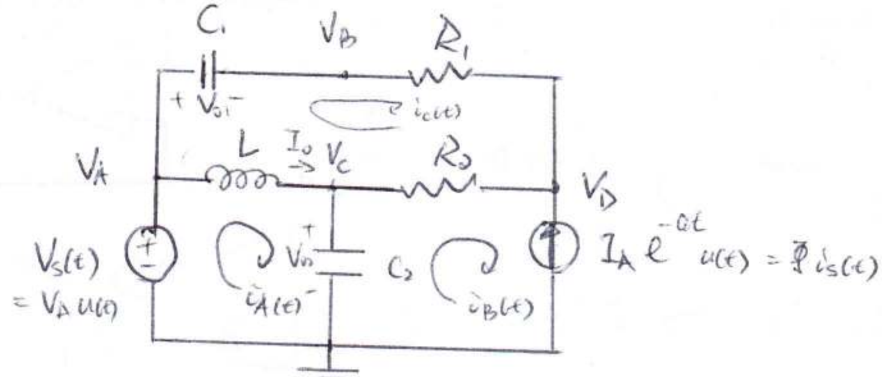
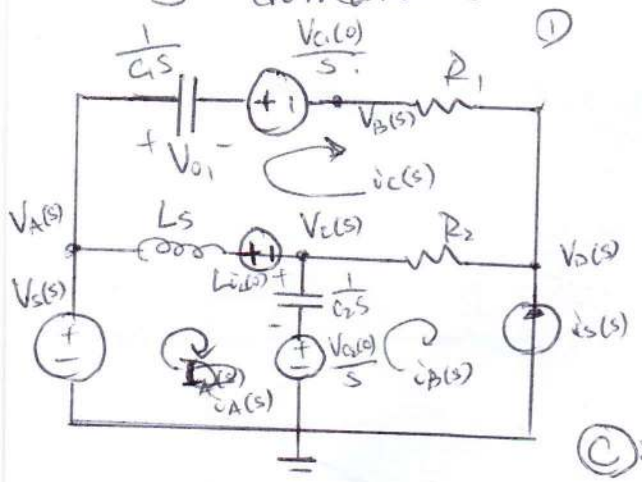
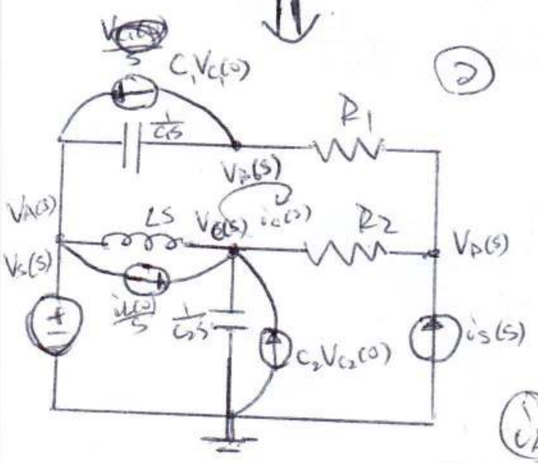
HW #9 Solution

10.76



a) S domain:





Node Voltage: (easier w/ ②)

(A): $V_A(s) = V_s(s) = V_A \cdot \frac{1}{s}$

(B): $\dfrac{V_D(s) - V_B(s)}{R_1} + \dfrac{V_A(s) - V_B(s)}{\frac{1}{C_1 s}} - C_1 V_{C_1}(0) = 0$

(C): $\dfrac{V_A(s) - V_C(s)}{Ls} + \dfrac{i_L(0)}{s} + \dfrac{0 - V_C(s)}{\frac{1}{C_2 s}} + \dfrac{V_D(s) - V_C(s)}{R_2}$

$+ C_2 V_{C_2}(0) = 0$

(D): $\dfrac{V_B(s) - V_D(s)}{R_1} + \dfrac{V_C(s) - V_D(s)}{R_2} + i_s(s) = 0$

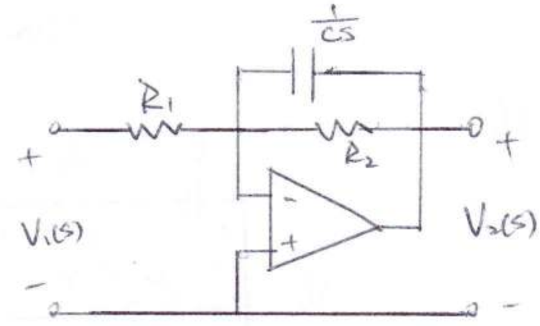
Mesh Current (easier w/ ①)

(i_A): $V_s(s) - i_A(s) \cdot Ls - Li_L(0) - \left(i_A(s) \cdot \frac{1}{C_2 s} - i_B(s) \cdot \frac{1}{C_2(s)}\right) - \frac{V_{C_2}(0)}{s} = 0$

b) (i_B): ~~$i_B(s) = i_s(s)$~~ $i_B(s) = -i_s(s)$

(i_C): $-i_C(s) \frac{1}{C_1 s} - \frac{V_{C_1}(0)}{s} - i_C(s) \cdot R_1 - R_2\left(i_C(s) - i_B(s)\right) + Li_L(0) - Ls\left(i_C(s) - i_A(s)\right) = 0$

11.6



a) $T_V(s) = \frac{V_2(s)}{V_1(s)} = -\frac{\frac{1}{Cs+\frac{1}{R_2}}}{R_1}$

$= \boxed{-\frac{\frac{1}{R_1 C}}{s + \frac{R_1}{R_1 R_2 C}}}$ inverting Op-Amp

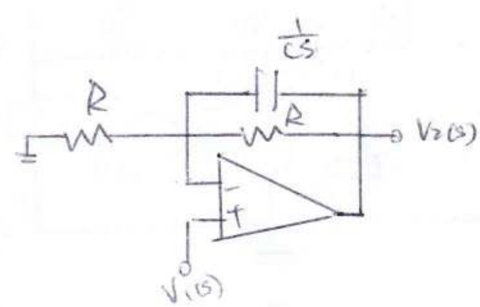
b) Input impedence 5000 Ω ⇒ $R_1 = 5000\,\Omega$

pole: $s = -\frac{R_1}{R_2 C} = -2000$ rad/s ⇒ $R_2 \cdot C = \frac{5}{2} = 2.5$ s

Driving point impedance: $Z_{in} = R_1$

⇒ $\cancel{R_2 \cdot C = \frac{1}{10^7} = 10^{-7}}$ any combination s.t. $R_2 C = 2.5$ s.

11.7



a) $T_V(s) = \frac{R + \frac{1}{Cs+\frac{1}{R}}}{R} = 1 + \frac{1}{RCs+1} = \frac{RCs+2}{RCs+1}$

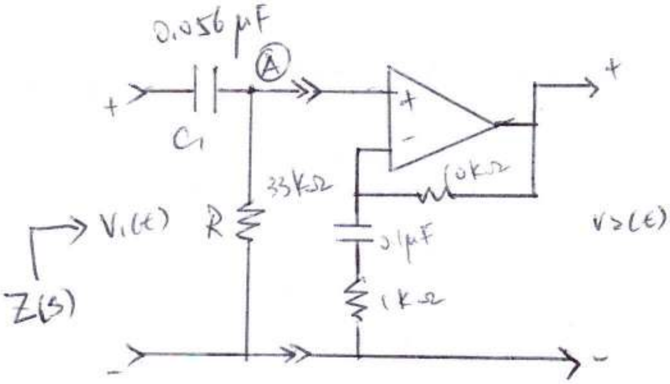
$= \frac{s + \frac{2}{RC}}{s + \frac{1}{RC}}$

pole: $s = -\frac{1}{RC} = -2000$ rad/s ⇒ any R, C s.t. $RC = \frac{1}{2000}$ s

zero: $s = -\frac{2}{RC} = -4000$ rad/s.

Driving point impedance: $Z_{in} = \infty$

11.15



Assume zero initial condition

$$V_A(s) = \frac{33\,k\Omega}{\frac{1}{(0.056\mu F)s} + 33\,k\Omega} \cdot V_1(s)$$

(voltage divider)

$$\frac{V_2(s)}{V_A(s)} = \frac{\frac{1}{(0.1\mu F)s} + 1\,k\Omega + 10\,k\Omega}{\frac{1}{(0.1\mu F)s} + 1\,k\Omega}$$

$$T_V(s) = \frac{V_2(s)}{V_1(s)} = \frac{V_2(s)}{V_A(s)} \cdot \frac{V_A(s)}{V_1(s)} = \frac{\frac{1}{0.1\times10^{-6}s} + 11\times10^3}{\frac{1}{0.1\times10^{-6}s} + 1\times10^3} \cdot \frac{33\times10^3}{\frac{1}{0.056\times10^{-6}s} + 33\times10^3}$$

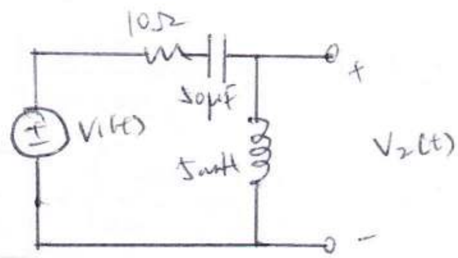
$$= \frac{1 + 11\times10^{-4}s}{1 + 10^{-4}s} \cdot \frac{33\times0.056\times10^{-3}s}{33\times0.056\times10^{-3}s + 1} = \boxed{\frac{11\cdot(s+909)\cdot s}{(s+10^4)(s+541.13)}}$$

pole: $s_1 = -10^4$, $s_2 = -541.13$. Zero: $z_1 = -909$. $z_2 = 0$

Input impedance: $Z_{in} = \frac{V_1(s)}{\frac{V_1 - V_A}{\frac{1}{C_1 s}}}$ $\quad V_A = \frac{33\,k\Omega}{33\,k\Omega + \frac{1}{0.056\mu F\cdot s}} \cdot V_1$

$$\boxed{Z_{in} = 33 + \frac{1}{0.056s}.}$$

11.37



$$V_2(s) = \frac{Ls}{R + \frac{1}{Cs} + Ls} V_1(s) \text{ assume zero IC}$$

$$\frac{V_2(s)}{V_1(s)} = \frac{Ls^2}{Ls^2 + Rs + \frac{1}{C}} = \frac{s^2}{s^2 + \frac{R}{L}s + \frac{1}{LC}}$$

$$V_2(s) = \frac{s^2}{s^2 + 2000s + 4\times10^6} \cdot 25 \cdot \frac{s}{s^2 + 2000^2} \Rightarrow V_2(t) = \mathcal{L}^{-1}\{\cdots\}$$

or $T_V = \frac{V_2(s)}{V_1(s)} = \frac{s^2}{s^2 + \frac{R}{L}s + \frac{1}{LC}}$. excited ~~the~~ system with $V_1(t) = 25\cos(2000t)$ V

$$T_V(j2000) = \frac{-2000^2}{\cancel{-2000^2} + 2000^2 j + \cancel{4\times10^6}} \quad |T_V(j2000)| = \left|\frac{-1}{j}\right| = 1$$

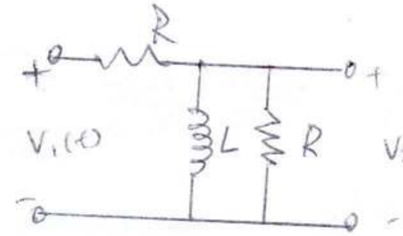
$$\measuredangle T_V(2000j) = \arg(-1) - \arg(j) = \pi - \frac{\pi}{2} = \frac{\pi}{2} \quad \therefore \boxed{V_{2ss} = 25\cos(2000t + \tfrac{\pi}{2})\ V = \boxed{25\sin(2000t)\ V}}$$

for $V_1(t) = 5V$, $\omega = 0$ $T_V(j0) = 0$

$\therefore$ $V_2 = 0$. (By inspection, constant V_1, then capacitor act as open circuits).

12.5

a) Voltage divider



$$\frac{V_2(s)}{V_1(s)} = \frac{\frac{1}{\frac{1}{Ls}+\frac{1}{R}}}{R + \frac{1}{\frac{1}{Ls}+\frac{1}{R}}} = \frac{1}{R\left(\frac{1}{Ls}+\frac{1}{R}\right)+1}$$

$$= \frac{\frac{1}{2}s}{s+\frac{R}{2L}} = T_V(s) = \frac{1}{2}\,\frac{s}{s+4700}$$ High pass filter

$|T_V(j0)| = 0$, $|T_V(j\infty)| = \left|\frac{\frac{1}{2}}{1+\frac{R}{2L}\frac{1}{j\infty}}\right| = \frac{1}{2}$, $\omega_{cutoff} = \left|\frac{R}{2L}\right| = 4700$

(cutoff frequency found by $|T_V(j\omega_c)| = |T_{V\,max}|\left(\frac{1}{\sqrt{2}} = \frac{1}{2\sqrt{2}}\right)$ or pole magnitude

c) $|T_V(j\frac{1}{4}\omega_c)| = \frac{1}{2}\left|\frac{j\frac{1}{4}\omega_c}{j\frac{1}{4}\omega_c+\omega_c}\right| = \frac{1}{2}\left|\frac{j}{4+j}\right| = \boxed{\frac{1}{2\sqrt{17}}} \simeq 0.1213$

$|T_V(j\frac{1}{2}\omega_c)| = \frac{1}{2}\left|\frac{j\frac{1}{2}\omega_c}{j\frac{1}{2}\omega_c+\omega_c}\right| = \frac{1}{2}\left|\frac{j}{2+j}\right| = \boxed{\frac{1}{2\sqrt{5}}} \simeq 0.2236$

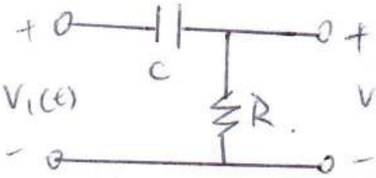
$|T_V(j\omega_c)| = \frac{1}{2}\left|\frac{j\omega_c}{j\omega_c+\omega_c}\right| = \boxed{\frac{1}{2\sqrt{2}}} \simeq 0.3536$

12.9

high pass filter in the form of $K\frac{s}{s+\omega_c}$, $\omega_c =$ ~~20k rad/s~~ $159\,Hz \cdot 2\pi$

pass-band gain of 5 $\Rightarrow$ $K=5$

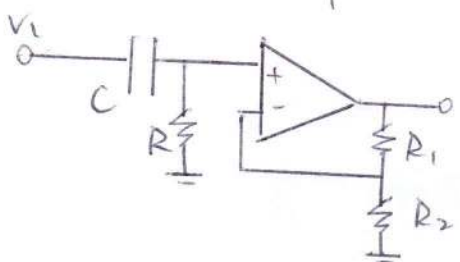
$\therefore$ high pass filter $\frac{5s}{s+\omega_c}$ $\rightarrow$ use simplest RC circuits.



$$\frac{V_2(s)}{V_1(s)} = \frac{R}{R+\frac{1}{sC}} = \frac{CRs}{CRs+1} = \frac{s}{s+\frac{1}{CR}}$$

select $\underline{C}, \underline{R}$ s.t. $\frac{1}{CR} = \omega_c$.

what about passband gain K? $\rightarrow$ could use Op-Amp (one choice)



select $\underline{\frac{R_1}{R_2} = 4}$.

12.2

$dB = 20 \log_{10}(\text{gain}) \Rightarrow 0\ dB \Rightarrow \text{gain} = 1.$

low pass filter : $F(s) = \frac{\omega_c}{s+\omega_c}$ so that pass band is unit gain.

$\omega_c = 5$ krad/s. $|F(j0)| = 1$ with in pass band.

$$|F(j500)| = \left|\frac{5000}{j500+5000}\right| = \left|\frac{10}{10+j}\right| = \frac{10}{\sqrt{101}} \simeq 0.995$$

as we can see, this is still w/i passband. 500 rad/s < 5 krad/s. but gain is not exactly one, which is because this is the actual case, not the straight line approximation.

$$|F(j50k)| = \left|\frac{5k}{50kj+5k}\right| = \left|\frac{1}{1+10j}\right| = \frac{1}{\sqrt{101}} \simeq 0.0995$$

$$|F(j500k)| = \left|\frac{5k}{500kj+5k}\right| = \left|\frac{1}{1+100j}\right| = \frac{1}{\sqrt{10001}} \simeq 0.01$$

12.22

a) low pass filter. $\omega_c = 2000$ rad/s, gain $= 5$

b) use matlab tf( ) command to build system, then, use bode( ). :

```
>> F = tf(10000, [1 2000]);
>> bode(F)
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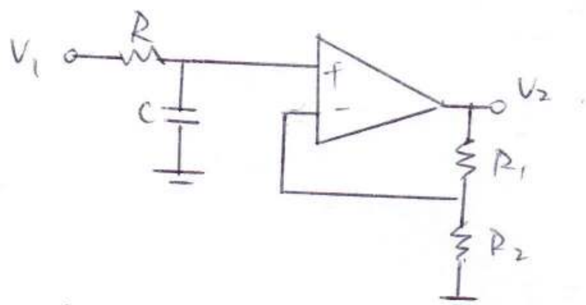
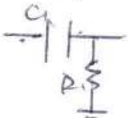
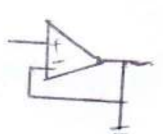
c) 

$$\begin{cases} \frac{1}{RC} = \omega_c = 2000 \text{ rad/s} \\ \frac{R_1}{R} = 4. \end{cases}$$

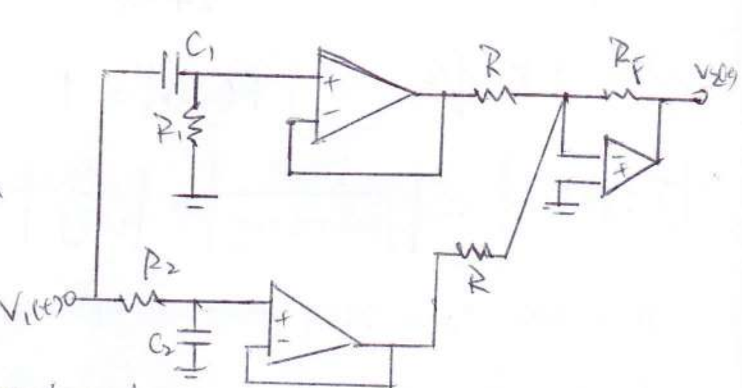
d) make a ω vector in matlab, then substitute $j\omega$ into transfer function $F(j\omega)$

12.34

a) the upper branch is just a ~~low~~ high pass filter 

, a buffer 

and an inverting op-amp w/ gain $-\frac{R_F}{R}$.



The lower branch is the same as upper branch but w/ a ~~hig~~ low pass filter.

$$V_2(s) = V_{up}(s) + V_{low}(s) = -\frac{R_F}{R} \cdot \frac{R_1}{R_1 + \frac{1}{C_1 s}} V_1(s) + \left(-\frac{R_F}{R}\right) \cdot \frac{\frac{1}{C_2 s}}{R_2 + \frac{1}{C_2 s}} V_1(s)$$

$$T_V(s) = \frac{V_2(s)}{V_1(s)} = -\frac{R_F}{R} \left(\frac{s}{s + \frac{1}{R_1 C_1}} + \frac{\frac{1}{R_2 C_2}}{s + \frac{1}{R_2 C_2}} \right)$$

b). $R_1 C_1$, R_2, C_2 control the two cutoff frequencies. $\frac{1}{R_1 C_1}$ $\frac{1}{R_2 C_2}$

let $\frac{1}{R_1 C_1} = 400$ krad/s, $\frac{1}{R_2 C_2} = 4000$ krad/s., $\omega_{center} = \frac{1}{2}(\omega_{c\text{-}LPF} + \omega_{c\text{-}HPF})$

$|T_V(j\omega_{center})| = 40\text{ dB} = 100.$ => select ω_{center} => R_1, C_1, R_2, C_2.