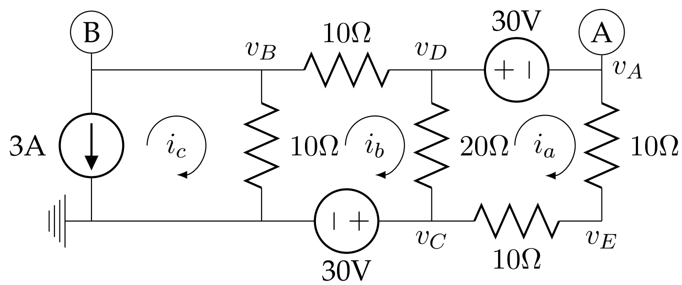
MAE140 - Linear Circuits - Fall 2018
Midterm, October 23rd

Instructions

1. This exam is open book. You may use whatever written materials you choose, including your class notes and textbook. You may use a hand calculator with no communication capabilities.
2. You have 70 minutes.
3. Write your answers on your "Blue Book".
4. Do not forget to write your name, student number, and instructor.
5. This exam has 3 questions, for a total of 30 points.



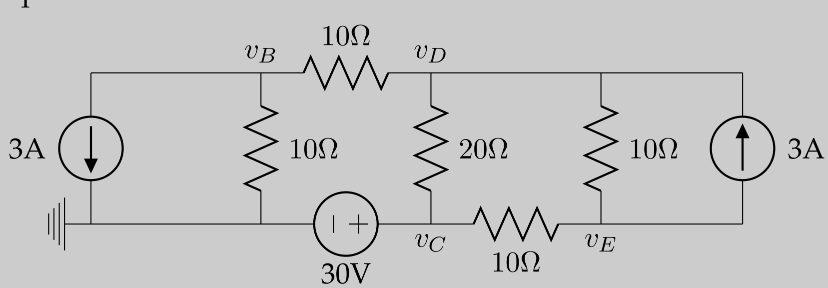
Hint:

$$\begin{bmatrix} 40 & -20 \\ -20 & 40 \end{bmatrix}^{-1} = \frac{1}{30}\begin{bmatrix} 1 & 1/2 \\ 1/2 & 1 \end{bmatrix}$$

1. **Node Voltage Analysis**

(a) (6 points) Apply a source transformation to eliminate v_A and formulate node-voltage equations for the circuit above. Use the node-voltages and labels provided in the figure and clearly indicate the final equations and circuit variable unknowns. Make sure your final equations involve only node-voltages. You do not need to solve any equations.

Solution:
After a source transformation is applied to the voltage source at node v_A we obtain the equivalent circuit:



(+1 point)

Using method #2 to handle the voltage source connected to node v_C we write the KCL equations at nodes B, D and E by inspection to obtain:

$$\begin{bmatrix} \frac{1}{10}+\frac{1}{10} & 0 & -\frac{1}{10} & 0 \\ -\frac{1}{10} & -\frac{1}{20} & \frac{1}{10}+\frac{1}{10}+\frac{1}{20} & -\frac{1}{10} \\ 0 & -\frac{1}{10} & -\frac{1}{10} & \frac{1}{10}+\frac{1}{10} \end{bmatrix} \begin{pmatrix} v_B \\ v_C \\ v_D \\ v_E \end{pmatrix} = \begin{pmatrix} -3 \\ 3 \\ -3 \end{pmatrix}$$

(+3 points)

plus the fact that

$$v_C = 30\text{V} \qquad \text{(+1 point)}$$

These are the node-voltage equations that need to be solved for the node voltages v_B, v_C, v_D, and v_E. (+1 point)

(b) (1 point) Explain how you could solve for the node voltage v_A.

Solution:
From the circuit diagram:

$$v_A = v_D - 30\text{V} \qquad \text{(+1 point)}$$

NOTE TO GRADERS: Multiple solutions are possible!

(c) (3 points) Explain how you could solve for the mesh-currents i_a, i_b, and i_c after having calculated the node-voltages.

Solution:
From the circuit diagram:

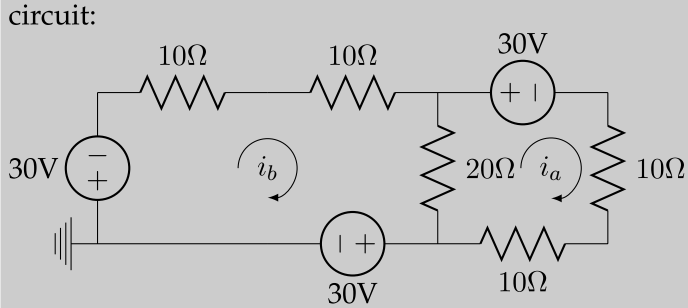
$$i_a = \frac{v_E - v_C}{10} \qquad \text{(+1 point)}$$
$$i_b = \frac{v_B - v_D}{10} \qquad \text{(+1 point)}$$
$$i_c = -3\text{A} \qquad \text{(+1 point)}$$

NOTE TO GRADERS: Multiple solutions are possible!

2. **Mesh Current Analysis**

(a) (4 points) Apply a source transformation to eliminate i_c and formulate mesh-current equations for the circuit above. Use the mesh-currents and labels provided in the figure and clearly indicate the final equations and circuit variable unknowns. Make sure your final equations involve only mesh-currents. You do not need to solve any equations.

Solution:
After a source transformation is applied to the current source we obtain the equivalent circuit:



(+1 point)

We write mesh-current equations by inspection to obtain:

$$\begin{bmatrix} 10+10+20 & -20 \\ -20 & 10+10+20 \end{bmatrix} \begin{pmatrix} i_a \\ i_b \end{pmatrix} = \begin{pmatrix} -30 \\ -30-30 \end{pmatrix} \qquad \text{(+2 points)}$$

which must be solved for i_a and i_b. (+1 point)

(b) (1 point) Explain how you could solve for the mesh current i_c.

Solution:
From the circuit diagram:

$$i_c = -3\text{A} \qquad \text{(+1 point)}$$

NOTE TO GRADERS: Multiple solutions are possible!

(c) (5 points) Explain how you could solve for the node voltages v_A through v_E after having calculated the mesh-currents.

Solution:
From the circuit diagram:

$$\begin{aligned} v_A &= 30 + 20(i_b - i_a) - 30 = 20(i_b - i_a) && \text{(+1 point)} \\ v_B &= -30 - 10i_b = 10(i_c - i_b) && \text{(+1 point)} \\ v_C &= 30\text{V} && \text{(+1 point)} \\ v_D &= 30 + 20(i_b - i_a) && \text{(+1 point)} \\ v_E &= 30 + 10i_a && \text{(+1 point)} \end{aligned}$$

NOTE TO GRADERS: Multiple solutions are possible!

3. **Equivalent Circuits**

Answer the following questions to calculate the equivalent as seen from terminals A and B.

(a) (4 points) Solve the mesh-current equations from Question 2 to show that $v_A = -10\text{V}$ and $v_B = -5\text{V}$. *Hint: use the hint!*

Solution:
We solve for i_a and i_b:

$$\begin{pmatrix} i_a \\ i_b \end{pmatrix} = \begin{bmatrix} 10+10+20 & -20 \\ -20 & 10+10+20 \end{bmatrix}^{-1} \begin{pmatrix} -30 \\ -60 \end{pmatrix} = \frac{1}{30} \begin{bmatrix} 1 & 1/2 \\ 1/2 & 1 \end{bmatrix} \begin{pmatrix} -30 \\ -60 \end{pmatrix} = \begin{pmatrix} -2 \\ -\frac{5}{2} \end{pmatrix}$$

(+2 points)

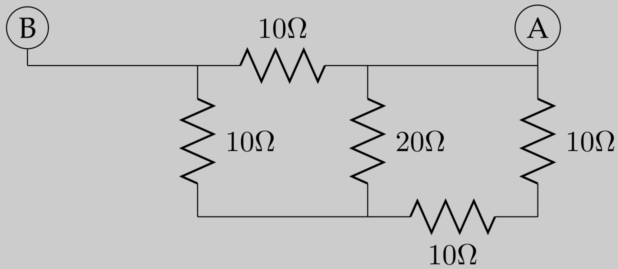
from which

$$\begin{aligned} v_A &= 20\frac{-5+4}{2} = -10\text{V} && \text{(+1 point)} \\ v_B &= -30 - 10i_b = -30 - 10\frac{-5}{2} = -5\text{V} && \text{(+1 point)} \end{aligned}$$

(b) (3 points) Set all sources to zero and calculate the equivalent resistance as seen from terminals (A) and (B).

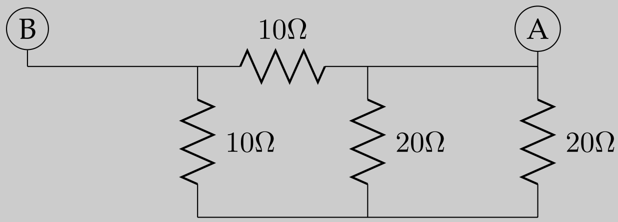
Solution:

After setting all sources to zero we obtain:



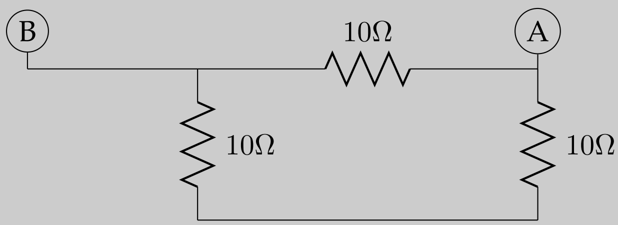
(+1 point)

Associating the 10Ω resistors in series:



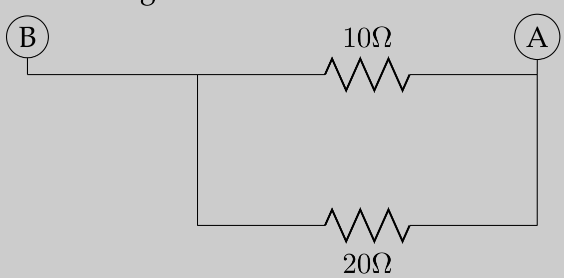
(+1/2 point)

Associating the 20Ω resistors in parallel:



(+1/2 point)

Associating the 10Ω resistors in series:



(+1/2 point)

Associating the 20Ω and 10Ω resistors in parallel:

(+1/2 point)

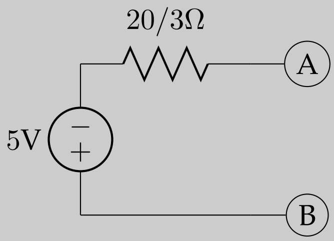
(c) (3 points) Use the answers to Parts (a) and (b) to calculate the Thevenin and Norton equivalent circuits as seen from terminals (A) and (B).

Solution:

From parts (a) and (b)

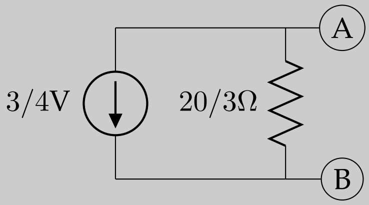
$$v_T = v_A - v_B = -10 - (-5) = -5\text{V}, \quad R = \frac{20}{3}\Omega, \quad i_N = \frac{v_T}{R} = \frac{-5}{20/3} = \frac{-3}{4}\text{A} \quad \text{(+1 point)}$$

which is associated with the Thevenin equivalent circuit:



(+1 point)

and the corresponding Norton equivalent circuit:



(+1 point)

NOTE TO GRADERS: Make sure the polarities/sign are marked correctly regarding the nodes A and B!