MAE140 - Linear Circuits - Fall 2017
Midterm, November 2nd

Instructions

1. This exam is open book. You may use whatever written materials you choose, including your class notes and textbook. You may use a hand calculator with no communication capabilities.
2. You have 70 minutes.
3. Write your answers on your "Blue Book".
4. Do not forget to write your name, student number, and instructor.
5. This exam has 4 questions, for a total of 20 points.

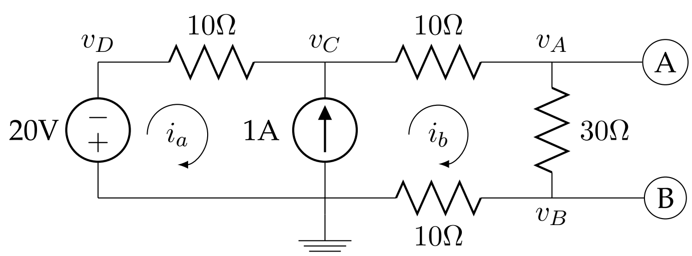
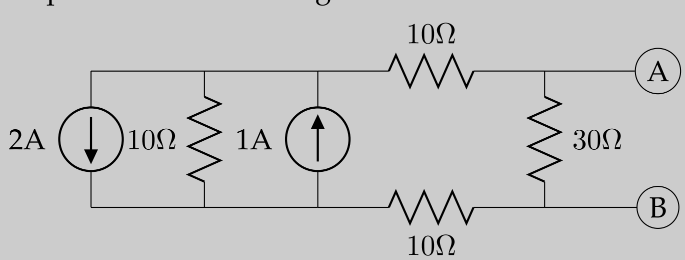


Figure 1: Circuit for Questions 1, 2, and 3

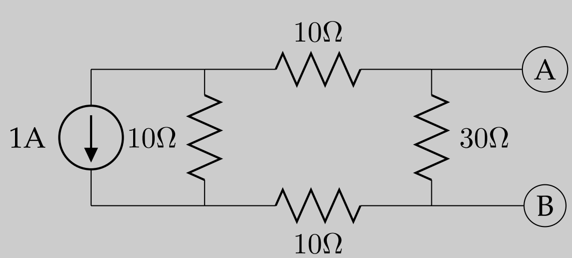
1. **Equivalent Circuits**

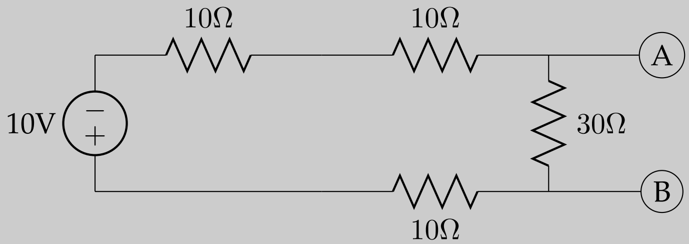
   (a) (5 points) Use circuit reduction to find the Thevenin and Norton equivalent circuits as seen from terminals A and B.

Solution:
NOTE TO GRADERS: Correct answer must use only source transformations!
We perform the following transformations:



(+1 point)





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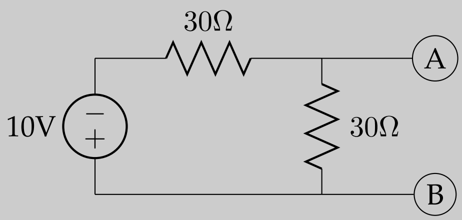
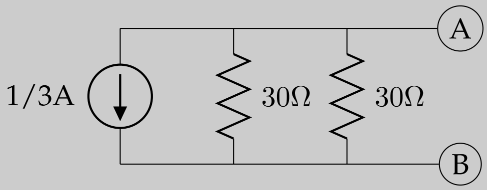
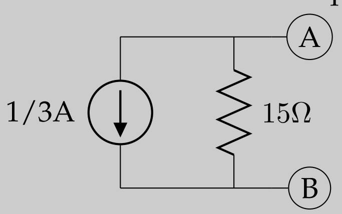




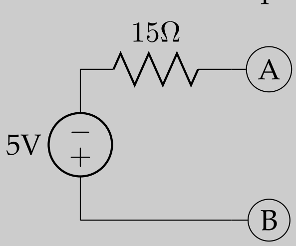
(+1 point)

From which the Norton equivalent is:



(+1 point)

and the Thevenin equivalent is:

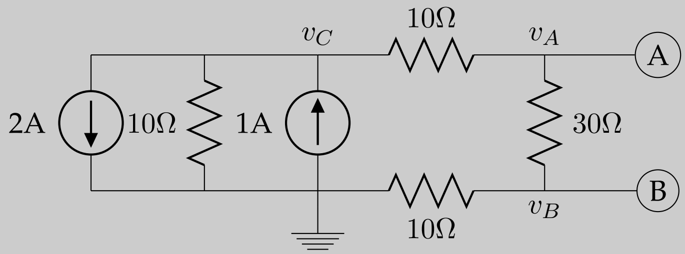


(+1 point)

2. **Node Voltage Analysis**

   (a) (4 points) Formulate node-voltage equations for the circuit in Fig. 1. Use the node-voltages and labels provided in the figure and clearly indicate the final equations and circuit variable unknowns. Make sure your final equations involve only node-voltages.

   Solution:
   In order to account for the voltage source we first use a source transformation:



(+1 point)

NOTE TO GRADERS: Using Method #2 to write $v_D = -20$ V is also fine.
Writing the KCL equations at nodes A, B and C by inspection we obtain:

$$\begin{bmatrix} \frac{1}{10} + \frac{1}{30} & -\frac{1}{30} & -\frac{1}{10} \\ -\frac{1}{30} & \frac{1}{10} + \frac{1}{30} & 0 \\ -\frac{1}{10} & 0 & \frac{1}{10} + \frac{1}{10} \end{bmatrix} \begin{pmatrix} v_A \\ v_B \\ v_C \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 1-2 \end{pmatrix} \quad \text{(+2 points)}$$

These are the node-voltage equations that need to be solved for the node voltages v_A, v_B, and v_C. (+1 point)

(b) (1 point) Explain how you could solve for the mesh-current i_a after having calculated the node-voltages.

Solution:
The mesh-current i_a is the current at the 10Ω resistor, so it could be calculated as

$$i_a = \frac{v_D - v_C}{10} = \frac{-20 - v_C}{10} \quad \text{(+1 point)}$$

3. **Mesh Current Analysis**

(a) (4 points) Formulate mesh-current equations for the circuit in Fig. 1. Use the mesh-currents and labels provided in the figure and clearly indicate the final equations and circuit variable unknowns. Make sure your final equations involve only mesh-currents.

Solution:
In this case it is not possible to use a simple source transformation so we write KVL by inspection:

$$\begin{bmatrix} 10 & 0 \\ 0 & 10+10+30 \end{bmatrix} \begin{pmatrix} i_A \\ i_B \end{pmatrix} = \begin{pmatrix} -20 - v_C \\ v_C \end{pmatrix} \quad \text{(+2 points)}$$

where v_C denotes the voltage across the current source.
Adding these two equations to eliminate v_C we obtain:

$$10 i_a + 50 i_b = -20 \quad \text{(+1 point)}$$

which, together with

$$i_b - i_a = 1 \quad \text{(+1 point)}$$

must be solved for i_a and i_b.

(b) (1 point) Explain how you could solve for the node voltage v_C after having calculated the mesh-currents.

Solution:

One way to obtain the node voltage v_C can be obtained by writing KVL around the first mesh

$$+20 + 10 \times i_a + v_C = 0 \qquad \text{(+1/2 point)}$$

from which

$$v_C = -20 - 10 \times i_a \qquad \text{(+1/2 point)}$$

NOTE TO GRADERS: Using the other mesh is another possibility.

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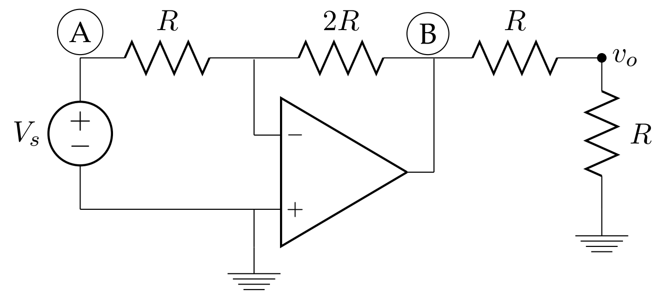


Figure 2: Circuit for Question 4

4. **OpAmp Circuit**

(a) (2 points) Calculate the voltage gain from V_s to v_o on the OpAmp circuit from Fig. 2.

Solution:
The voltage at the ouput of the OpAmp can be calculated as:

$$v_B = -\frac{2R}{R}V_s = -2V_s \qquad \text{(+1/2 point)}$$

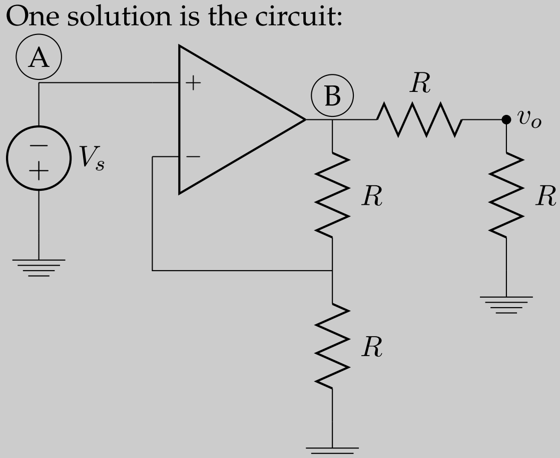
from which v_o can be calculated using voltage division:

$$v_o = \frac{R}{2R}V_B = -\frac{1}{2} \times 2V_s = -V_s \qquad \text{(+1/2 point)}$$

so the gain is $v_o/V_s = -1$. (+1 point)

(b) (3 points) Replace the inverting OpAmp circuit by a non-inverting OpAmp circuit connected between nodes A and B so that the voltage gain from V_s to v_0 has the same magnitude as you calculated in part (a).

Solution:
One solution is the circuit:



(+2 points)

for which the overall gain is:

$$\frac{v_o}{V_s} = \frac{1}{2} \times \frac{2R}{R} = \frac{1}{2} \times 2 = 1. \qquad \text{(+1 point)}$$