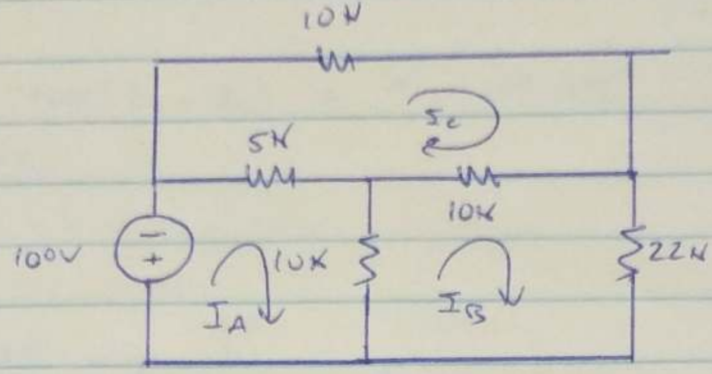
HW4 solutions

3.14



We start by formulating the mesh currents in the circuit

I_A

$$-100 - 5K(I_A - I_C) - 10K(I_A - I_B) = 0$$

I_B $$-10K(I_B - I_C) - 22K\, I_B - 10K(I_B - I_A) = 0$$

I_C $$-10K\, I_C - 10K(I_C - I_B) - 5K(I_C - I_A) = 0$$

With these three equations, we can formulate a Matrix

$$\begin{bmatrix} 15K & -10K & -5K \\ -10K & 42K & -10K \\ -5K & -10K & 25K \end{bmatrix} \begin{bmatrix} I_A \\ I_B \\ I_C \end{bmatrix} = \begin{bmatrix} -100 \\ 0 \\ 0 \end{bmatrix}$$

Using Matlab to solve, we get

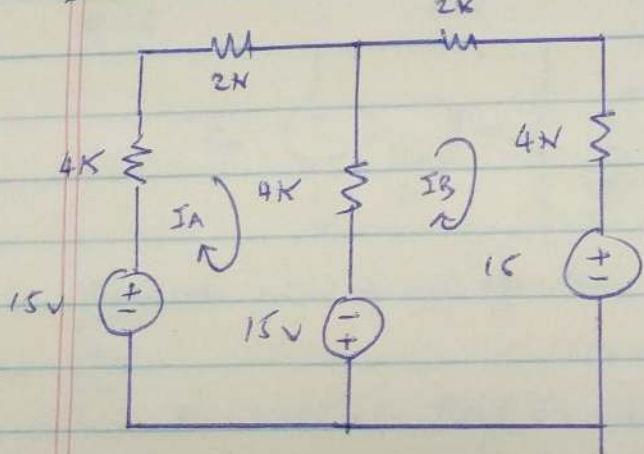
$$\begin{bmatrix} I_A \\ I_B \\ I_C \end{bmatrix} = \begin{bmatrix} -9.7938 \\ -3.0928 \\ -3.1957 \end{bmatrix} mA$$

Now, we can solve for V_x and i_x.

$V_x = -10K(I_B - I_c) = -1.0309$ volts.

$i_x = i_c$.

3.18



Formulate mesh current equations

I_A

$$15 - 4KI_A - 2KI_A - 4K(I_A - I_B) + 15 = 0$$

I_B

$$-2KI_B - 4KI_B - 15 - 15 - 4K(I_B - I_A) = 0$$

From which we can formulate our matrix equation

$$\begin{bmatrix} 10K & -4K \\ -4K & 10K \end{bmatrix} \begin{bmatrix} I_A \\ I_B \end{bmatrix} = \begin{bmatrix} 30 \\ -30 \end{bmatrix}$$

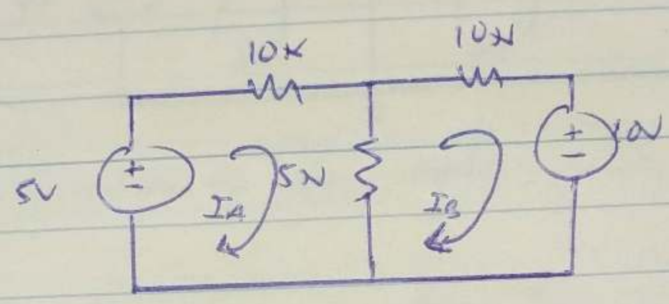
Using Matlab, we get

$$\begin{bmatrix} I_A \\ I_B \end{bmatrix} = \begin{bmatrix} 2.1429 \\ -2.1429 \end{bmatrix} mA$$

$V_x = 4K \times I_B$
$= -8.5714$ volts.

$i_x = I_B - I_A$
$= -4.2857 mA$

3.16



Formulate mesh current equations

I_A

$$5 - 10K I_A - 5K(I_A - I_B) = 0$$

I_B

$$-10 - 10K I_B - 5K(I_B - I_A) = 0$$

From here we can formulate our matrix equation

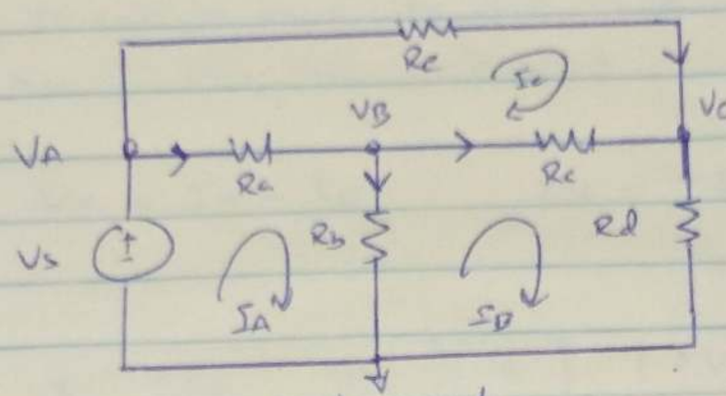
$$\begin{bmatrix} 15K & -5K \\ -5K & 15K \end{bmatrix} \begin{bmatrix} I_A \\ I_B \end{bmatrix} = \begin{bmatrix} 5 \\ -10 \end{bmatrix}$$

$$\begin{bmatrix} I_A \\ I_B \end{bmatrix} = \begin{bmatrix} 0.1250 \\ -0.1250 \end{bmatrix} mA$$

$V_x = 10K \cdot I_A = 1.25$ volts

$I_x = I_A - I_B = \underline{\underline{0.75 mA}}$

3.17



Let us formulate the ~~node voltage~~ Mesh current equations

IA

$$V_s - R_a(I_a - I_c) - R_b(I_a - I_b) = 0$$

IB

$$-R_b(I_b - I_a) - R_c(I_b - I_c) - R_d I_b = 0$$

Ic

$$-I_c R_e - R_c(I_c - I_b) - R_a(I_c - I_a) = 0$$

Now let's formulate the node-voltage equations.

$V_D = 0$

$V_A = V_s$

Equation at B

$$\frac{V_s - V_B}{R_a} = \frac{V_B}{R_b}$$

Equation at C

$$\frac{V_A - V_c}{R_e} + \frac{V_b - V_c}{R_c} = \frac{V_c}{R_d}$$

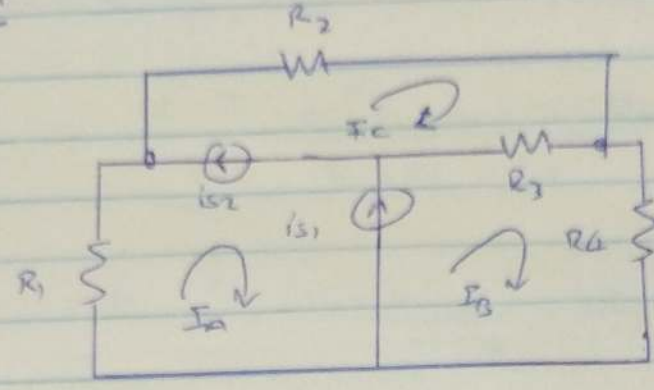
(c) It will be easier to solve node-voltage equations as there are only two unknowns

(d) $V_x = I_A - I_B$, $I_x = -I_B$

where:

$$\begin{bmatrix} R_a + R_b & -R_b & -R_a \\ -R_b & R_b + R_c + R_d & -R_c \\ -R_a & -R_c & R_a + R_c + R_e \end{bmatrix} \begin{bmatrix} I_A \\ I_B \\ I_C \end{bmatrix} = \begin{bmatrix} +v_s \\ 0 \\ 0 \end{bmatrix}$$

3.19



we have the following super-mesh current relationships:

$$I_B - I_A = i_{s1}$$

$$I_C - I_A = i_{s2}$$

and we can now formulate a super mesh by removing these current sources and formulating the mesh equation

$$R_1 I_A + R_2 I_C + R_4 I_B = 0$$

Now, let's write expressions for I_a, I_b & I_c.

I_A

$$R_1 I_A + R_2 (i_{s2} + I_A) + R_4 (i_{s1} + I_A) = 0$$

$$I_A (R_1 + R_2 + R_4) = -R_4 i_{s1} - R_2 i_{s2}$$

$$I_a = \frac{-i_{s1} R_4 - R_2 i_{s2}}{R_1 + R_2 + R_4}$$

In a similar way, we have

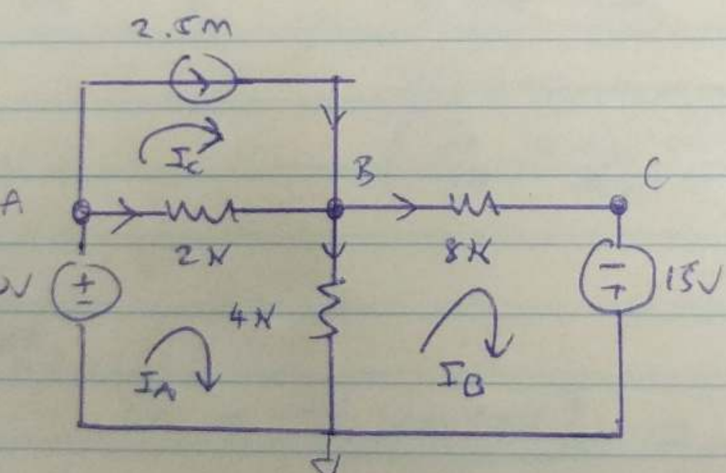
$$i_B = \frac{i_{s1}(R_1+R_2) - i_{s2}R_2}{R_1+R_2+R_4}$$

$$i_C = \frac{-i_{s1}R_4 + i_{s2}(R_1+R_4)}{R_1+R_2+R_4}$$

and we have

$$v_x = -i_B R_4$$
$$= \left(\frac{i_{s1}(R_1+R_2) - i_{s2}R_2}{R_1+R_2+R_4}\right) R_4$$

3.22



$$\left.\begin{aligned} I_C &= 2.5mA \\ 10 - 2K(I_A - I_C) - 4K(I_A - I_B) &= 0 \\ -4K(I_B - I_A) - 8K\,I_B + 15 &= 0 \end{aligned}\right\}$$ Mesh current equations

(c) If we had used the node voltage equations, we would only have a single unknown, vs 2 unknowns for mesh currents

If we formulate node-voltage equations we have,

$V_A = 10V$ $\quad V_D = 0$

$V_C = -15V$,

and

$$\frac{V_A - V_B}{2K} + 2.5m = \frac{V_B}{4K} + \frac{V_B - V_C}{8K}$$

so.

$$V_B\left(\frac{1}{4K} + \frac{1}{8K} + \frac{1}{2K}\right) = 2.5m + \frac{10}{2K} - \frac{15}{8K}$$

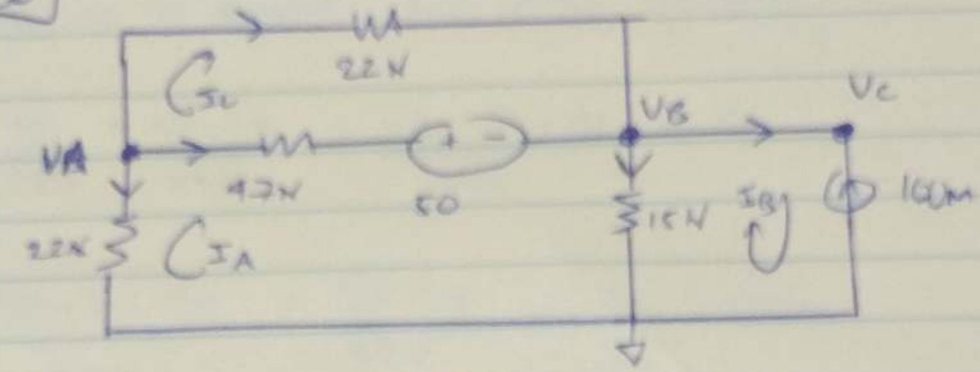
$$V_B = \frac{2.5m + \frac{10}{2K} - \frac{15}{8K}}{\left(\frac{1}{4K} + \frac{1}{8K} + \frac{1}{2K}\right)} = \underline{\underline{6.429V}}$$

so that

$$i_x = \frac{-(V_B + 15)}{8K} = \underline{-2.679\,mA}$$

$$V_x = V_D =$$

3.26



(a) $-22K\,I_A - 47K(I_A - I_C) - 50 - 15K(I_A - I_B) = 0$

$I_B = -100m\,A$

$-22K\,I_C + 50 - 47K(I_C - I_A) = 0$

$$\begin{bmatrix} 22K + 47K + 15K & -47K \\ -47K & 22K + 47K \end{bmatrix}\begin{bmatrix} I_A \\ I_C \end{bmatrix} = \begin{bmatrix} -50 + 15K(100m) \\ 50 \end{bmatrix}$$

$$\begin{bmatrix} I_A \\ I_C \end{bmatrix} = \begin{bmatrix} -29.2 \\ -19.1 \end{bmatrix} mA$$

Now we can formulate node-voltage equations

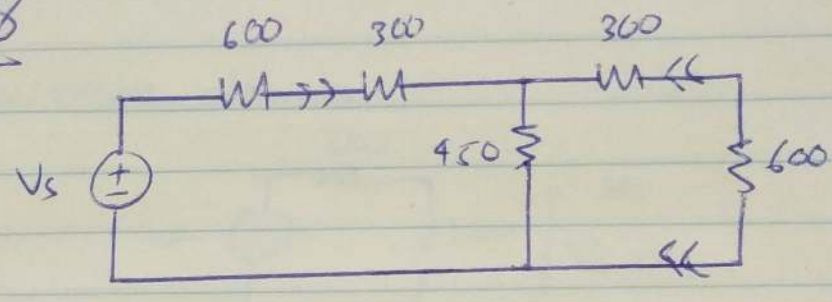
$$\frac{VA}{22K} + \frac{VA - VB}{22K} + \frac{VA - 50 - VB}{47K} = 0$$

$$\frac{VA - 50 - VB}{47K} + \frac{VA - VB}{22K} = \frac{VB}{15K} - 100m$$

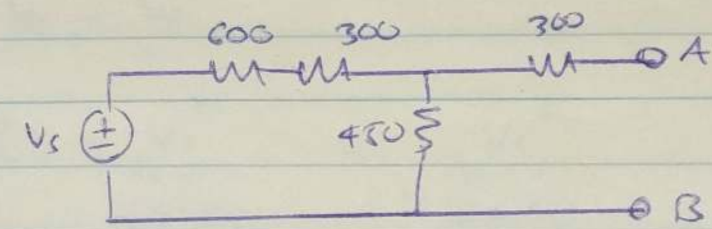
$$\begin{bmatrix} \frac{1}{22K} + \frac{1}{47K} + \frac{1}{22K} & \frac{-1}{47K} - \frac{1}{22K} \\ \frac{-1}{22K} - \frac{1}{47K} & \frac{1}{22K} + \frac{1}{15K} + \frac{1}{47K} \end{bmatrix} \begin{bmatrix} VA \\ VB \end{bmatrix} = \begin{bmatrix} 50/47K \\ -50/47K + 100m \end{bmatrix}$$

$$\begin{bmatrix} VA \\ VB \end{bmatrix} = \begin{bmatrix} 641.5 \\ 1062.6 \end{bmatrix} \text{ volts}$$

3.98



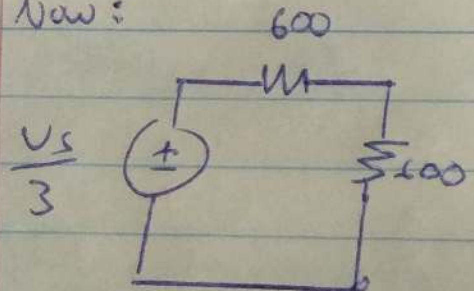
Find Thevenin circuit without load



$$R_{th} = 300 + 900 \| 450 = 600$$

$$V_{th} = \frac{V_s}{1350} \cdot 450 = \frac{V_s}{3}$$

Now:

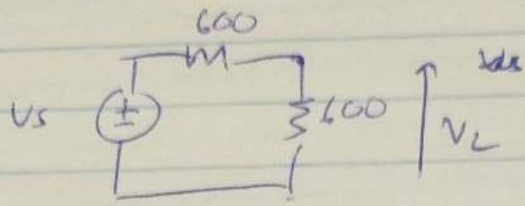


after we add load back.

$$V_L = \frac{V_s}{6}, \qquad P_L = \frac{V^2}{R} = \frac{V_s^2}{36 \times 600} = \frac{V_s^2}{21600}$$

(12)

Now, remove attenuator



so $V_L = \frac{V_s}{2}$

$$P_L = \frac{V_s^2}{4 \times 600} = \frac{V_s^2}{2400}$$

This is the power gain if we remove attenuator from circuit.

$$\frac{P_{L,\text{noatten}}}{P_{L,\text{atten}}} = \frac{V_s^2}{2400} \times \frac{21600}{V_s^2} = \underline{\underline{9}}$$

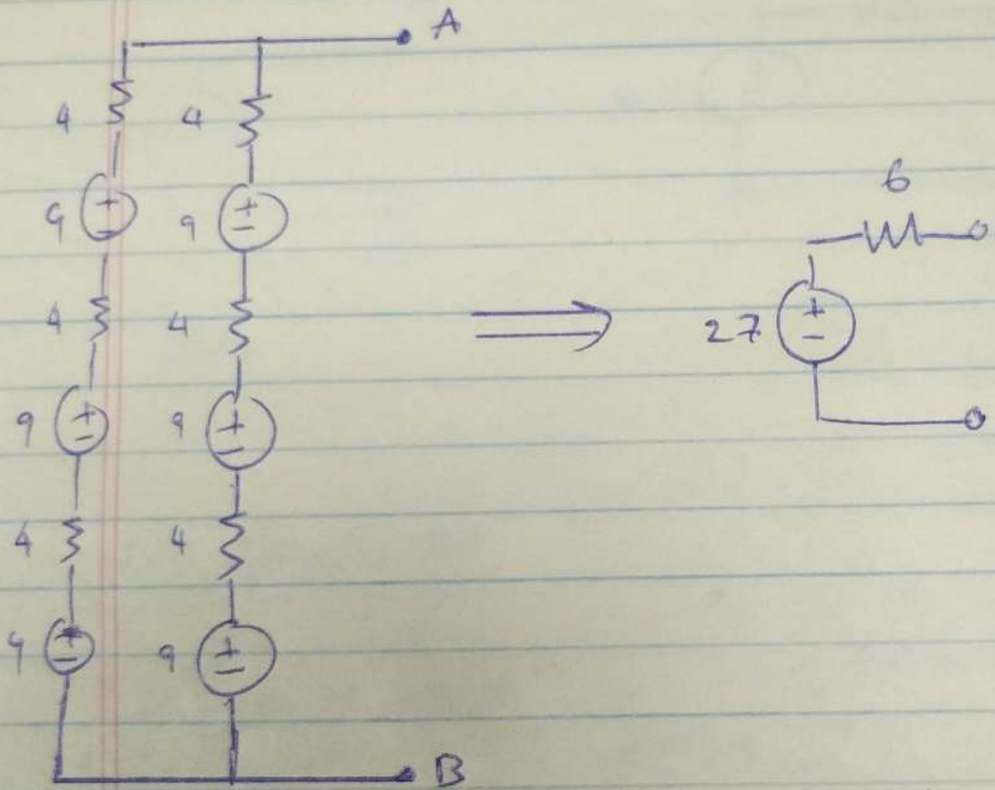
$$20\log_{10}(9) = \underline{\underline{19.0849 \text{ dB}}}$$

(or can compute $\frac{P_{L,\text{atten}}}{P_{L,\text{no atten}}}$)

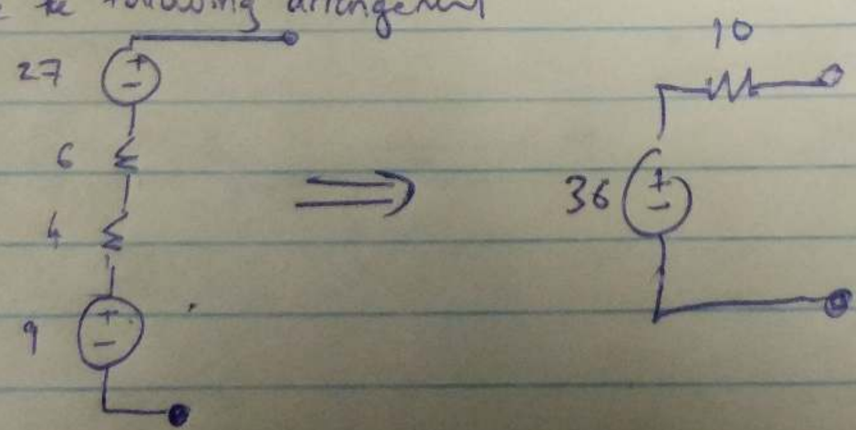
3(0)

Two possible solutions are shown here. For full points, students need two solutions with the corresponding weight of each one.

Alternative # 1



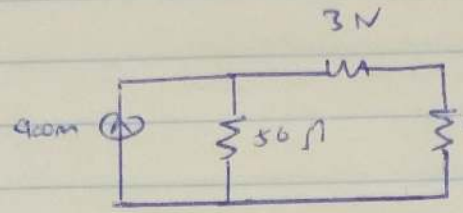
So we have the following arrangement



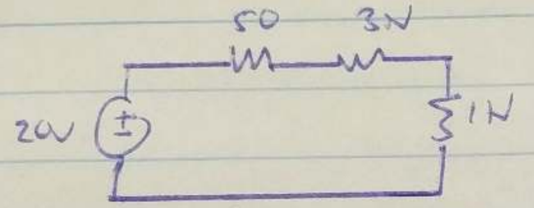
which has a weight of $(30 \times 6) + 30 = \underline{\underline{210g}}$

3-103

Team A

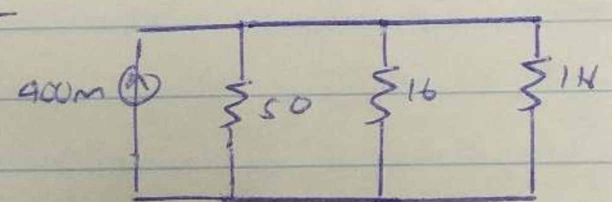


To find P_L, lets do a source transformation



$P_L = 24.3865$ mW

Team B



$P_L = 22.9481$ mW

so for 10% tolerance, both circuits are acceptable but only team A's solution is feasible for 5% tolerance

- Number of parts is same for both designs.
- If we consider E12 resistance values, the 3 kΩ resistor can either be 2.7 kΩ (which gives $P_L = 28.4$ mW or 3.3 kΩ (which gives $P_L = 21.14$ mW)
- 16 Ω resistor can be 15 Ω ($P_L = 20.8$ mW or 18 Ω ($P_L = 27.30$ mW)