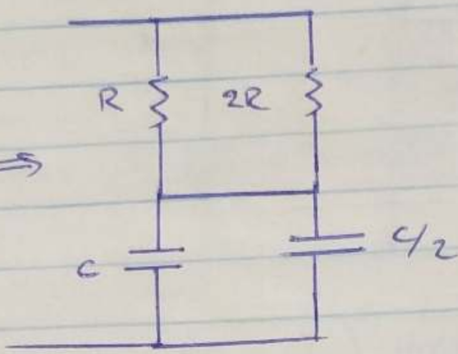
HW #8 Solutions

10.4

$Z_{eq} \Rightarrow$



$$Z_{eq}(s) = \frac{2R^2}{3R} + \frac{\frac{1}{sC}\cdot\frac{2}{sC}}{\frac{1}{sC}+\frac{2}{sC}} = \frac{2R}{3} + \frac{2}{3Cs}$$

$$= \frac{6RCs + 6}{9Cs} = \frac{2RCs + 2}{3sC}$$

which has a pole at $s = 0$ and a zero at:

$2RCs + 2 = 0$

$\Rightarrow \quad s = \frac{-1}{RC}$

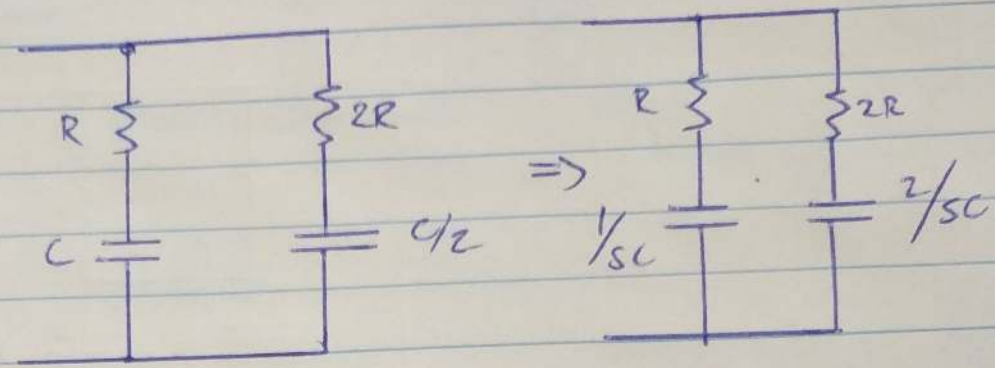
we want to select, $\quad \frac{1}{RC} = 330,$

$$\Rightarrow \quad 1 = 330RC$$

One possible selection: $R = 1, \quad C = \frac{1}{330}$

10.5

$Z_{eq} \Rightarrow$



So, $Z_{eq}(s) = \left(R + \frac{1}{sC}\right) \| \left(2R + \frac{2}{sC}\right)$

$$= \frac{Rsc+1}{sc} \| \frac{2RCs+2}{sC} = \frac{Rsc+1}{sc} \| 2\left(\frac{Rsc+1}{sc}\right)$$

$$= \frac{2}{3} \cdot \frac{Rsc+1}{sC}$$

which has a pole at $s=0$ again and a zero at
$sRC+1=0$

$$\Rightarrow s = \frac{-1}{RC}$$

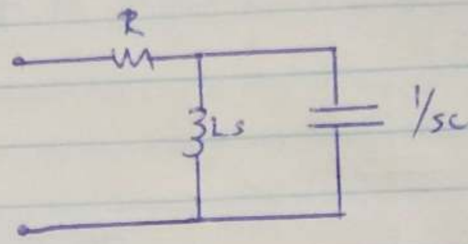
so we want $470 = \frac{1}{RC}$

$$\Rightarrow 470RC = 1$$

$R=1$, $C=\frac{1}{470}$ is a possible selection

10.10

$Z_{eq}(s) \Rightarrow$



$$Z_{eq}(s) = R + Ls \parallel \frac{1}{sC}$$

$$= R + \frac{\frac{L}{C}}{(Ls) + \frac{1}{sC}} = R + \frac{L/C}{\frac{s^2LC+1}{sC}}$$

$$= \frac{R(s^2LC+1) + \frac{L}{C}sC}{s^2LC+1} = \underline{\underline{\frac{s^2RLC + sL + R}{s^2LC+1}}}$$

Now we can locate the poles and zeros of our expression

Zeros

we have $s^2RLC + sL + R = 0$

$$\Rightarrow \quad s = \frac{-L \pm \sqrt{L^2 - 4R^2LC}}{2RLC}$$

poles: $s^2LC + 1 = 0$

$$s^2 = \frac{-1}{LC}, \quad s = \underline{\pm j\frac{1}{LC}}$$

R = 15K, and we want to select L & C s.t. poles are at $\pm j$ 200 Krad/s.

so, $$\frac{1}{LC} = 200{,}000$$

let $C = \cancel{\text{}}\ 1mF$

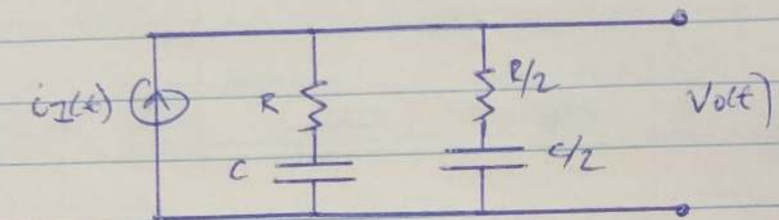
$L = 5mH$

Now, we can compute the location of the zeros

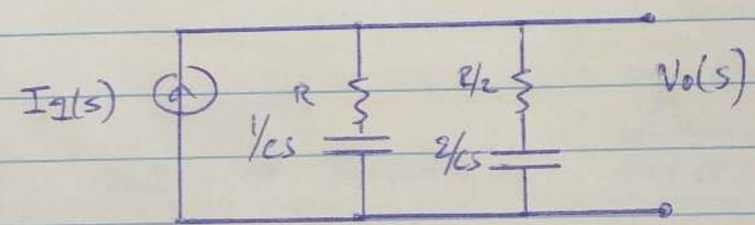
$$s = \frac{-5m \pm \sqrt{(5m)^2 - 4(15K)^2 \times 5m \times 1m}}{2 \times 15K \times 5m \times 1m}$$

$$= -0.033 \pm 447.2\,j$$

10.35



Let's convert to the s-domain representation



$$Z_{eq}(s) = \left(R + \frac{1}{Cs}\right) \Big|\Big| \left(\frac{R}{2} + \frac{2}{Cs}\right)$$

$$= \frac{\left(R + \frac{1}{Cs}\right)\left(\frac{R}{2} + \frac{2}{Cs}\right)}{R + \frac{R}{2} + \frac{1}{Cs} + \frac{2}{Cs}} = \frac{\left(R + \frac{1}{Cs}\right)\left(\frac{R}{2} + \frac{2}{Cs}\right)}{\frac{3}{2}R + \frac{3}{Cs}}$$

$$Z_{eq}(s) = \frac{(RCs + 1)(RCs + 4)}{(Cs)(3RCs + 6)} = \frac{V_o(s)}{I_1(s)}$$

$$V_o(s) = I_1(s) \cdot Z_{eq}(s)$$

Poles: $s = 0,\ 3RCs + 6 = 0 \Rightarrow s = -2/RC$

Zeros: $s = \frac{-1}{RC},\ \frac{-4}{RC}$

10.48

$$Z_{eq}(s) = 2000 \cdot \frac{s+3000}{s+2000}$$

$$v(t) = 10e^{-10t}u(t), \quad V(s) = \frac{10}{s+10}$$

$$I(s) = \frac{V(s)}{Z(s)} = \frac{10}{s+10} \cdot \frac{s+2000}{s+3000} \cdot \frac{1}{2000}$$

$$= \frac{s+2000}{(200)(s+10)(s+3000)} = \frac{A}{s+10} + \frac{B}{s+3000}$$

so,

$$s+2000 = 200A(s+3000) + 200B(s+10)$$
$$= 200As + (3000)(200)A + 200Bs + 2000B$$

$$s+2000 = 200As + (3000)(200)A + 200Bs + 2000B$$

$\Rightarrow$

$$1 = 200A + 200B$$
$$1000 = 3000(200)A + 2000B$$

$$\Rightarrow A = 1990/598000 = 199/59800$$
$$B = 1/598$$

$$\Rightarrow 2000 = 3000(200)A + \frac{2000}{200}(1-200A)$$

$$2000 = 3000(200)A + 10 - 2000A$$

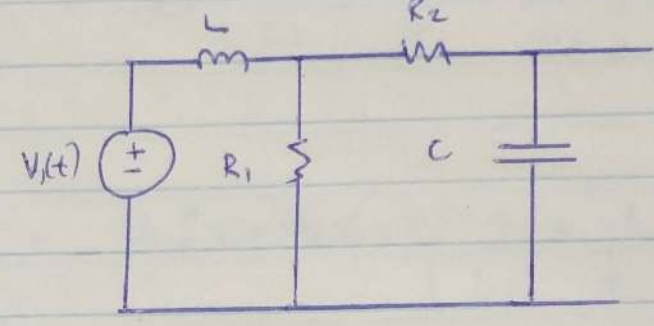
$$1990 = 598000A$$

$$A = \frac{1990}{598000}$$

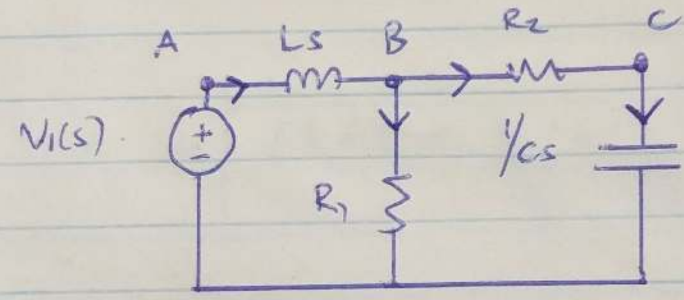
7

So,
$$i(t) = \left(\frac{199}{59800} e^{-10t} + \frac{1}{598} e^{-3000t} \right) u(t)$$

Q-52



(a)



(A) $V_A = V_1(s)$

(B) $$\frac{V_A - V_B}{Ls} = \frac{V_B}{R_1} + \frac{V_B - V_C}{R_2}$$

(C) $$\frac{V_B - V_C}{R_2} = \frac{V_C}{(1/Cs)} \Rightarrow \frac{V_B - V_C}{R_2} = \frac{CsV_C}{1}$$

$$\Rightarrow \frac{V_B}{R_2} = V_C\left(\frac{1}{R_2} + Cs\right)$$

$$\frac{V_B}{R_2} = V_C\left(\frac{1 + R_2Cs}{R_2}\right)$$

$$\Rightarrow V_C = \frac{V_B}{1 + R_2Cs}$$

$$\frac{V_A}{Ls} = V_B\left(\frac{1}{Ls} + \frac{1}{R_1} + \frac{1}{R_2}\right) - \frac{V_C}{R_2}$$

$$= V_C(1 + R_2Cs)\left(\frac{1}{Ls} + \frac{1}{R_1} + \frac{1}{R_2}\right) - \frac{V_C}{R_2}$$

$$= V_C\left[(1 + R_2Cs)\left(\frac{1}{Ls} + \frac{1}{R_1} + \frac{1}{R_2}\right) - \frac{1}{R_2}\right]$$

$$= V_C\left[(1 + R_2Cs)\left(\frac{R_1R_2 + LsR_2 + LsR_1}{LsR_1R_2}\right) - \frac{1}{R_2}\right]$$

$$\frac{V_A}{Ls} = V_C\left[\frac{R_2(1 + R_2Cs)(R_1R_2 + LsR_2 + LsR_1) - LsR_1R_2}{LsR_1R_2^2}\right]$$

$$V_A = V_C\left[\frac{R_2(1 + R_2Cs)(R_1R_2 + LsR_2 + LsR_1) - LsR_1R_2}{R_1R_2^2}\right]$$

$$V_A = \frac{V_C}{R_1R_2^2} \cdot \left[s^2\left(R_2^2CL(R_1 + R_2)\right) + s\left(CR_1R_2^3 + LR_2^2 + LR_1R_2 - LR_1R_2\right) + R_2^2R_1\right]$$

So,

$$V_i(s) R_1 = V_o(s)\left[s^2 CL(R_1 + R_2) + s(CR_1 + L) + R_1\right]$$

$$\Rightarrow V_o(s) = \frac{V_i(s) R_1}{s^2 CL(R_1 + R_2) + s(CR_1R_2 + L) + R_1}$$

(c) The natural poles are $s : s^2 CL(R_1 + R_2) + s(CR_1R_2 + L) + R_1 = 0$

Without knowing $V_i(s)$, we do not know the forced poles of the circuit

(d)

$$V_2(s) = \frac{500 \cdot 5}{s(0.0005s^2 + 0.75s + 500)}$$

$$= \frac{2500}{s(0.0005s^2 + 0.75s + 500)}$$

$$\cancel{= \frac{A}{s} + \frac{Bs + C}{0.0005s^2 + 0.75s + 500}}$$

$$= \frac{2500}{s(0.0005s^2 + 0.75s + 500)}$$

$$= \frac{5 \times 10^6}{s(s^2 + 1500s + 1 \times 10^6)}$$

$$= \frac{A}{s} + \frac{Bs + C}{s^2 + 1500s + 1 \times 10^6}$$

s.t.

$$A(s^2 + 1500s + 1 \times 10^6) + Bs^2 + Cs = 5 \times 10^6$$

$$\Rightarrow A + B = 0 \qquad A = 5,\ B = -5$$

$$1 \times 10^6 A = 5 \times 10^6 \qquad \Rightarrow C = -7500$$

$$1500 A + C = 0$$

$$V_2(s) = \frac{5}{s} + \frac{-5s - 7500}{s^2 + 1500s + 1 \times 10^6}$$

$$= \frac{5}{s} - \frac{5s + 7500}{s^2 + 1500s + 1 \times 10^6}$$

$$= \frac{5}{s} - \frac{5(s + 1500)}{s^2 + 1500s + 1 \times 10^6} = \frac{5}{s} - \frac{5(s + 750 + 750)}{(s + 750)^2 + 43750}$$

$$= \frac{5}{s} - \frac{5(s+750)}{(s+750)^2 + \left(\sqrt{437500}\right)^2} - \frac{5(750)}{(s+750)^2 + \left(\sqrt{437500}\right)^2}$$

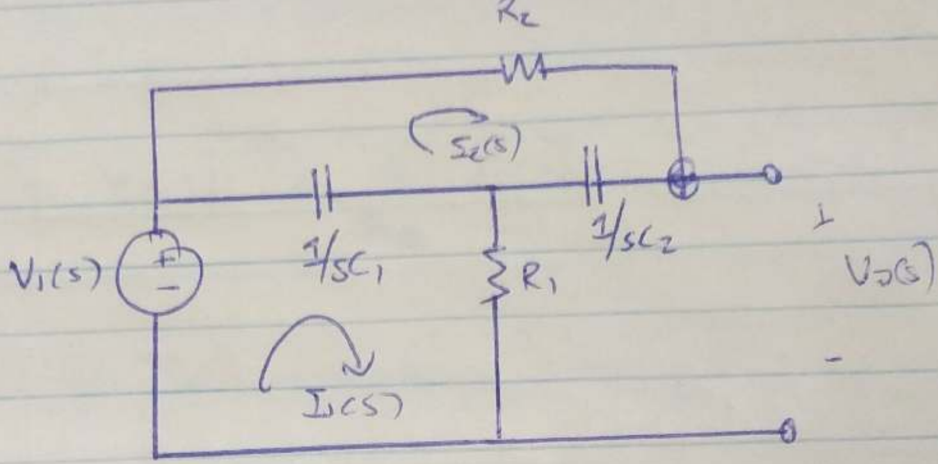
so,

$$v_2(t) = \Big[5 - 5e^{-750t}\cos\left(\sqrt{437500}\,t\right)$$

$$- \frac{5\ 750}{\sqrt{437500}}\, e^{-750t} \sin\left(\sqrt{437500}\,t\right)\Big]$$

$$u(t))$$

10-5C



$$V_1(s) - \left(I_1(s) - I_2(s)\right)\frac{1}{sC_1} - I_1(s)R_1 = 0$$

$$-R_2 I_2(s) - \frac{1}{sC_2} I_2(s) - \left(I_2(s) - I_1(s)\right)\frac{1}{sC_1} = 0$$

(1) $\Rightarrow V_1(s) = I_1(s)\left(R_1 + \frac{1}{sC_1}\right) + I_2(s)\frac{1}{sC_1}$

(2) $\cancel{I_1(s)R_1 + I_2(s)(R_2 + R_1 +}$

(2) $-I_1(s)\frac{1}{sC_1} + I_2(s)\left(R_2 + \frac{1}{sC_1} + \frac{1}{sC_2}\right) = 0$

$$\boxed{V_o(s) = I_1(s) R_1 + \frac{1}{sC_2} I_2(s)}$$

Combine with previous expressions to get:

$$\frac{V_o(s)}{V_1(s)} = \frac{R_1 R_2 C_1 C_2 s^2 + (C_1 R_1 + R_1 C_2) s + 1}{R_1 R_2 C_1 C_2 s^2 + (R_1 C_1 + R_1 C_2 + R_2 C_2) s + 1}$$

10-77

$$V_{oc}(t) = \left(10e^{-10t} - 10e^{-50t}\right)u(t) \text{ Volts}$$

$$I_{sc}(t) = \left(100e^{-10t} + 200e^{-50t} + 50e^{-100t}\right)u(t) \text{ mA}$$

Let's first take the Laplace transform of these expressions

$$V_{oc}(s) = \frac{10}{s+10} - \frac{10}{s+50} = \frac{10(s+50) - 10(s+10)}{(s+10)(s+50)}$$

$$I_{sc}(s) = \frac{100}{s+10} + \frac{200}{s+50} + \frac{50}{s+100}$$

$$\cancel{= 100(s+50)(s+100) + 200(s+10)(s+100)}$$

$$= \frac{100(s+50)(s+100) + 200(s+10)(s+100) + 50(s+10)(s+50)}{(s+10)(s+50)(s+100)}$$

$$= \frac{50(7s^2 + 800s + 14500)}{(s+10)(s+50)(s+100)}$$

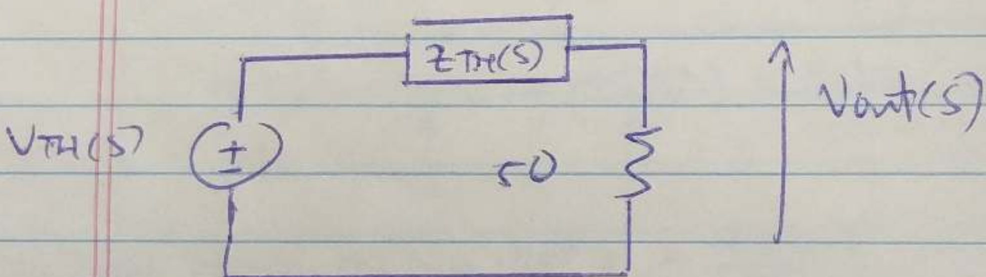
$$Z_{TH}(s) = \frac{V_{oc}(s)}{I_{sc}(s)} = \frac{400}{(s+10)(s+50)} \times \frac{(s+10)(s+50)(s+100)}{50(7s^2 + 800s + 14500)}$$

convert from mA to A.

$$Z_{TH}(s) = \frac{8(s+100)}{7s^2 + 800s + 14500} \times 1000$$

$$= \frac{8000(s+100)}{7s^2 + 800s + 14500} = \frac{\frac{8000}{7}(s+100)}{s^2 + \frac{800s}{7} + \frac{14500}{7}}$$

So we have



$$V_{out}(s) = \left(\frac{V_{TH}(s)}{Z_{TH}(s) + 50}\right) 50$$

$=$

$$V_L(s) = \left(\frac{50}{50 + Z_{TH}(s)}\right) V_{TH}(s)$$

$$= \left(\frac{50}{50 + \dfrac{\frac{8000}{7}(s+100)}{s^2 + \frac{800}{7}s + \frac{14500}{7}}}\right) \times \frac{400}{(s+50)(s+10)}$$

$$= \frac{50\left(s^2 + 800/7\, s + 14500/7\right)}{50\left(s^2 + \frac{800}{7}s + \frac{14500}{7}\right) + \frac{8000}{7}(s+100)} \times \frac{400}{(s+50)(s+10)}$$

$$= \frac{20{,}000\left(s^2 + \frac{800}{7}s + \frac{14500}{7}\right)}{\left(50s^2 + \frac{48000}{7}s + \frac{1525000}{7}\right)(s+50)(s+10)}$$

$$= \frac{400\left(s^2 + \frac{800}{7}s + \frac{14500}{7}\right)}{\left(s^2 + \frac{960}{7}s + \frac{30500}{7}\right)(s+50)(s+10)}$$

Use Matlab to find inverse Laplace

$$v(t) = 400\left(\frac{10}{13}e^{-50t}t + \frac{7e^{-610/7t}}{2535} - \frac{15e^{-50t}}{1352} + \frac{e^{-10t}}{120}\right)$$

10-78

(a) $$Z_1(s) = \frac{s+10}{s+20}$$

$$\frac{1}{Z_1(s)} = \frac{1}{R} + \frac{1}{Z_2(s)}$$

$$\frac{s+20}{s+10} = \frac{1}{20} + \frac{1}{Z_2(s)}$$

$$\frac{s+20}{s+10} = \frac{Z_2(s)+20}{20\,Z_2(s)}$$

$$20(s+20)Z_2(s) = 20(s+10) + Z_2(s)(s+10)$$

$$Z_2(s)\left[20(s+20) - (s+10)\right] = 20(s+10)$$

$$Z_2(s)(19s+390) = 20(s+10)$$

$$Z_2(s) = \frac{20s+200}{19s+390}$$

(b)

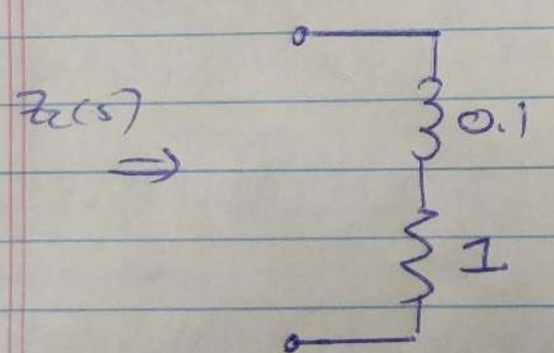
$$Y_L(s) = \frac{1}{Z_L(s)} = \frac{s+20}{s+10} = \frac{s+10+10}{s+10} = 1 + \frac{10}{s+10}$$

$$Y_L(s) = Y_2(s) + Y_R(s) = \frac{1}{Z_2(s)} + \frac{1}{R}$$

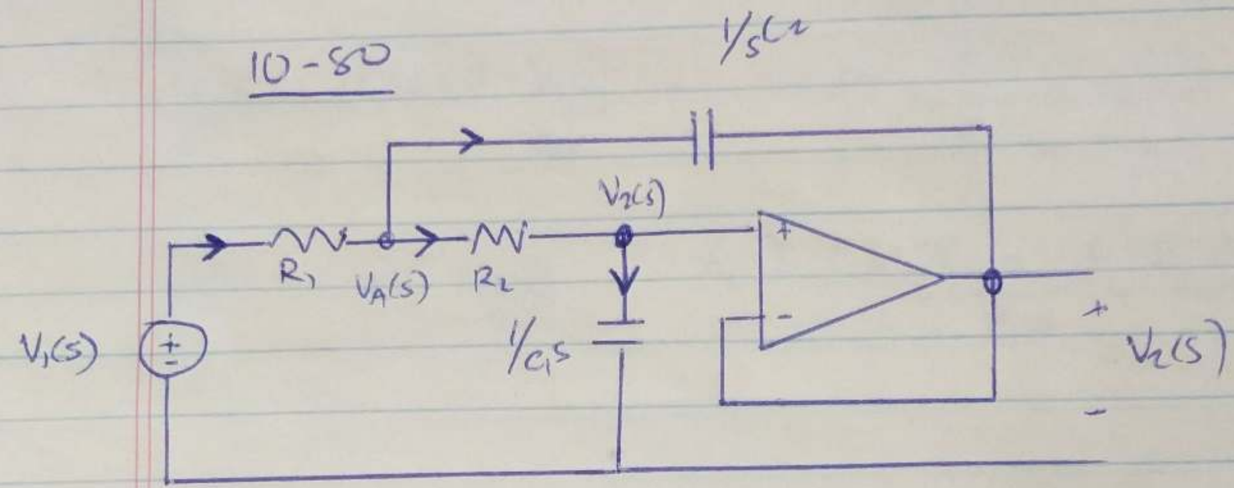
so $$\frac{1}{Z_2(s)} + \frac{1}{R} = 1 + \frac{10}{s+10}$$

s.t. $$Z_2(s) = \frac{s+10}{10} = \frac{s}{10} + 1$$

which is the same as :



10-80



(1) $$\frac{V_1(s) - V_A(s)}{R_1} = (V_A(s) - V_2(s))\, sC_2 + \frac{(V_A(s) - V_2(s))}{R_2}$$

(2) $$\frac{V_A(s) - V_2(s)}{R_2} = V_2(s) \cdot sC_1$$

$$\Rightarrow V_2(s)\left[sC_1 + \frac{1}{R_2}\right] = \frac{V_A(s)}{R_2}$$

$$V_2(s)\left[\frac{sR_2C_1 + 1}{R_2}\right] = \frac{V_A(s)}{R_2} \Rightarrow \underline{\underline{V_A(s) = V_2(s)(1 + sC_1R_2}}$$

$$\frac{V_1(s)}{R_1} - \frac{V_A(s)}{R_1} = V_A(s)\, sC_2 - V_2(s)\, sC_2 + \frac{V_A(s)}{R_2} - \frac{V_2(s)}{R_2}$$

$$\frac{V_1(s)}{R_1} + V_2(s) \cdot sC_2 + \frac{V_2(s)}{R_2} = V_A(s)\left[\frac{1}{R_1} + sC_2 + \frac{1}{R_2}\right]$$

$$\frac{V_1(s)}{R_1} + sC_2V_2(s) + \frac{V_2(s)}{R_2} = V_A(s)\left[\frac{R_1 + R_2 + sC_2R_1R_2}{R_1R_2}\right]$$

$$\frac{V_1(s)}{R_1} + sC_2V_2(s) + \frac{V_2(s)}{R_2} = V_2(s) \cdot \frac{(1+sC_1R_2)(R_1+R_2+sC_2R_1R_2)}{R_1R_2}$$

$$\frac{V_1(s)}{R_1} = V_2(s)\left[\frac{(1+sC_1R_2)(R_1+R_2+sC_2R_1R_2)}{R_1R_2} - \frac{1}{R_2} - sC_2\right]$$

$$\frac{V_1(s)}{R_1} = V_2(s)\left[\frac{(1+sC_1R_2)(R_1+R_2+sC_2R_1R_2) - R_1 - sR_1R_2C_2}{R_1R_2}\right]$$

$$V_1(s) = \frac{V_2(s)}{R_2}\Big[R_1 + R_2 + sC_2R_1R_2 + sC_1R_1R_2 + sC_1R_2^2 + s^2C_1C_2R_1R_2^2$$
$$- R_1 - sR_1R_2C_2\Big]$$

$$V_1(s) = \frac{V_2(s)}{R_2}\left[s^2C_1C_2R_1R_2^2 + sR_2(\cancel{s}C_1R_1 + \cancel{s}C_1R_2) + R_2\right]$$

$$\Rightarrow \quad \frac{V_2(s)}{V_1(s)} = \frac{1}{s^2C_1C_2R_1R_2 + sC_1(R_1+R_2) + 1}$$