MAE140 - Linear Circuits - Fall 2016
Midterm, October 27th

Instructions

1. This exam is open book. You may use whatever written materials you choose, including your class notes and textbook. You may use a hand calculator with no communication capabilities.
2. You have 70 minutes.
3. Write your answers on your "Blue Book".
4. Do not forget to write your name, student number, and instructor.

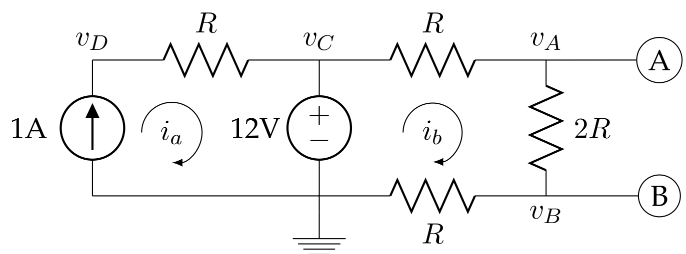


Figure 1: Circuit for Questions 1, 2, and 3

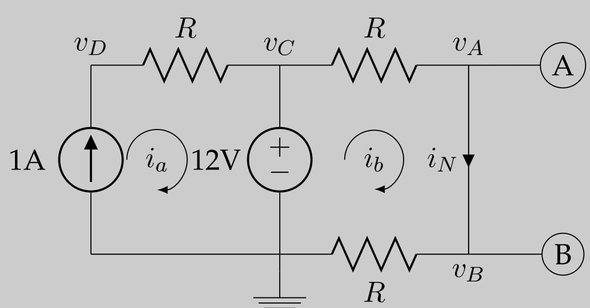
1. **Equivalent Circuits**

(a) (5 points) For the circuit in Fig. 1, find the Thevenin and Norton equivalent circuits as seen from terminals A and B by performing open-circuit and short-circuit experiments.

Solution:
First determine the Thevenin voltage using circuit division:

$$v_T = v_A - v_B = \frac{2R}{4R}12\text{V} = 6\text{V}. \qquad \text{(+1 point)}$$

Note that $v_C = 12$ V no matter what!
Short-circuit A-B and calculate the Norton current on the circuit:



(+1 point)

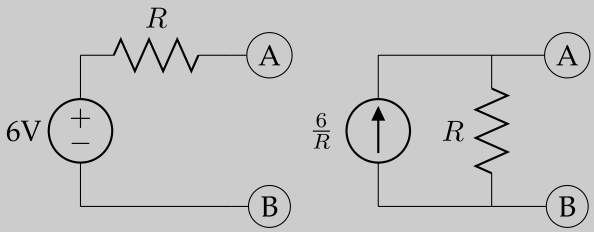
to obtain

$$i_N = \frac{12}{2R} = \frac{6}{R}. \qquad \text{(+1 point)}$$

The equivalent resistance is

$$R = \frac{v_T}{i_N} = \frac{6}{\frac{6}{R}} = R \qquad \text{(+1 point)}$$

The Thevenin and Norton equivalent circuits are therefore:



(+1 point)

(b) (1 point (bonus)) Explain the role played by the current source in the equivalent circuits.

Solution: It plays no role since the voltage v_c is completely determined by the voltage source. (+1 point)

2. **Node Voltage Analysis**

(a) (4 points) Formulate node-voltage equations for the circuit in Fig. 1. Use the node-voltages and labels provided in the figure and clearly indicate the final equations and circuit variable unknowns. Make sure your final equations involve only node-voltages.

Solution:
Perhaps the easiest approach here is to use Method #2 to write

$$v_C = 12\text{V}. \qquad \text{(+1 point)}$$

NOTE TO GRADERS: A source transformations is also fine.
Writing the KCL equations at nodes A, B and D by inspection we obtain:

$$\begin{bmatrix} \frac{1}{R} + \frac{1}{2R} & -\frac{1}{2R} & -\frac{1}{R} & 0 \\ -\frac{1}{2R} & \frac{1}{R} + \frac{1}{2R} & 0 & 0 \\ 0 & 0 & -\frac{1}{R} & \frac{1}{R} \end{bmatrix} \begin{pmatrix} v_A \\ v_B \\ v_C \\ v_D \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 1\text{A} \end{pmatrix} \qquad \text{(+2 points)}$$

These are the node-voltage equations that need to be solved for the node voltages v_A, v_B, v_C, and v_D. (+1 point)

(b) (1 point) Explain how you could solve for the mesh-current i_b after having calculated the node-voltages.

Solution:
The mesh-current i_b is the current at the R resistor, so it could be calculated as

$$i_b = \frac{v_B}{R} \qquad \text{(+1 point)}$$

NOTE TO GRADERS: Using the $2R$ or the top R resistor is also possible.

3. **Mesh Current Analysis**

   (a) (4 points) Formulate mesh-current equations for the circuit in Fig. 1. Use the mesh-currents and labels provided in the figure and clearly indicate the final equations and circuit variable unknowns. Make sure your final equations involve only mesh-currents.

   Solution:
   Again the easiest approach here is to use Method #2 to write

   $$i_a = 1\text{A}. \qquad \text{(+1 point)}$$

   Writing the KVL equations for mesh b by inspection we obtain:

   $$\begin{bmatrix} 0 & R + R + 2R \end{bmatrix} \begin{pmatrix} i_A \\ i_B \end{pmatrix} = \begin{pmatrix} 12\text{V} \end{pmatrix} \qquad \text{(+2 points)}$$

   These are the trivial mesh-current equations that need to be solved for the mesh currents i_a and i_b. (+1 point)

   (b) (1 point) Explain how you could solve for the node voltages v_A and v_B after having calculated the mesh-currents.

   Solution:
   The node voltage v_B is the voltage across the resistor R, that is

   $$v_B = Ri_b \qquad \text{(+1/2 point)}$$

   and the node voltage v_A is obtained by going over the $2R$ resistor:

   $$v_A = v_B + 2Ri_b = 3Ri_b \qquad \text{(+1/2 point)}$$

   NOTE TO GRADERS: Using the top R resistor is also possible.

4. **OpAmp Circuit**

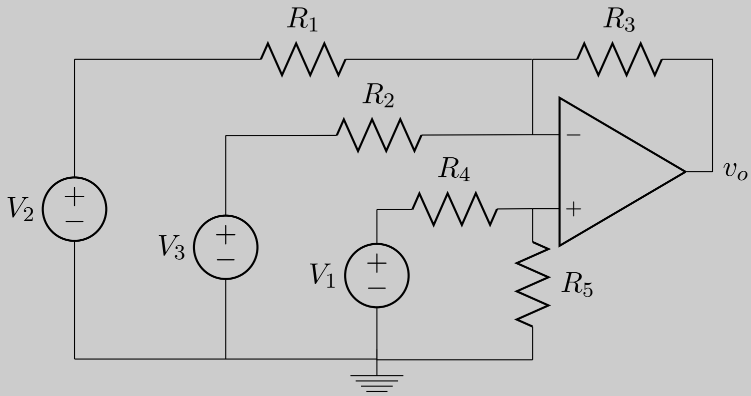
   (a) (5 points) Design an OpAmp circuit that realizes the following arithmetic operation:

   $$v_o = v_1 - v_2 - v_3$$

   where v_1, v_2 and v_3 are given voltage sources. Do not forget to select values for resistors!

   (b) (1 point (bonus)) You get a bonus point if your circuit uses only one OpAmp.

   Solution:
   NOTE TO GRADERS: Many solutions are possible.
   One solution using a differentiator is as follows:



(+2 points)

Using linearity, the output v_o is

$$v_o = \frac{R_5}{R_4 + R_5} \frac{R_3 + R_1//R_2}{R_1//R_2} v_1 - \frac{R_3}{R_1} v_2 - \frac{R_3}{R_2} v_3 \qquad \text{(+1 point)}$$

where $R_1//R_2$ is the parallel association of R_1 and R_2.
The desired operation can be realized by having

$$R_1 = R_2 = R_3 = R \qquad \text{(+1 point)}$$

and

$$\frac{R_5}{R_4 + R_5} = \frac{R_1//R_2}{R_3 + R_1//R_2} = \frac{R/2}{R + R/2} = \frac{1}{3}$$

that can be achieved with

$$R_4 = 2R, \qquad R_5 = R. \qquad \text{(+1 point)}$$

Select R to be, say, $100K$ to complete the design.