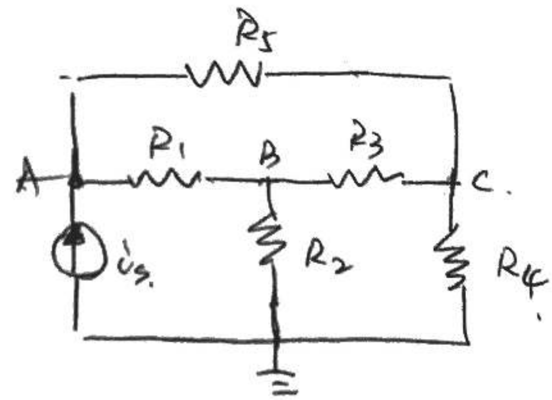
HW #3 Solution.

3.1.

$$\begin{bmatrix} \frac{1}{R_1}+\frac{1}{R_5} & -\frac{1}{R_1} & -\frac{1}{R_5} \\ -\frac{1}{R} & \frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3} & -\frac{1}{R_3} \\ -\frac{1}{R_5} & -\frac{1}{R_3} & \frac{1}{R_3}+\frac{1}{R_4}+\frac{1}{R_5} \end{bmatrix} \begin{pmatrix} v_A \\ v_B \\ v_C \end{pmatrix} = \begin{pmatrix} i_{s1} \\ 0 \\ 0 \end{pmatrix}$$



3.3

a) $$\begin{bmatrix} \frac{1}{5}+\frac{1}{10}+\frac{1}{15} & -\frac{1}{15} \\ -\frac{1}{15} & \frac{1}{5}+\frac{1}{15} \end{bmatrix} \begin{pmatrix} v_A \\ v_B \end{pmatrix} = \begin{pmatrix} 0 \\ i_s \end{pmatrix}$$ w/ $i_s = 3A$.

b) Solve: $\boxed{v_A = 2.1429\ V}$, $\boxed{v_B = 11.7857\ V}$

c) Current flow 5Ω (connected to B):

$i_1 = \dfrac{V_B}{5\Omega} = 2.3571\ A$,

i_2 through 15Ω : $i_2 = 3A - 2.3571\ A = 0.6429\ A$

i_3 through 10Ω : $i_3 = \dfrac{V_A}{10\Omega} = 0.2143\ A$.

$i_x = i_2 - i_3 = \boxed{0.4286\ A}$.

$\boxed{v_x = V_B = 11.7857\ V}$

3.5

a)

$\underline{A}$: $\dfrac{v_A}{10k\Omega} + \dfrac{v_A - v_B}{10k\Omega} = 10\,mA$

$\underline{B}$: $v_B = 5\,V$

C : $\dfrac{v_C}{10k\Omega} + \dfrac{v_C - v_B}{10k\Omega} + 10\,mA = 0$

$$\begin{bmatrix} \frac{1}{10}+\frac{1}{10} & 0 \\ 0 & \frac{1}{10}+\frac{1}{10} \end{bmatrix} \begin{pmatrix} v_A \\ v_C \end{pmatrix} = \begin{pmatrix} 10mA + v_B/10k\Omega \\ -10mA + v_B/10k\Omega \end{pmatrix}$$

b) $v_A = 52.5\ V$, $v_C = -47.5\ V$

c) $V_x = V_A - V_C = 100\ V$

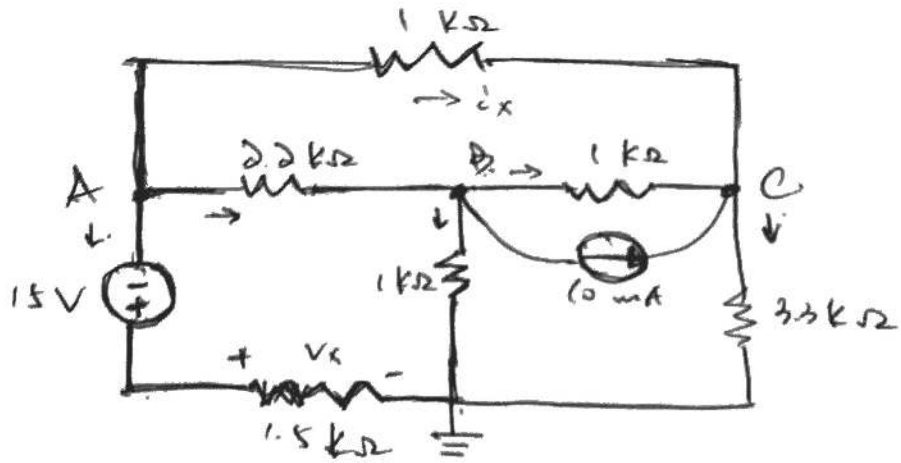
$$-i_x + \frac{V_A - V_B}{10k\Omega} + \left(-\frac{v_B - v_C}{10k\Omega}\right) = 0$$

$i_x = 4.75\,mA + 5.25\,mA$

$\boxed{i_x = 10\,mA}$

3.8

a)



b)

$$\underline{A}:\ \frac{v_A - v_B}{2.2\,k\Omega} + \frac{v_A - v_C}{1\,k\Omega} + \frac{v_A + 15\,V}{1.5\,k\Omega} = 0$$

$$\underline{B}:\ \frac{v_B}{1\,k\Omega} + \frac{v_B - v_A}{2.2\,k\Omega} + \frac{v_B - v_C}{1\,k\Omega} + 10\,mA = 0$$

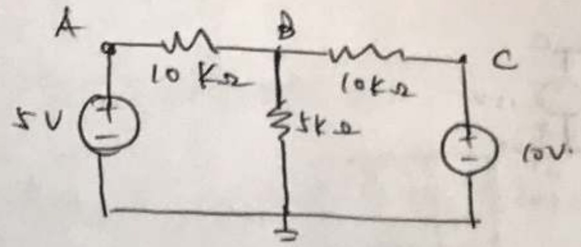
$$\underline{C}:\ \frac{v_C - v_A}{1\,k\Omega} + \frac{v_C - v_B}{1\,k\Omega} + \frac{v_C}{3.3\,k\Omega} - 10\,mA = 0$$

$$\begin{bmatrix} \frac{1}{2.2}+\frac{1}{1}+\frac{1}{1.5} & -\frac{1}{2.2} & -\frac{1}{1} \\ -\frac{1}{2.2} & \frac{1}{1}+\frac{1}{2.2}+\frac{1}{1} & -\frac{1}{1} \\ -\frac{1}{1} & -\frac{1}{1} & \frac{1}{1}+\frac{1}{1}+\frac{1}{3.3} \end{bmatrix} \begin{pmatrix} v_A \\ v_B \\ v_C \end{pmatrix} = \begin{pmatrix} -\frac{15\,V}{1.5\,k\Omega} \\ -10\,mA \\ 10\,mA \end{pmatrix}$$

$$v_x = v_A + 15\,V = 8.714\ V$$

$$i_x = \frac{v_A - v_C}{1\,k\Omega} = -5.482\ mA.$$

3.16



a)

$\underline{A}$: $V_A = 5\,V$

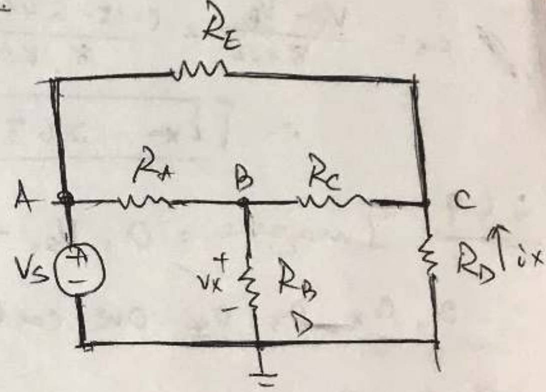
$\underline{C}$: $V_C = 10\,V$

B: $\dfrac{V_A - V_B}{10k\Omega} + \dfrac{0 - V_B}{5k\Omega} + \dfrac{V_C - V_B}{10k\Omega} = 0$

$\Rightarrow V_B\left(\frac{1}{10k\Omega} + \frac{1}{5k\Omega} + \frac{1}{10k\Omega}\right) = \frac{V_A}{10k\Omega} + \frac{V_C}{10k\Omega}$

or $\left[\frac{1}{10k\Omega} + \frac{1}{5k\Omega} + \frac{1}{10k\Omega}\right](V_B) = \left(\frac{V_A}{10k\Omega} + \frac{V_C}{10k\Omega}\right)$

3.17 $\underline{A}$: $V_A = V_S$



$\underline{B}$: $\dfrac{V_B - V_A}{R_A} + \dfrac{V_B}{R_B} + \dfrac{V_B - V_C}{R_C} = 0$

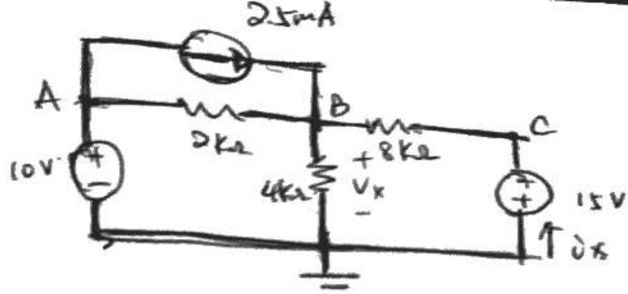
$\underline{C}$: $\dfrac{V_C - V_B}{R_C} + \dfrac{V_C}{R_D} + \dfrac{V_C - V_A}{R_E} = 0$

$$\begin{bmatrix} \frac{1}{R_A} + \frac{1}{R_B} + \frac{1}{R_C} & -\frac{1}{R_C} \\ -\frac{1}{R_C} & \frac{1}{R_C} + \frac{1}{R_D} + \frac{1}{R_E} \end{bmatrix} \begin{pmatrix} V_B \\ V_C \end{pmatrix} = \begin{pmatrix} V_A/R_A \\ V_A/R_E \end{pmatrix}$$

$$\boxed{\begin{aligned} V_x &= V_B \\ i_x &= \frac{0 - V_C}{R_D} \end{aligned}}$$

3.22

b)



$\underline{A}$: $V_A = 10\,V$

$\underline{C}$: $V_C = -15\,V$

$\underline{B}$: $\dfrac{V_B - V_A}{2\,k\Omega} + \dfrac{V_B - 0}{4\,k\Omega} + \dfrac{V_B - V_C}{8\,k\Omega} - 2.5\,mA = 0$

rewrite B :

$$\left(\frac{1}{2k\Omega} + \frac{1}{4k\Omega} + \frac{1}{8k\Omega}\right)(V_B) = \left(2.5\,mA + \frac{10\,V}{2k\Omega} + \frac{-15\,V}{8k\Omega}\right)$$

d) solve for V_B : $V_B = \frac{45}{7}\,V = 6.43\,V$

$V_B = \boxed{V_x = 6.43\,V}$

$$i_x = \frac{V_C - V_B}{8\,k\Omega} = \frac{(-15 - 6.43)\,V}{8\,k\Omega} \approx -2.68\,A$$

$\therefore \boxed{i_x = -2.68\,A}$

3.49 Imagine : $a_1 V_{s1} + a_2 V_{s2} + a_3 V_{s3} + a_4 V_{s4} = V_o$

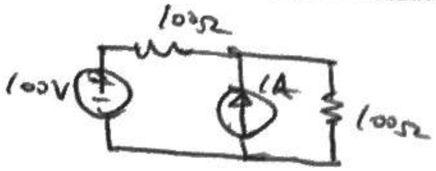
a_1, a_2, a_3, a_4 are coefficients for input voltage.

$$\begin{pmatrix} 2 & 4 & -4 & 1 \\ 1 & 2 & 2 & 1.5 \\ 1 & 4 & 2 & 2 \\ 0 & 5 & 3 & -1 \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{pmatrix} = \begin{pmatrix} 20 \\ -4 \\ -1 \\ 3 \end{pmatrix} \Rightarrow \begin{cases} a_1 = 1 \\ a_2 = 2 \\ a_3 = -3 \\ a_4 = -2 \end{cases}$$

$\therefore \boxed{V_o = V_{s1} + 2V_{s2} - 3V_{s3} - 2V_{s4}}$

3.50

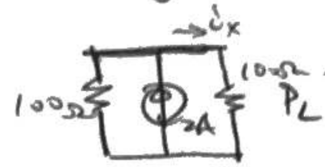
Source transformation:



$\rightarrow$

$$i_S = \frac{100\,V}{100\,\Omega} = 1\,A.$$

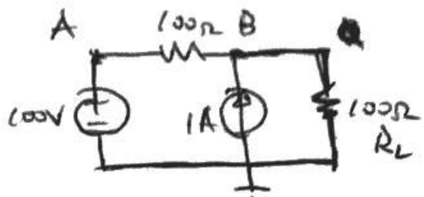
$\Downarrow$ 2 current source in parallel



$$i_x = \frac{1}{2} \cdot 2A = 1A.$$

$$\boxed{P_L = i_x^2 \cdot R_L = 100\,W.}$$

or Nodal Analysis:



A:

$V_A = 100\,V.$

B:

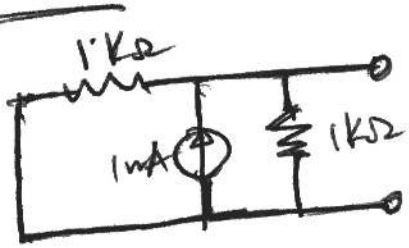
$$\frac{V_B - V_A}{100\,\Omega} + \frac{V_B}{100\,\Omega} - 1A = 0$$

$$\Rightarrow \left(\frac{1}{100\,\Omega} + \frac{1}{100\,\Omega}\right)(V_B) = \left(V_A/100\,\Omega + 1A\right)$$

$$V_B = 100\,V$$

$$P_L = \frac{V_B^2}{R_L} = \frac{(100\,V)^2}{100\,\Omega} = \boxed{100\,W}$$

3.52



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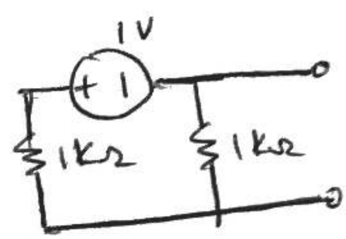


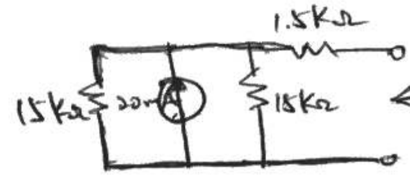
$R_T = 1k\Omega \parallel 1k\Omega$

$$\boxed{R_T = 0.5\,k\Omega}$$

$$V_T = \frac{1\,k\Omega}{1k\Omega + 1k\Omega} \cdot 1V = \boxed{0.5\,V.}$$

$$\boxed{i_N = \frac{V_T}{R_T} = 1\,mA}$$

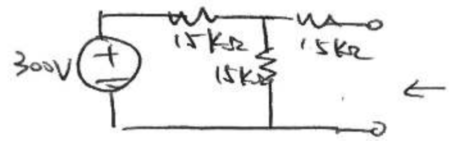
3.57



$R_N = R_T = 15\,k\Omega \,||\, 15\,k\Omega + 1.5\,k\Omega$

$\boxed{R_N = 9\ k\Omega}$

source transform:

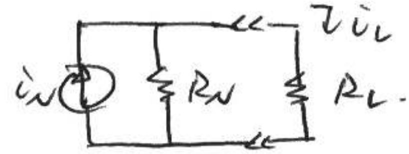


$$V_T = \frac{15\,k\Omega}{15\,k\Omega + 15\,k\Omega} \cdot 300\,V$$

$$V_T = 150\ V$$

$$i_N = \frac{V_T}{R_T} = 16.67\ mA \;\Rightarrow$$

Norton equivalent:



$$i_L = \frac{G_L}{G_N + G_L} i_N = \frac{i_N}{1 + \frac{R_L}{R_N}}$$

when R_L =

$\boxed{\begin{array}{ll} 4.7\ k\Omega, & i_L = 10.95\ mA \\ 15\ k\Omega, & i_L = 6.25\ mA \\ 68\ k\Omega. & i_L = 1.95\ mA \end{array}}$