MAE140 - Linear Circuits - Fall 2018
Midterm, November 13th

Instructions

1. This exam is open book. You may use whatever written materials you choose, including your class notes and textbook. You may use a hand calculator with no communication capabilities.
2. You have 70 minutes.
3. Write your answers on your "Blue Book".
4. Do not forget to write your name, student number, and instructor.
5. This exam has 3 questions, for a total of 27 points.

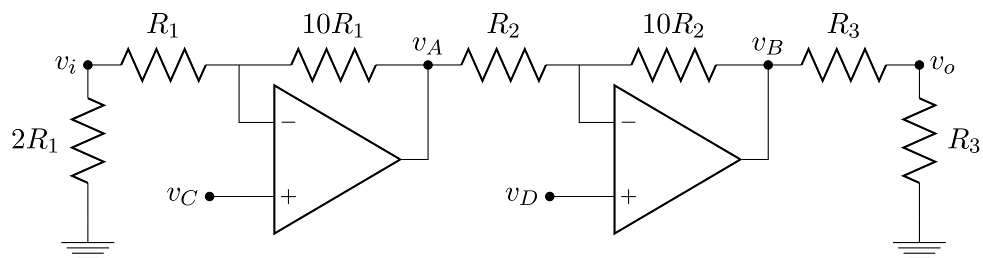


Figure 1: Circuit for Questions 1 and 2

1. **Gain Analysis**

   Connect v_C and v_D to the ground in the circuit in Fig. 1 and answer the following questions:

   (a) (3 points) Calculate the gains: v_i to v_A, v_A to v_B, v_B to v_o.

   Solution:
   Because the first opAmp is a standard inverting opAmp circuit:

   $$\frac{v_A}{v_i} = -\frac{10R_1}{R_1} = -10. \qquad \text{(+1 point)}$$

   Because second opAmp is a standard inverting opAmp circuit:

   $$\frac{v_B}{v_A} = -\frac{10R_2}{R_2} = -10. \qquad \text{(+1 point)}$$

   From voltage division

   $$\frac{v_o}{v_B} = \frac{R_3}{R_3 + R_3} = \frac{1}{2}. \qquad \text{(+1 point)}$$

   (b) (1 point) Calculate the gain: v_i to v_o.

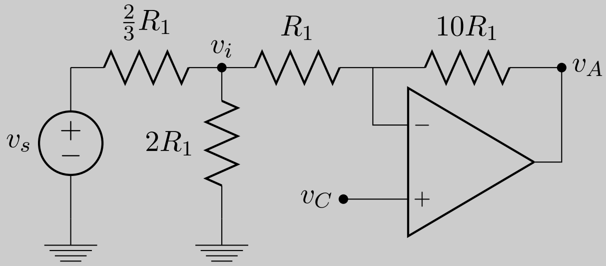
   Solution:
   Because there is no loading of any circuit

   $$\frac{v_o}{v_i} = \frac{v_o}{v_B}\frac{v_B}{v_A}\frac{v_A}{v_i} = (-10)(-10)\frac{1}{2} = 50 \qquad \text{(+1 point)}$$

(c) (4 points) Calculate the gain v_s to v_o if a circuit with Thevenin voltage v_s and equivalent resistance $2R_1/3$ is connected to the input v_i.

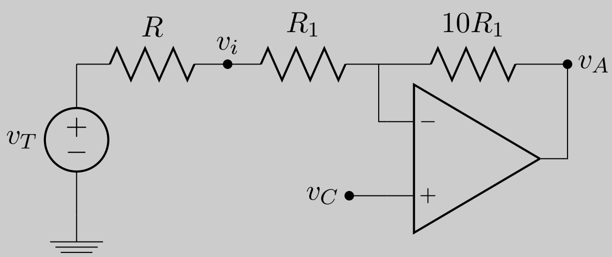
Solution:

It is enough to analyze the first stage:



(+1 point)

Replace the source, $2R_1/3$ and $2R_1$ by its Thevenin equivalent:



where

$$v_T = \frac{2R_1}{2R_1 + \frac{2}{3}R_1} v_s = \frac{3}{4} v_s, \qquad R = \frac{1}{\frac{1}{2R_1} + \frac{3}{2R_1}} = \frac{R_1}{2}$$

(+1 point)

so that

$$v_A = -\frac{10R_1}{\frac{R_1}{2} + R_1} v_T = -\frac{20}{3}\frac{3}{4} v_s = -5v_s$$

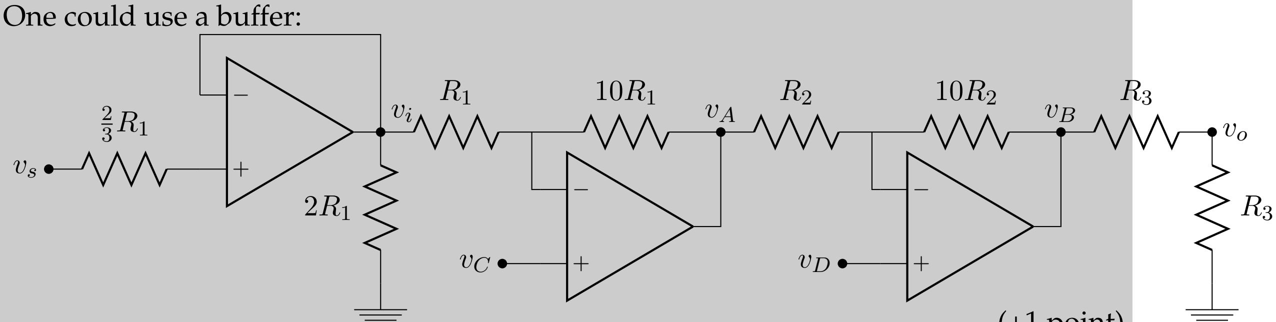
(+1 point)

which has half the gain of the original circuit so that $v_o/v_i = 50/2 = 25$. (+1 point)

(d) (1 point) If a circuit is to be connected to v_i as in part (c), show how you could add an opAmp circuit before connecting the circuit to v_i to make the gain from v_s to v_o equal to the gain from v_i to v_o.

Solution:

One could use a buffer:



(+1 point)

2. **Voltage Offset**

Let $v_C = v_D = 1\text{V}$ and answer the following questions regarding the opAmp circuit in Fig. 1:

(a) (3 points) Show that $v_A = 11 - 10v_i$, $v_B = 100v_i - 99$, $v_o = 50v_i - 49.5$.

Solution: Calculate using the substractor formulas or linearity:

$$v_A = \frac{11R}{R}v_C - \frac{10R}{R}v_i = 11 - 10v_i, \qquad \text{(+1 point)}$$

$$v_B = \frac{11R}{R}v_D - \frac{10R}{R}v_A = 100v_i - 99, \qquad \text{(+1 point)}$$

$$v_o = \frac{R}{2R}v_B = 50v_i - \frac{99}{2} \qquad \text{(+1 point)}$$

(b) (3 points) Let the signal $v_i = 1 + v_r$ in which $-0.01\text{V} \le v_r \le 0.01\text{V}$. Calculate the corresponding range for the signal v_o and the the gain from v_r to v_o.

Solution:
Calculate

$$v_o = 50(1 + v_r) - 49.5 = 50v_r + 0.5 \qquad \text{(+1 point)}$$

from which the gain is 50 and (+1 point)

$$0 \le v_o = 50(1 + v_r) - 49.5 = 50v_r + 0.5 \le 1 \qquad \text{(+1 point)}$$

(c) (2 points) Calculate the largest possible value of $-v_{\text{CC}}$ and the smallest possible value of $+v_{\text{CC}}$ so that the voltages v_A and v_B do not saturate for a signal $v_i = 1 + v_r$ as in part (b).

Solution:
Because $-0.01 \le v_r \le 0.01$

$$0.9 \le v_A = 11 - 10(1 + v_r) = 1 - 10v_r \le 1.1$$

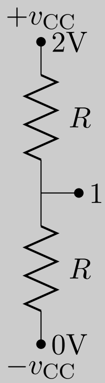
and v_A does not saturates if $+v_{\text{CC}} \ge 1.1$ and $-v_{\text{CC}} \le 0.9$. (+1/2 point)
Likewise

$$0 \le v_B = 100(1 + v_r) - 99 = 1 + 100v_r \le 2$$

and v_B not to saturate if $+v_{\text{CC}} \ge 2$ and $-v_{\text{CC}} \le 0$. (+1/2 point)
Therefore $+v_{\text{CC}} = 2$ and $-v_{\text{CC}} = 0$ are the largest and smallest possible. (+1 point)

(d) (1 point) Let $+v_{\text{CC}} = 2\text{V}$ and $-v_{\text{CC}} = 0\text{V}$ and design an additional circuit with access only to theses voltages that when connected to the circuit in Fig. 1 is such that $v_C = v_D = 1\text{V}$.

Solution:
A simple voltage-divider with equal resistors suffices:



because v_C and v_D are directly connected to the $+$ terminals of the opAmps. (+1 point)

3. **OpAmp Design**

   Design an opAmp circuit that implements the following operations $v_o = 2(v_1 - v_2) + v_3$

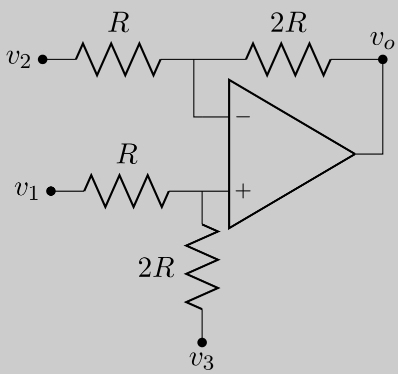
   (a) (3 points) Using any number of opAmps.

   (b) (3 points) Using at most two opAmps. *Not necessary if part (a) has two opAmps or less*

   (c) (3 points) Using at most one opAmp. *Not necessary if part (b) has one opAmp*

Solution:

Here is one solution with 1 opAmp:



Using linearity

$$v_o = -2v_2 + 3\frac{2}{3}v_1 + 3\frac{1}{3}v_3 = 2(v_1 - v_2) + v_3.$$

NOTE TO GRADERS: Any correct solution with more than 2 opAmps earns 3 points.
NOTE TO GRADERS: Any correct solution with 2 opAmps earns 6 points.
NOTE TO GRADERS: Any correct solution with 1 opAmp earns 9 points.
NOTE TO GRADERS: If a solution is correct but uses inverted sources or signals, discount 1 point per inverted source.