HW7 Solution

9.2

$f(t) = 20\times10^3 \cdot \sin(60\pi t)\cdot u(t)$

from table, $\mathcal{L}\{\sin(60\pi t)\} = \frac{60\pi}{s^2+(60\pi)^2}$ thus

$$\boxed{\mathcal{L}\{f(t)\} = 20\times10^3 \cdot \frac{60\pi}{s^2+(60\pi)^2}}$$

pole: $s_1 = 60\pi j$, $s_2 = -60\pi j$

9.6 $f(t) = A\left[(B+\alpha t)e^{-\alpha t}\right]u(t)$

$= ABe^{-\alpha t}u(t) + A\alpha t\, e^{-\alpha t}u(t)$

$$\mathcal{L}\{f(t)\} = \frac{AB}{s+\alpha} + \frac{A\alpha}{(s+\alpha)^2} = \cancel{A\frac{B+\alpha B}{(s+\alpha)^2}} = \boxed{A\frac{Bs+B\alpha+\alpha}{(s+\alpha)^2}}$$

poles: $s=-\alpha,-\alpha$, zeros: $\cancel{s=\frac{B}{\alpha}}$ $s=-\alpha-\frac{\alpha}{B}$

9.10 $f(t) = \left[2-5t-2e^{-25t}\right]u(t)$

$$\mathcal{L}\{f(t)\} = \frac{2}{s} - \frac{5}{s^2} - \frac{2}{s+25} = \boxed{\frac{45s-125}{s^2(s+25)}}$$

poles: $s=0,0,-25$, zeros: $s=\frac{125}{45}$

9.20

a) $$\boxed{f(t) = e^{-\frac{t-\frac{T_c}{2}}{T_c/2}}\cdot u\left(t-\frac{T_c}{2}\right)}$$

b) let $\tilde{f}(t) = e^{-\frac{t}{T_c/2}}$ ~~$u(t)$~~, then, $f(t) = \tilde{f}\left(t-\frac{T_c}{2}\right)$

$$\mathcal{L}\{f(t)\} = \mathcal{L}\Big\{\underbrace{\tilde{f}\left(t-\tfrac{T_c}{2}\right)}_{\text{delay}}\cdot \underbrace{u\left(t-\tfrac{T_c}{2}\right)}_{\text{delay}}\Big\} = e^{-\frac{T_c}{2}s}\cdot\tilde{F}(s)$$

$$\tilde{F}(s) = \mathcal{L}\{\tilde{f}(t)\} = \mathcal{L}\left\{e^{-\frac{t}{T_c/2}}\right\} = \frac{1}{s+\left(\frac{T_c}{2}\right)^{-1}} \Rightarrow \boxed{\mathcal{L}\{f(t)\} = e^{-\frac{T_c}{2}s}\frac{1}{s+\left(\frac{T_c}{2}\right)^{-1}}}$$

c) $\mathcal{L}\{f(t)\} = \int_0^{\infty} e^{-\frac{t-T_c/2}{T_c/2}} u(t-\frac{T_c}{2}) \cdot e^{-st}\, dt$

$u(t-\frac{T_c}{2}) = \begin{cases} 1, & t > \frac{T_c}{2} \\ 0, & t < \frac{T_c}{2} \end{cases}$ then,

$\mathcal{L}\{f(t)\} = \int_{T_c/2}^{\infty} e^{-\frac{t-\frac{T_c}{2}}{T_c/2}} e^{-st}\, dt = e \int_{\frac{T_c}{2}}^{\infty} e^{-(\frac{2}{T_c}+s)t}\, dt$

$= -\frac{e}{s+(\frac{T_c}{2})^{-1}} \left(e^{-(\frac{1}{T_c/2}+s)t} \Big|_{\frac{T_c}{2}}^{\infty} \right) = -\frac{e \cdot (-e^{-(\frac{1}{T_c/2}+s)\cdot\frac{T_c}{2}})}{s+(\frac{T_c}{2})^{-1}}$

$= \boxed{e^{-\frac{T_c}{2}s} \frac{1}{s+(\frac{T_c}{2})^{-1}}}$ $\therefore$ verified

9.22

a) $F_1(s) = \frac{s}{(s+10)(s+40)} = \frac{A_1}{s+10} + \frac{B_1}{s+40}$.

$A_1 = \lim_{s \to -10} (s+10) \cdot \frac{s}{(s+10)(s+40)} = -\frac{1}{3}$ (residue formula)

$B_1 = \lim_{s \to -40} (s+40) \frac{s}{(s+10)(s+40)} = \frac{4}{3}$ $\therefore F_1(s) = \frac{-1/3}{s+10} + \frac{4/3}{s+40}$.

$\mathcal{L}^{-1}\{F_1(s)\} = \boxed{\left(-\frac{1}{3}e^{-10t} + \frac{4}{3} \cdot e^{-40t}\right) u(t)}$

b) $F_2(s) = \frac{(s+1)(s+10)}{s(s+100)(s+1000)} = \frac{A_2}{s} + \frac{B_2}{s+100} + \frac{C_2}{s+1000}$

$A_2 = \lim_{s \to 0} s \cdot F_2(s) = \frac{1}{10000}$

$B_2 = \lim_{s \to -100} (s+100) F_2(s) = \frac{(-99)(-90)}{(-100)(900)} = -\frac{99}{1000}$

$C_2 = \lim_{s \to -1000} (s+1000) F_2(s) = \cancel{(-999)(-990)} = \frac{(-999)(-990)}{(-1000)(-900)} = \frac{10989}{10000}$

$$\therefore F_2(s) = \frac{1/10000}{s} + \frac{-99/1000}{s+100} + \frac{10989/10000}{s+1000}$$

$$\mathcal{L}^{-1}\{F_2(s)\} = \boxed{\left[\frac{1}{10000} + \left(-\frac{99}{1000}\right)\cdot e^{-100t} + \frac{10989}{10000}\cdot e^{-1000t}\right] u(t)}$$

9.25

a) $F_1(s) = \dfrac{9000}{(s+10)^2+(30)^2} = \boxed{300 e^{-10t}\cdot \sin(30t)\, u(t)}$ from table.

b) $F_2(s) = \dfrac{5(s+10)}{(s+10)^2+(30)^2} = \boxed{5\cdot e^{-10t}\cos(30t)\, u(t)}$

9.27

a) $F_1(s) = \dfrac{\alpha^2}{s^2(s+\alpha)} = \dfrac{A_1}{s} + \dfrac{B_1}{s^2} + \dfrac{C_1}{(s+\alpha)}$

$$A_1 = \frac{1}{(2-1)!}\lim_{s\to 0}\frac{d^{2-1}}{ds^{2-1}}\left(s^2\cdot F_1(s)\right) = -1$$

Note:
residue formula w/ multiplicity > 1 :
$$Res = \frac{1}{(n-1)!}\lim_{s\to c}\frac{d^{n-1}}{ds^{n-1}}\left((s-c)^n F(s)\right)$$

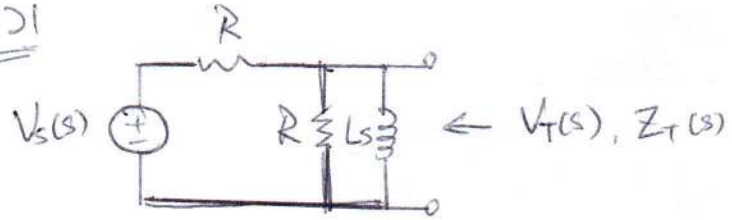
$$B_1 = \lim_{s\to 0} s^2 F_1(s) = \alpha$$

$$C_1 = \lim_{s\to -\alpha}(s+\alpha)F_1(s) = 1$$

$$\therefore F_1(s) = -\frac{1}{s} + \frac{\alpha}{s^2} + \frac{1}{s+\alpha}$$

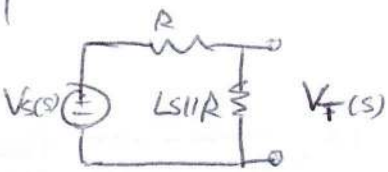
$$\mathcal{L}^{-1}\{F_1(s)\} = \boxed{\left(-1 + \alpha t + e^{-\alpha t}\right) u(t)}$$

10.21



$$Z_T(s) = R \parallel R \parallel Ls = \frac{1}{\frac{1}{R}+\frac{1}{R}+\frac{1}{Ls}} = \frac{Ls}{\frac{2}{R}Ls+1}$$

$$V_T(s) = \frac{R\parallel Ls}{R + R\parallel Ls} V_s(s)$$, voltage divider



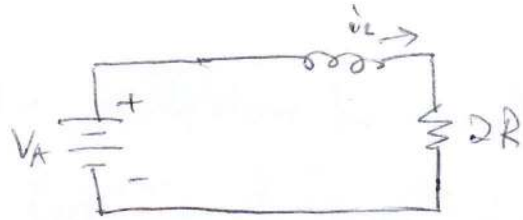
$$= \frac{\frac{1}{\frac{1}{R}+\frac{1}{Ls}}}{R + \frac{1}{\frac{1}{R}+\frac{1}{Ls}}} = \frac{1}{1+\frac{R}{Ls}+1} = \frac{Ls}{2Ls+R}$$

pole: $s = -\frac{R}{2L} = -24$ krad/s $\Rightarrow$ any combination of R & L s.t. $\frac{R}{L} = 48000$.

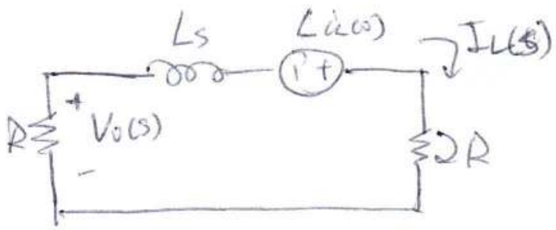
10.25

Initial condition

$i_L(0) = \frac{V_A}{2R}$



Circuits in S-domain

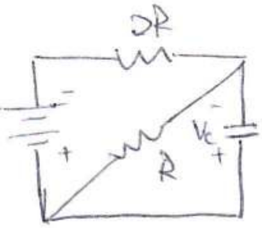


KVL: $-(3R+Ls)I_L(s) + Li_L(0) = 0$

$$I_L(s) = \frac{Li_L(0)}{3R+Ls} = \boxed{\frac{V_A L/2R}{Ls+3R}} \Rightarrow i_L(t) = \boxed{\frac{V_A}{2R} e^{-\frac{3R}{L}t} u(t)}$$

$$\& \quad V_0(s) = -R \cdot I_L(s) = \boxed{\frac{-V_A L/2}{Ls+3R}} \Rightarrow v_0(t) = \boxed{\frac{-V_A}{2} e^{-\frac{3R}{L}t} u(t)}$$

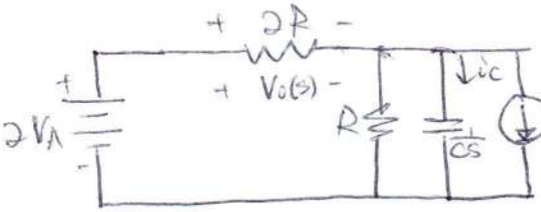
10.28 Initial Condition



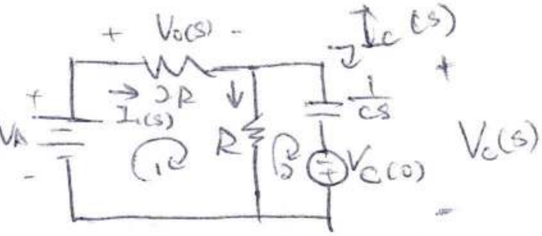
$$V_c(0) = \frac{R}{2R+R} V_A$$

$$= \frac{1}{3} V_A$$

Circuits in s-domain



$C \cdot V_c(0) \Rightarrow$



KVL at loop 1:

$$2V_A - I_1(s)\cdot 2R - \left(I_1(s) - I_c(s)\right)\cdot R = 0 \qquad ①$$

KVL at loop 2.

$$V_c(0) + \left(I_1(s) - I_c(s)\right)\cdot R - I_c(s)\cdot \frac{1}{cs} = 0 \qquad ②$$

Solve for $I_c(s)$:

$$I_c(s) = \frac{3}{2}\frac{V_A}{R}\cdot\frac{s}{s+\frac{3}{2RC}} \Rightarrow V_c(s) = -\left(V_c(0) + I_c(s)\cdot\frac{1}{cs}\right)$$

$$= \boxed{-\frac{1}{3}V_A - \frac{3}{2}\frac{V_A}{CR}\frac{1}{s+\frac{3}{2RC}}}$$

Solve for $I_1(s)$:

$$I_1(s) = \frac{2}{3R}V_A + \frac{V_A}{2R}\,\frac{s}{s+\frac{3}{2RC}}$$

$$V_o(s) = I_1(s)\cdot 2R = \boxed{\frac{4}{3}V_A + V_A\cdot\frac{s}{s+\frac{3}{2RC}}}$$

$$V_c(t) = \mathcal{L}^{-1}\{V_c(s)\} = \cancel{-\frac{1}{3}V_A\delta(t) - \frac{3}{2}\frac{V_A}{RC}e^{-\frac{3}{2RC}t}} = \boxed{-\frac{1}{3}V_A\,\delta(t) - \frac{3}{2}\frac{V_A}{CR}\frac{1}{s+\frac{3}{2RC}}\,u(t)}$$

$$V_o(t) = \mathcal{L}^{-1}\{V_o(s)\} = \frac{4}{3}V_A\cdot\delta(t) + V_A\,\delta(t) - V_A\frac{3}{2RC}e^{-\frac{3}{2RC}t}\,u(t)$$

$$= \boxed{\frac{7}{3}V_A\,\delta(t) - \frac{3V_A}{2RC}e^{-\frac{3}{2RC}t}\,u(t)}$$