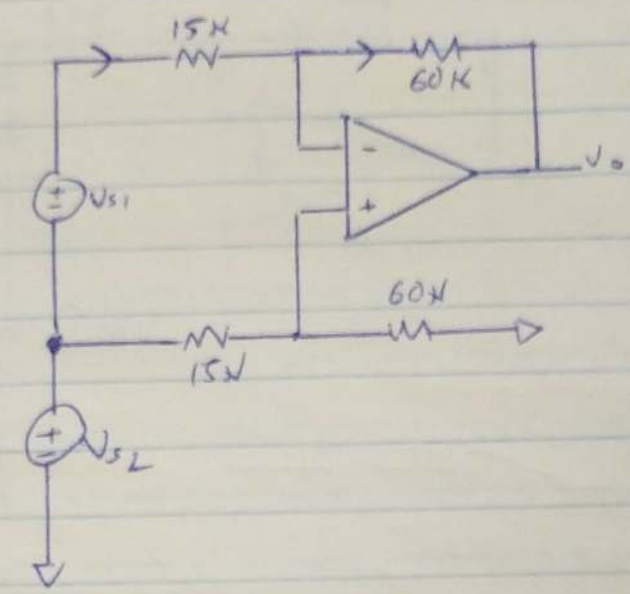
HW6 Solution

4.44

Switch open



First, let us compute the voltage at the positive terminal

$$V_+ = \frac{V_{s2}}{75K} \cdot 60K = \frac{4V_{s2}}{5}$$

Now, we know that $V_+ = V_-$, so after assigning current directions we have that:

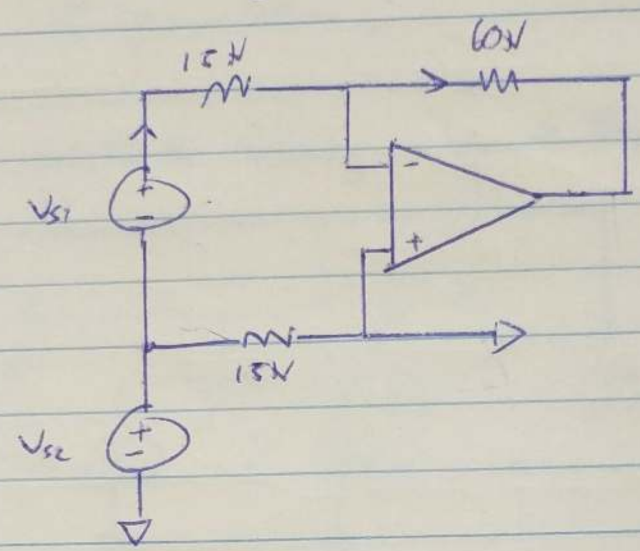
$$\frac{V_{s1} + V_{s2} - \frac{4}{5}V_{s2}}{15K} = \frac{\frac{4}{5}V_{s2} - V_o}{60K}$$

$$\frac{V_{s1}}{15K} + \frac{V_{s2}}{15K} - \frac{4V_{s2}}{75K} - \frac{4V_{s2}}{300K} = \frac{-V_o}{60K}$$

$$\frac{V_{s1}}{15K} = \frac{-V_o}{60K}$$

$$\Rightarrow V_o = -\frac{60K}{15K} V_{s1} = \underline{\underline{-4V_{s1}}}$$

Now, let us repeat the analysis with the switch closed



Now $V_+ = V_- = 0V$, and

$$\frac{V_{S1} + V_{S2}}{15K} = \frac{-V_O}{60K}$$

$$\Rightarrow \quad V_O = \underline{-4(V_{S1} + V_{S2})}$$

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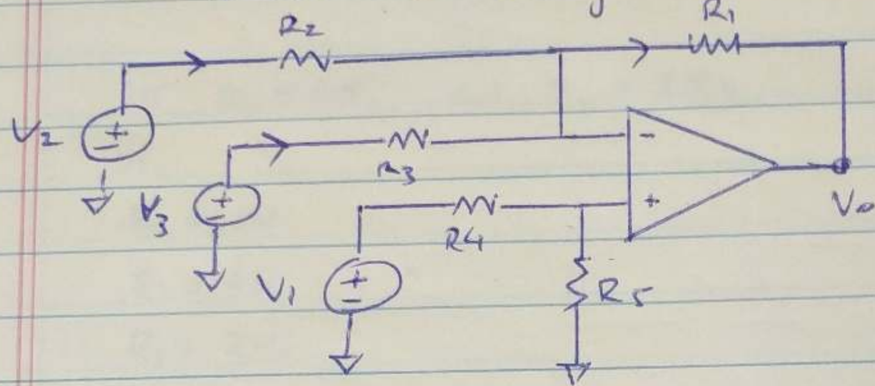
One possible solution using a single op-amp is shown below.

We want to realize the following: $V_o = 5V_1 - 4V_2 - 2V_3$



Let's use superposition to solve for the output of the circuit.

$V_1 = V_3 = 0,$

$$V_o = -\frac{R_1}{R_2}V_2$$

$V_1 = V_2 = 0$

$$V_o = -\frac{R_1}{R_3}V_3$$

$V_2 = V_3 = 0$

$$V_+ = \frac{V_1}{R_4+R_5} R_5 \text{, and} \quad \frac{-V_+}{R_2 \| R_3} = \frac{V_+ - V_o}{R_1}$$

$$\frac{V_o}{R_1} = \frac{V_+}{R_1} + \frac{V_+}{R_2 \| R_3}$$

$$V_o = V_+ R_1 \cdot \frac{(R_1 + R_2 \| R_3)}{R_1(R_2 \| R_3)} = \frac{V_1 R_5}{R_4 + R_5} \cdot \frac{(R_1 + R_2 \| R_3)}{R_2 \| R_3}$$

Summing gives

$$V_o = -\frac{R_1}{R_2}V_2 - \frac{R_1}{R_3}V_3 + \frac{V_1 R_5}{R_4+R_5}\left(\frac{R_1 + R_2 \| R_3}{R_2 \| R_3}\right)$$

let $R_1 = 4R_2$, and $R_1 = 2R_3$.

s.t.

$R_2 = 1K$

$R_1 = 4K$

$R_3 = 2K$,

now : $V_o = -4V_2 - 2V_3 + \frac{V_1 R_5}{R_4+R_5} \cdot 7$

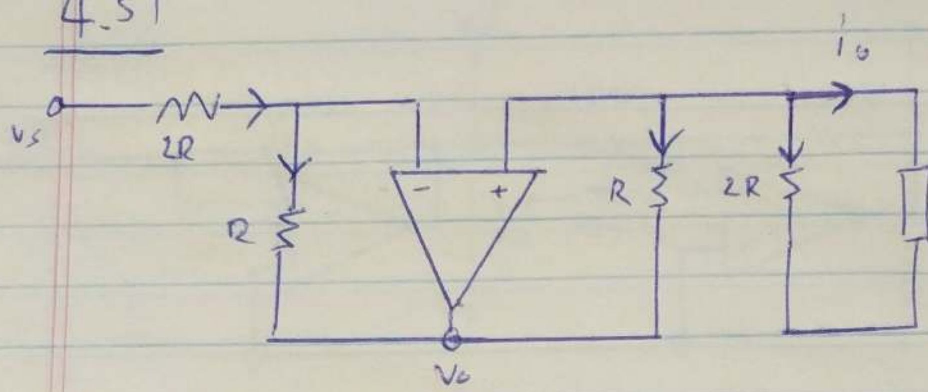
so: $\frac{7R_5}{R_4+R_5} = 5$, let $\underline{R_5 = 1K}$

s.t. $7000 = 5R_4 + 5000$

$$\underline{\underline{R_4 = 400}}$$

which achieves our objective

4.51



$$i_o = \frac{-V_+}{2R} - \frac{(V_+ - V_o)}{R} \Rightarrow i_o = \frac{-V_+}{2R} - \frac{V_+}{R} + \frac{V_o}{R}$$

$$\boxed{i_o = -V_+\left(\frac{3}{2R}\right) + \frac{V_o}{R}}$$

Now, lets do a nodal analysis at the negative terminal

$$\frac{V_s - V_+}{2R} = \frac{V_+ - V_o}{R}$$

$$\Rightarrow \frac{V_s}{2R} + \frac{V_o}{R} = \frac{V_+}{R} + \frac{V_+}{2R}$$

$$\Rightarrow \frac{V_s}{2R} + \frac{V_o}{R} = \frac{3}{2R} V_+$$

$$V_+ = \left(\frac{V_s}{2R} + \frac{V_o}{R}\right)\frac{2R}{3}$$

$$V_+ = \frac{RV_s + 2RV_o}{3R}$$

Now combine the equations

$$i_o = -\frac{(RV_s + 2RV_o)}{3R} \cdot \frac{3}{2R} + \frac{V_o}{R}$$

$$= -\frac{RV_s}{2R^2} - \frac{2RV_o \cdot 3}{3 \cdot 2 \cdot R^2} + \frac{V_o}{R}$$

$$= \underline{\underline{-\frac{V_s}{2R}}}$$

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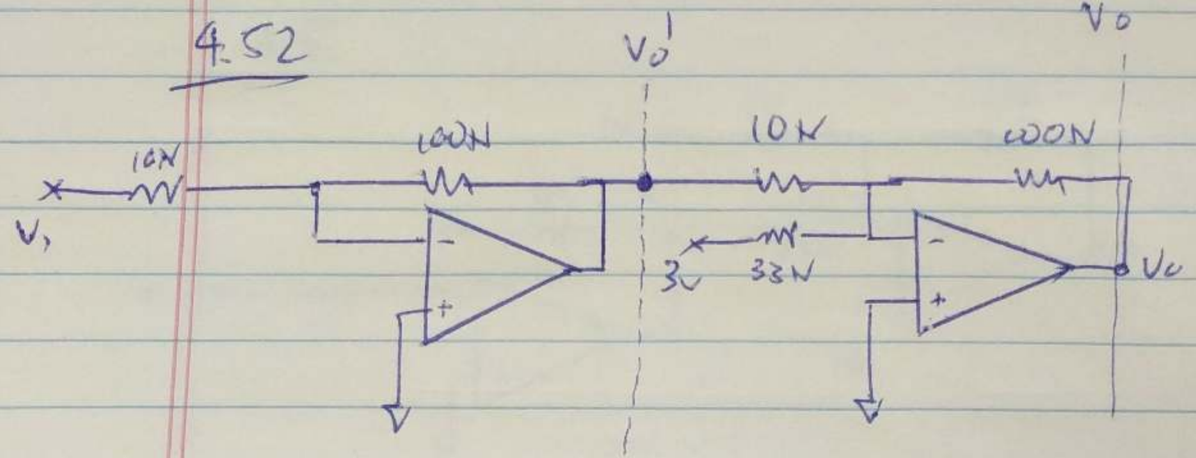
Inverting opamp | Inverting Summer

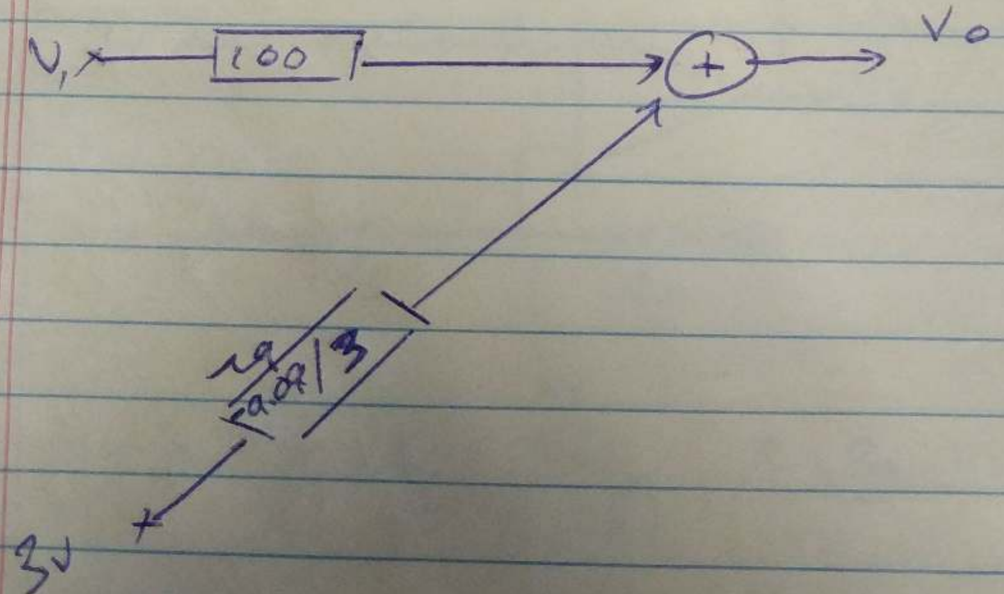
4.52



$V_o' = -10V_1$

$V_o = -10V_o' - \frac{100K}{33K} \cdot 3V$

$= 100V_1 - 9.09$



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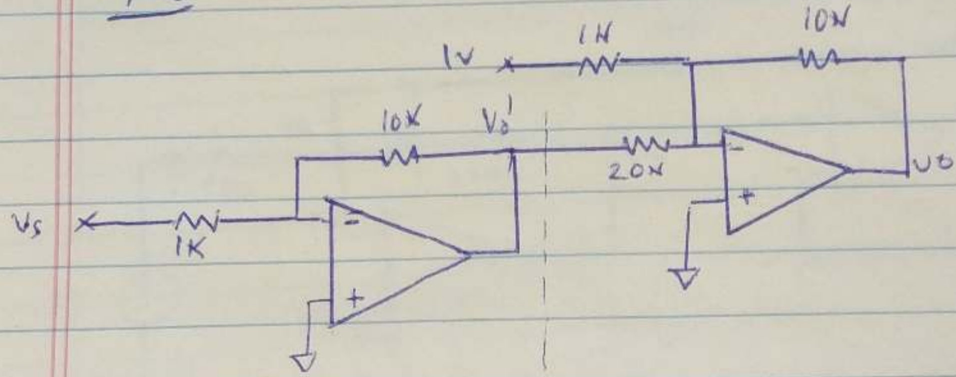
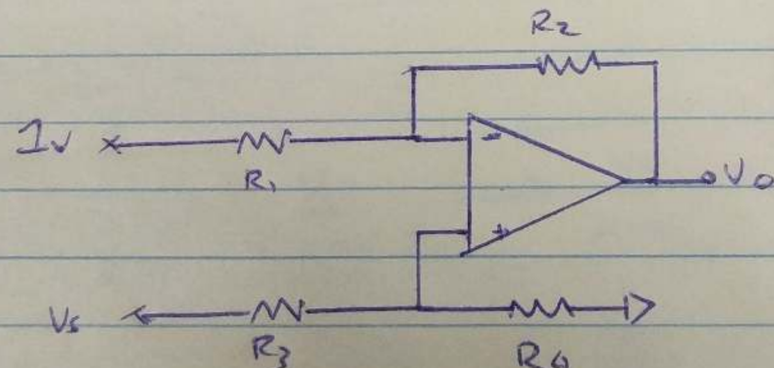


$$V_o' = -10V_s$$

$$V_o = -\frac{1}{2}V_o' - \frac{10}{1}\cdot 1$$

$$\underline{\underline{V_o = 5V_s - 10}}$$

Now, we can redesign using a single opamp.



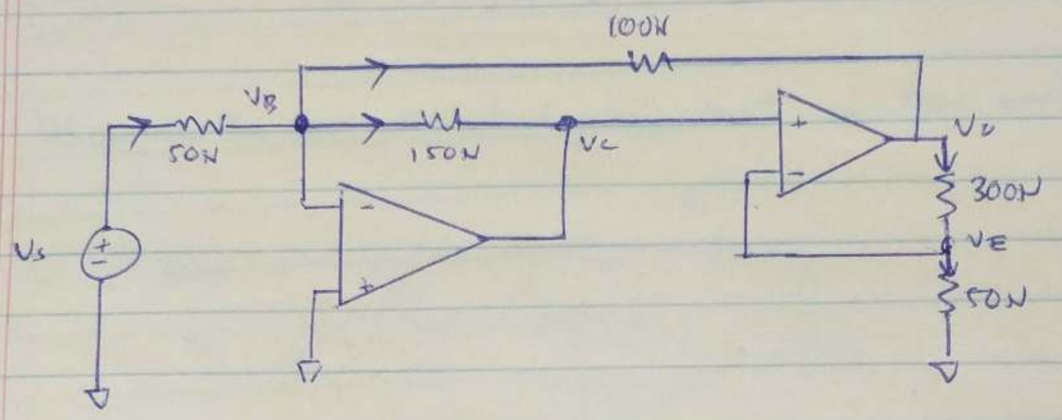
$$V_o = \frac{-R_2}{R_1}\cdot 1 + \underbrace{\frac{R_4}{R_3+R_4}}_{5'K}\cdot \overbrace{\frac{R_1+R_2}{R_1}}^{11V}\cdot V_s$$

(where we can follow the same steps as before to get this)

set $R_2 = 10K$ $R_4 = 5K$

$R_1 = 1K$ $R_3 = 6K$ to achieve desired function

4-60



Nodal analysis at V_B

$$\frac{V_s}{50k} = -\frac{V_o}{100k} - \frac{V_c}{150k} \Rightarrow V_o = -100k\left(\frac{V_c}{150k} + \frac{V_s}{50k}\right)$$

Nodal analysis at V_E.

$$\frac{V_o - V_c}{300k} = \frac{V_E}{50k} \Rightarrow \frac{V_o}{300k} = \frac{V_c}{300k} + \frac{V_c}{50k}$$

$$\frac{V_o}{300k} = \frac{7V_c}{300k} \Rightarrow V_c = \frac{V_o}{7}$$

Sub into first equation

$$V_o = -100k\left(\frac{V_o}{7 \cdot 150k} + \frac{V_s}{50k}\right)$$

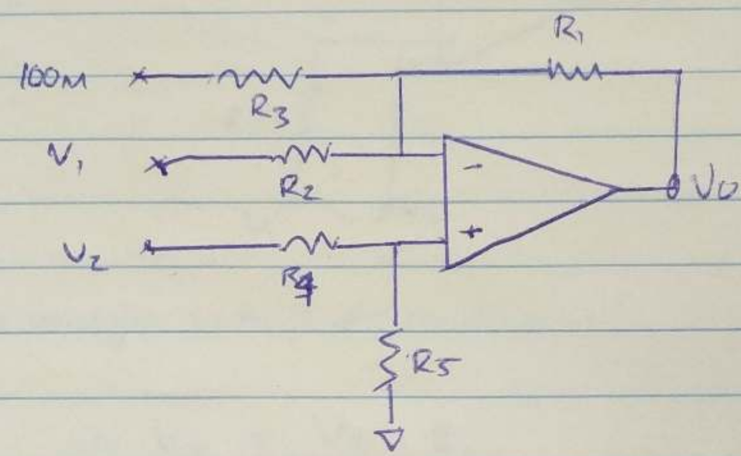
$$V_o\left(1 + \frac{100k}{7 \cdot 150k}\right) = -2V_s$$

$$V_o\left(\frac{23}{21}\right) = -2V_s \Rightarrow \boxed{V_o = -\frac{42}{23}V_s}$$

4.66

$$V_o = -3V_1 + 2V_2 - 300mV$$

We can use the same circuit topology that we have used before



$$V_o = \frac{-R_1}{R_3} \cdot 100m - \frac{V_1 R_1}{R_2} + \frac{V_2 \cdot R_5}{R_4 + R_5} \cdot \frac{R_1 + R_2 \| R_3}{R_2 \| R_3}$$

let. $R_1 = 6000$, $R_2 = 3000$, $R_3 = 2000$

$$\frac{R_5}{R_4 + R_5} \cdot 6 = 2 \Rightarrow \frac{R_5}{R_4 + R_5} = \frac{2}{6} = \frac{1}{3}$$

so, $\boxed{\begin{array}{l} R_5 = 1k \\ R_4 = 2k \end{array}}$

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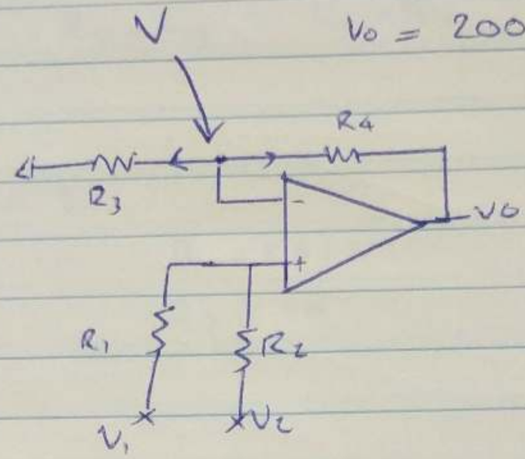
$V_o = 200(V_1+V_2)$



We can analyze with super position.

$V_1 = 0 \Rightarrow V_+ = \frac{V_2}{R_1+R_2} R_1.$

Now, let's apply nodal analysis at negative terminal ($V_- = V$

$$\frac{V}{R_3} + \frac{V - V_o}{R_4} = 0 \Rightarrow \frac{V_o}{R_4} = V\left(\frac{1}{R_3} + \frac{1}{R_4}\right)$$

$$V_o = VR_4\left(\frac{R_3+R_4}{R_3R_4}\right)$$

$$= V\frac{(R_3+R_4)}{R_3}$$

$$V_o = \frac{V_2 R_1}{R_1+R_2} \frac{(R_3+R_4)}{R_3}$$

Applying the same procedure with $V_2 = 0$, gives

$$V_o = \frac{V_2 R_1}{R_1+R_2} \cdot \frac{R_3+R_4}{R_3} + \frac{V_1 R_2}{R_1+R_2} \cdot \frac{(R_3+R_4)}{R_3}$$

Now let, $R_3 + R_4 = R_1 + R_2$, $\frac{R_1}{R_3} = 200$, $\frac{R_2}{R_3} = 200$

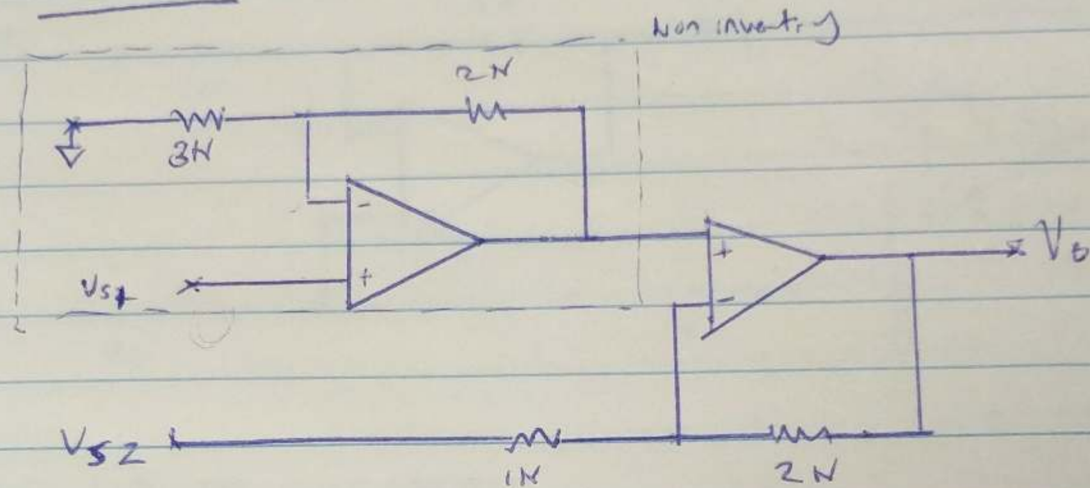
For instance :

$$R_1 = 200K, \quad R_2 = 200K$$

$$R_3 = 1K \quad R_4 = 399K$$

4.75

Circuit #1



Let's use superposition.

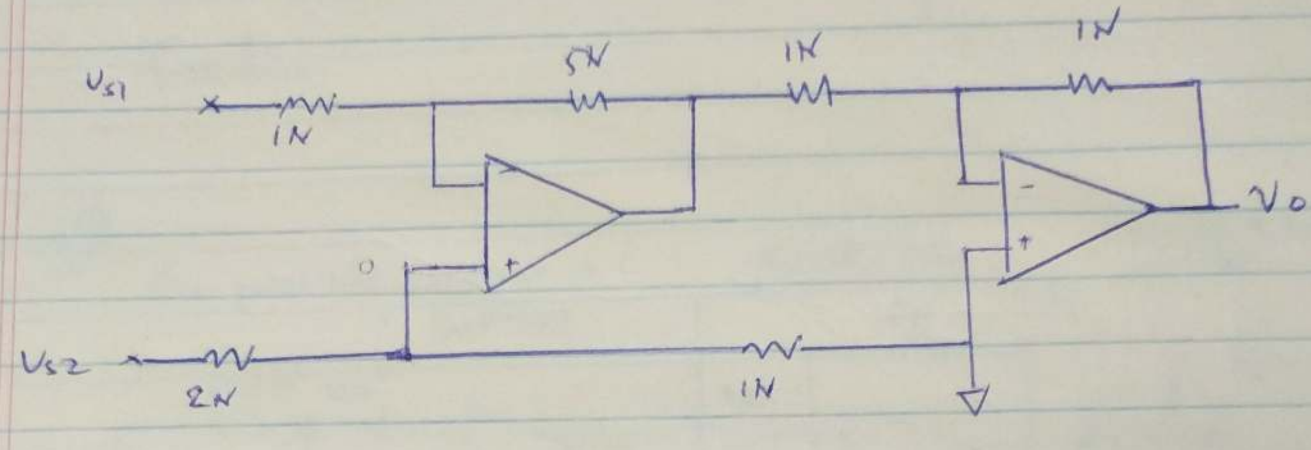
$$V_{s2} = 0 \Rightarrow V_o = \left(1 + \frac{2}{3}\right)\left(1 + \frac{2}{1}\right) V_{s1}$$

$$= 5V_{s1}$$

$$V_{s1} = 0 \Rightarrow -2V_{s2}$$

$$\Rightarrow V_o = 5V_{s1} - 2V_{s2}$$

Circuit 2/



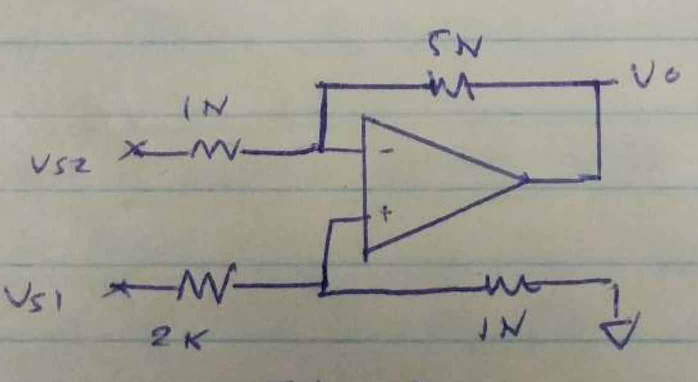
Again, let's use superposition

$V_{s2} = 0 \Rightarrow V_o = (-5)(-1) V_{s1}$

$V_{s1} = 0 \Rightarrow \left(\frac{V_{s2}}{3}\right)(6)(-1)$

so, $V_o = \underline{5V_{s1} - 2V_{s2}}$

Circuit 3

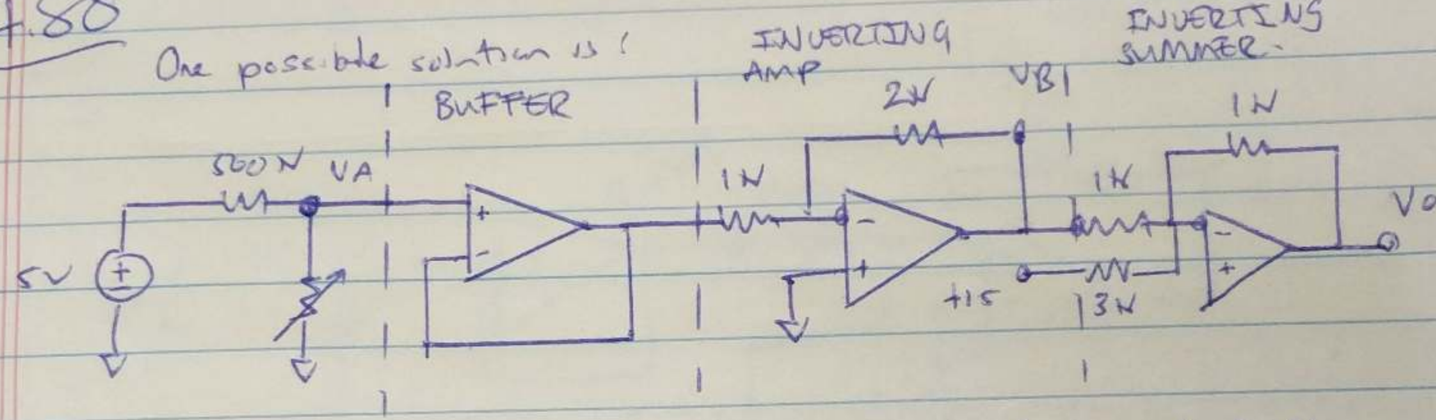


$V_{s1} = 0 \Rightarrow V_o = -5V_{s2}$ } so we see already that this circuit will ~~not~~ work

Circuit #1 is best, has least number of resistors and is able to perform required function.

4.88

One possible solution is:



- Here [photoresistor symbol] represents the photoresistor. When dark, the output VA is 2.5V, when bright it's 9.9998×10^{-5} volts.

- Voltage buffer so that the amplifier gains are not affected by the photoresistor voltage divider circuit

- Next is inverting amplifier circuit with a gain of 2. If dark, the $V_B = -5$, if bright, then 1.99996×10^{-4}.

- Next is inverting summer which adds $-5V$ to output.

So, if dark then

- $V_O = -(-5) - 5 = 0V$

If bright

$V_O = -1.9996 \times 10^{-4} - 5 \approx \underline{-5 \text{ Volts}}$