db,

$$T(20000) = \frac{1000}{20000j + 2000} = 0.04975\angle{-84.289}$$

So gain is 0.04975 or $\underline{-26.0638 \text{ db}}$

$$T(2000) = \frac{1000}{j2000 + 2000} = 0.35355\angle{-45}$$

So gain is $\underline{0.35355 \text{ or } -9.0308999 \text{ db}}$

(e) ~~1 octave down is half the cutoff frequency, so $\omega = 1000$ rad/s.~~

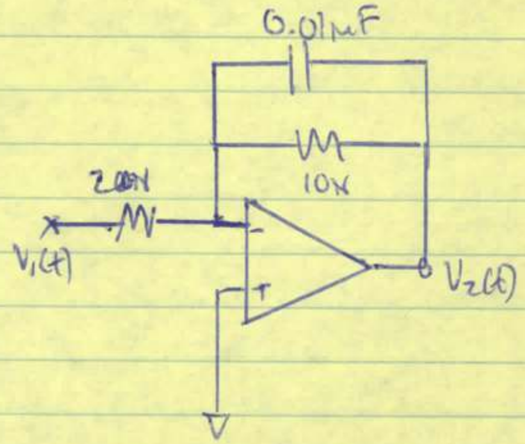
$$\cancel{T(1000) = \frac{1000}{j1000 + 2000} = 0.4 - 0.2j = 0.4472\angle{-26.565^\circ}}$$

1 octave past cutoff is $2\omega_c = 4000$.

$$T(4000) = \frac{1000}{4000j + 2000} = 0.2236\angle{-63.435}$$

Gain is 0.2236 or $\underline{\underline{-13.0103 \text{ db}}}$, At the passband the gain is -6.0638 db, so the dB difference is $13.0103 - 6.0638 = \underline{6.9465 \text{ db down}}$

12.7



We know the transfer function of this circuit from our prior analysis

$$T(s) = -\frac{R_2}{R_1} \cdot \frac{1}{1 + R_2 C s}$$ where $R_1 = 20K$, $R_2 = 10K$, $C = 0.01\mu F$

=

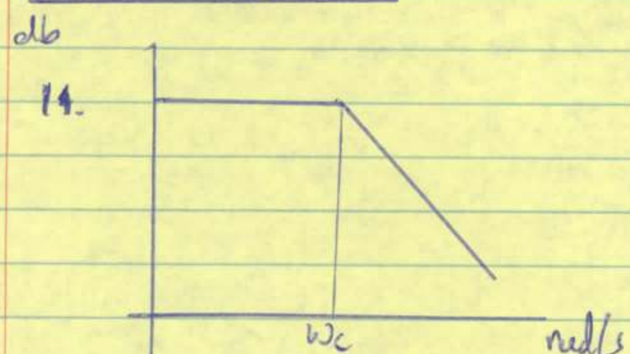
We can again see from the form of the transfer function that this circuit has a first order low pass response.

DC gain $= \frac{+R_2}{R_1} = \underline{\underline{-.5}}$

Infinite frequency gain, $= 0$

Cutoff frequency $\omega_c = \frac{1}{R_2 C} = \underline{\underline{10000 \text{ rad/s}}}$

(b) Straight line approximation



Compute gain at $0.1\omega_c$, ω_c and $10\omega_c$

$0.1\omega_c$

$$|T(j\,0.1\omega_c)| = \left| -5 \cdot \frac{1}{1 + 0.1m \times (1000j)} \right| = 4.975$$

so gain is 4.975 or $\underline{\underline{13.936db}}$

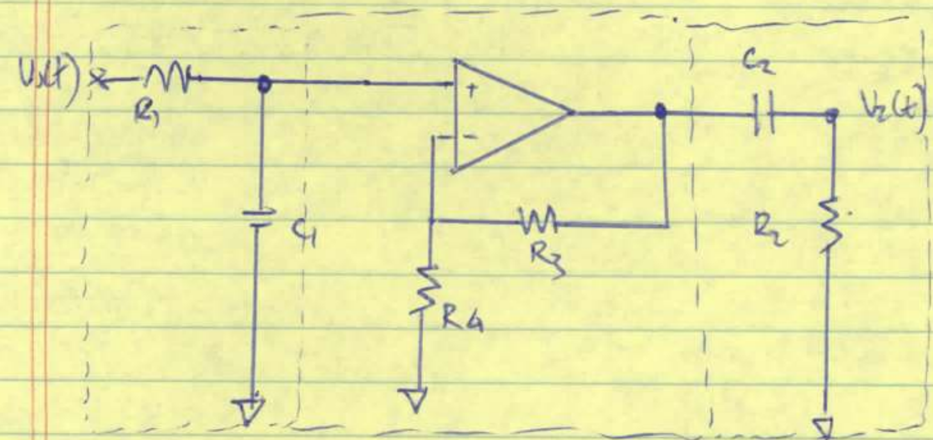
ω_c

$$|T(j\omega_c)| = 3.5355 \text{ or } \underline{10.97db}$$

$10\omega_c$ $|T(j10\omega_c)| = 0.4975$ or $-6.06db$

(e) You would change R_1 by a factor of 5.

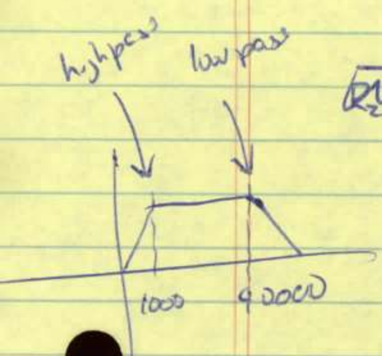
12.31



R_1 & C_1 control the cutoff frequency of a ~~high~~ low pass filter while R_2 & C_2 control the cutoff frequency of a ~~low~~ high pass filter. The passband gain is controlled by R_3 & R_4 (which we see are arranged in an non inverting amplifier configuration

~~We want the highpass lowpass filter to have a cutoff frequency of 40000 rad/s,~~

~~so $\frac{1}{R_2C_2} = 40000, \Rightarrow R_2 = 25k, C_2 = 1nF$~~



~~$\frac{1}{R_2C_2} =$~~

$$\frac{1}{R_1C_1} = 40000, \Rightarrow R_1 = 25k, \quad C_1 = 1nF$$

$$\frac{1}{R_2C_2} = 1000, \Rightarrow R_2 = 10k, \quad C_2 = 0.1\mu F$$

$$1 + \frac{R_3}{R_4} = 100, \Rightarrow R_3 = 99k, \quad \underline{\underline{R_4 = 1k}}$$

Passband gain is $\left(\frac{R_2}{R_1}\right) \cdot \left(\frac{R_4}{R_3}\right)$ so $\underline{R_4 = 10^4\ \Omega}$