MAE140 – Linear Circuits – Fall – 2017
Final, December 11th

Instructions

1. This exam is open book. You may use whatever written materials you choose, including your class notes and textbook. You may use a handheld calculator with no communication capabilities.
2. You have 170 minutes.
3. Do not forget to write your name and student number.
4. This exam has 6 questions, for a total of 60 points and 2 bonus points.

1. (10 points) **OpAmp Circuit Design**

   Design an OpAmp circuit that can be used to realize the following transfer-function

   $$T(s) = \frac{10{,}000s}{s^2 + 10{,}010s + 100{,}000}$$

   calculated between an input voltage v_i and an output voltage v_o. Show your work.

2. **Frequency Response**

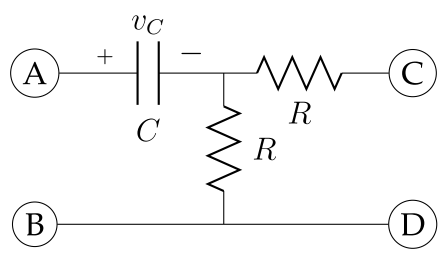
   In this question you will analyze the circuit you designed in Question 1.

   (a) (3 points) Evaluate $|T(j\omega)|$ and $\angle T(j\omega)$ for $\omega = \{0, 1000, \infty\}$rad/s.

   (b) (3 points) Sketch the magnitude and phase of the frequency response $T(j\omega)$.

   (c) (2 points) Classify your circuit as a filter based on its frequency response.

   (d) (2 points) Use the frequency response method or a direct calculation to compute the steady-state response of the circuit to an input of the form $v_i(t) = -\cos(1000t) = \cos(\pi + 1000t)$V.



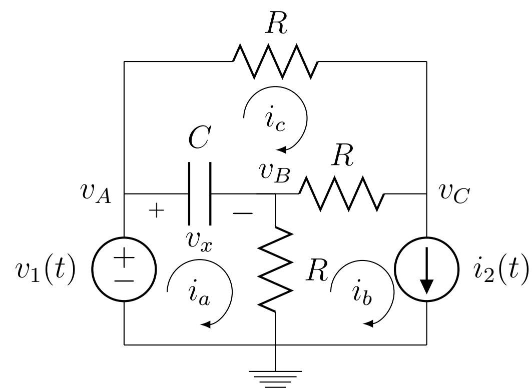


Figure 1: Circuits for Questions 3 and 4

3. **Circuit Equivalence**

   IMPORTANT: All impedances should be given as a ratio of two polynomials!

   (a) (3 points) Assume zero-initial conditions and find the s-domain equivalent circuit as seen from terminals Ⓐ and Ⓑ in Fig. 1a.

   (b) (3 points) Assume zero-initial conditions and find the s-domain equivalent circuit as seen from terminals Ⓒ and Ⓓ in Fig. 1a.

   (c) (4 points) Connect a voltage source $v_s(t)$ between terminals Ⓐ and Ⓑ, assume zero-initial conditions and calculate the s-domain Thevenin equivalent as seen from terminals Ⓒ and Ⓓ.

4. **Circuit Analysis**

   (a) (4 points) Convert the circuit in Fig. 1b to the s-domain and formulate its node-voltage equations. Use the node-voltage and labels provided in the figure and clearly indicate the final equations and circuit variable unknowns. Do not assume zero initial conditions. Make sure your final equations only involve node-voltages.

   (b) (4 points) Convert the circuit in Fig. 1b to the s-domain and formulate its mesh-current equations. Use the mesh-currents and labels provided in the figure and clearly indicate the final equations and circuit variable unknowns. Do not assume zero initial conditions. Make sure your final equations only involve mesh-currents.

   (c) (2 points) Express the s-domain voltage $V_x(s)$ in terms of the node voltages and mesh currents from parts (a) and (b).

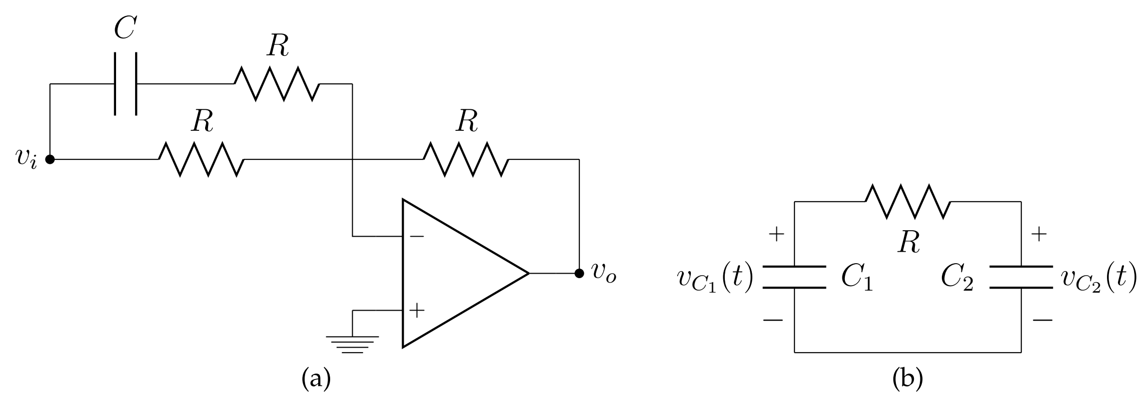


Figure 2: Circuits for Questions 5 and 6

5. **OpAmp Circuit Analysis**

   A student claims that the circuit in Fig. 2a is an amplifier that has a higher gain at DC than it has at higher frequencies. Answer the following questions to prove or disprove this claim.

   (a) (3 points) Assume zero-initial conditions and calculate the transfer-function from the input v_i to the output v_o.

   (b) (2 points) Calculate the DC gain and the gain at $\omega \to \infty$.

   (c) (3 points) Sketch the magnitude of the frequency response of the transfer-function.

   (d) (2 points) Is the student right or wrong? Explain.

6. **Transient Analysis**

   In the circuit in Fig. 2b the initial voltages are $v_{C_1}(0) = 5\text{V}$ and $v_{C_2}(0) = 0\text{V}$.

   (a) (1 point) Convert the circuit to the s-domain.

   (b) (6 points) Use the Laplace transform to show that when $C_1 = C_2 = C$ then

   $$v_{C_1}(t) = 2.5 + 2.5\, e^{-2t/(RC)}\text{V}, \quad t \geq 0$$

   and

   $$v_{C_2}(t) = 2.5 - 2.5\, e^{-2t/(RC)}\text{V}, \quad t \geq 0$$

   (c) (3 points) Plot the voltages $v_{C_1}(t)$ and $v_{C_2}(t)$ and calculate the power absorbed by the resistor for $t \geq 0$.

   (d) (2 points (bonus)) Explain the values of the voltages v_{C_1} and v_{C_2} at $t \to \infty$ based on what you know about the relationship between charge and the voltage in capacitors. Without performing any circuit calculations, predict what would happen if $C_1 = C$ and $C_2 = C/2$?