

s sense physically. When the gain is high enough, the active circuit can produce the energy needed to sustain an unbounded output. If the gain is almost always undesirable, we usually state the conclusion another way around. That is, this active RC circuit is stable provided $\beta < 3$. It is common for active circuits to be stable when circuit parameters are in one range and unstable when they are outside this range. For multi-pole circuits the stable range can be found by relating the pole locations to circuit parameters. For single-pole circuits the stable range is that the pole lies on the negative real axis.

The laws apply to voltage and current waveforms in the time domain and to the corresponding transforms in the s domain.

Main models for the passive elements include initial conditions and the element impedance or admittance. Impedance is the proportionality factor in the expression $V(s) = Z(s)I(s)$ relating the voltage and current transforms. Admittance is the reciprocal of impedance.

Impedances of the three passive elements are $Z_R(s) = R$, $Z_L(s) = Ls$, and $Z_C(s) = 1/Cs$.

Main circuit analysis techniques closely parallel the analysis techniques developed for resistance circuits. Basic analysis techniques, such as circuit reduction, Thévenin's and Norton's theorems, the unit step method, or superposition, can be used in simple circuits. More complex networks require a general approach, such as the node-voltage or mesh-current methods.

Transforms are rational functions whose poles are zeros of the denominator or poles of the transform of the input driving forces. Poles introduced by the circuit determinant are called natural poles and poles introduced by the input are called forced poles and lead to the forced response.

In circuits, response transforms and waveforms can be separated into steady-state and zero-input components. The zero-state component is obtained by setting the initial capacitor voltages and inductor currents to zero. The zero-input component is found by setting all input driving forces to zero.

The purpose of s -domain circuit analysis is to gain insight into circuit behavior without necessarily finding the time-domain response. The poles reveal the form, stability, and observability of the circuit response. The number of natural poles is never greater than the number of energy storage elements in the circuit.

A circuit is stable if all of its natural poles are in the left half of the s -plane.

PROBLEMS

OBJECTIVE 10-1 EQUIVALENT IMPEDANCE (SECTS. 10-1, 10-2)

Given a linear circuit, use series and parallel equivalences to find the poles and zeros of the equivalent impedance at specified terminal pairs. Select element values to obtain specified pole locations.

See Examples 10-2, 10-4, 10-5, 10-6 and Exercise 10-1, 10-2

10-1 Find $Z_{EQ}(s)$ in Figure P10-1. Express $Z_{EQ}(s)$ as a rational function and locate its poles and zeros.

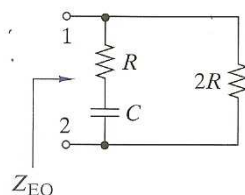


FIGURE P10-1

10-2 Find $Z_{EQ}(s)$ in Figure P10-2. Express $Z_{EQ}(s)$ as a rational function and locate its poles and zeros.

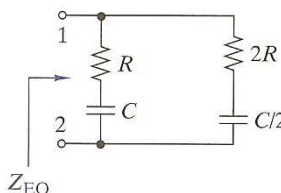


FIGURE P10-2

10-3 Find $Z_{EQ}(s)$ in Figure P10-3. Express $Z_{EQ}(s)$ as a rational function and locate its poles and zeros for $R = 4 \text{ k}\Omega$, $L = 0.4 \text{ H}$, and $C = 100 \text{ nF}$.

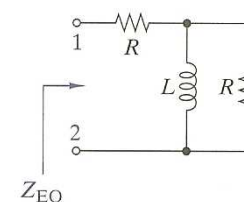
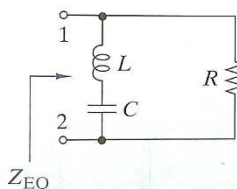


FIGURE P10-4

10-5 Find $Z_{EQ}(s)$ in Figure P10-5. Express $Z_{EQ}(s)$ as a rational function and locate its poles and zeros.

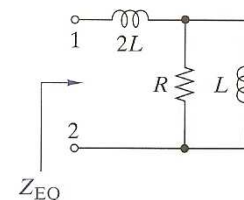


FIGURE P10-5

10-6 Find the poles and zeros of Z_{EQ} in Figure P10-6 for $R = 1 \text{ k}\Omega$, $L = 1 \text{ H}$, and $C = 500 \text{ nF}$.

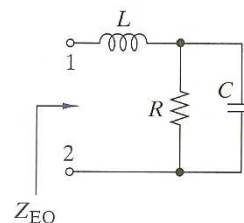
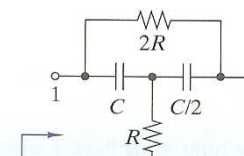


FIGURE P10-6

10-7 Find $Z_{EQ}(s)$ in Figure P10-7. Express $Z_{EQ}(s)$ as a rational function and locate its poles and zeros.



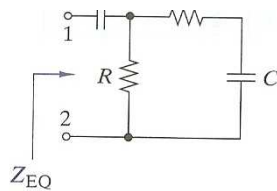


FIGURE P10-8

10-9 Find $Z_{EQ}(s)$ in Figure P10-9. Express $Z_{EQ}(s)$ as a rational function and locate its poles and zeros.

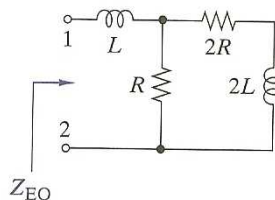


FIGURE P10-9

10-10 Find $Z_{EQ}(s)$ in Figure P10-10. Select values of R and L so that $Z_{EQ}(s)$ has a pole at $s = -1000$ rad/s. Locate the zeros of $Z_{EQ}(s)$ for your choice of R and L .

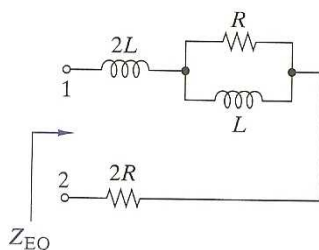


FIGURE P10-10

OBJECTIVE 10-2 BASIC CIRCUIT ANALYSIS TECHNIQUES (SECTS. 10-2, 10-3)

Given a linear circuit:

- Determine the initial conditions (if not given) and transform the circuit into the s domain.
- Solve for zero-state and/or zero-input response transforms and waveforms using basic analysis methods such as circuit reduction, unit output, Thévenin/Norton equivalent circuits, or superposition.
- Locate the forced and natural poles or select circuit parameters to place the natural poles at specified locations. See Examples 10-2, 10-3, 10-4, 10-5, 10-6, 10-7, 10-8, 10-9

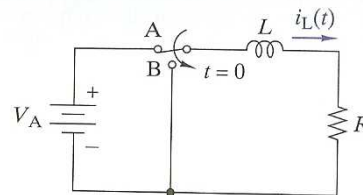


FIGURE P10-11

10-12 The switch in Figure P10-11 has been in position B for a long time and is moved to position A at $t = 0$. Transform the circuit into the s domain and solve for $I_L(s)$ and $i_L(t)$ in symbolic form.

10-13 The switch in Figure P10-13 has been in position A for a long time and is moved to position B at $t = 0$. Transform the circuit into the s domain and solve for $I_C(s)$ and $i_C(t)$ in symbolic form.

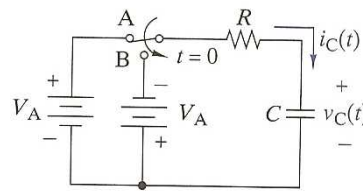


FIGURE P10-13

10-14 The switch in Figure P10-13 has been in position B for a long time and is moved to position A at $t = 0$. Transform the circuit into the s domain and solve for $V_C(s)$ and $v_C(t)$ in symbolic form.

10-15 There is no initial energy stored in the circuit in Figure P10-15. Transform the circuit into the s domain and find $I_L(s)$ and $i_L(t)$ when $v_1(t) = V_A u(t)$.

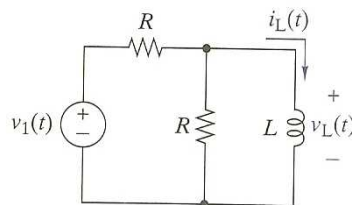


FIGURE P10-15

for a long time and is moved to position B at $t = 0$.

- Transform the circuit into the s domain and solve for $I_L(s)$ in symbolic form.
- Find $i_L(t)$ for $R_1 = R_2 = 500 \Omega$, $L = 250$ mH, $C = 4 \mu\text{F}$, and $V_A = 15$ V.

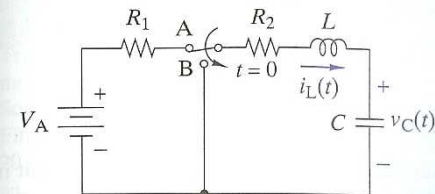


FIGURE P10-17

10-18 The switch in Figure P10-17 has been in position B for a long time and is moved to position A at $t = 0$.

- Transform the circuit into the s domain and solve for $V_C(s)$ in symbolic form.
- Find $v_C(t)$ for $R_1 = R_2 = 500 \Omega$, $L = 500$ mH, $C = 1 \mu\text{F}$, and $V_A = 15$ V.

10-19 The circuit in Figure P10-19 is in the zero state. Find the s -domain relationship between the input $I_1(s)$ and the output $I_R(s)$.

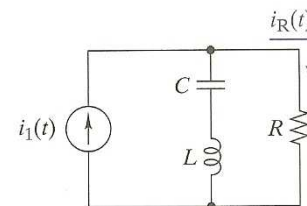


FIGURE P10-19

10-20 The circuit in Figure P10-20 is in the zero state. Find the s -domain relationship between the input $I_1(s)$ and the output $V_O(s)$.

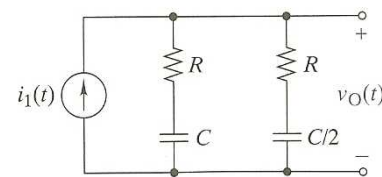


FIGURE P10-20

- 10-11 The switch in Figure P10-11 has been in position A for a long time and is moved to position B at $t = 0$. Transform the circuit into the s domain and solve for $i_L(s)$ and $i_L(t)$ in symbolic form.

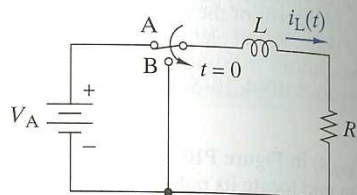


FIGURE P10-11

- 10-12 The switch in Figure P10-11 has been in position B for a long time and is moved to position A at $t = 0$. Transform the circuit into the s domain and solve for $i_L(s)$ and $i_L(t)$ in symbolic form.

- 10-13 The switch in Figure P10-13 has been in position A for a long time and is moved to position B at $t = 0$. Transform the circuit into the s domain and solve for $i_C(s)$ and $i_C(t)$ in symbolic form.

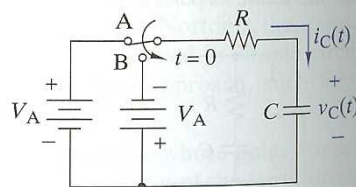
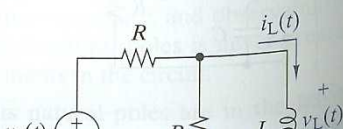


FIGURE P10-13

- 10-14 The switch in Figure P10-13 has been in position B for a long time and is moved to position A at $t = 0$. Transform the circuit into the s domain and solve for $v_C(s)$ and $v_C(t)$ in symbolic form.

- 10-15 There is no initial energy stored in the circuit in Figure P10-15. Transform the circuit into the s domain and find $i_L(s)$ and $i_L(t)$ when $v_1(t) = V_A u(t)$.



PROBLEMS

- 10-16 There is no initial energy stored in the circuit in Figure P10-15. Transform the circuit into the s domain and find $V_L(s)$ and $v_L(t)$ when $v_1(t) = V_A u(t)$.

- 10-17 The switch in Figure P10-17 has been in position A for a long time and is moved to position B at $t = 0$.

- (a) Transform the circuit into the s domain and solve for $i_L(s)$ in symbolic form.

- (b) Find $i_L(t)$ for $R_1 = R_2 = 500 \Omega$, $L = 250 \text{ mH}$, $C = 4 \mu\text{F}$, and $V_A = 15 \text{ V}$.

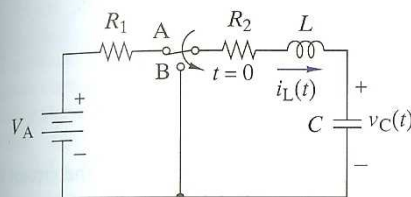


FIGURE P10-17

- 10-18 The switch in Figure P10-17 has been in position B for a long time and is moved to position A at $t = 0$.

- (a) Transform the circuit into the s domain and solve for $V_C(s)$ in symbolic form.

- (b) Find $v_C(t)$ for $R_1 = R_2 = 500 \Omega$, $L = 500 \text{ mH}$, $C = 1 \mu\text{F}$, and $V_A = 15 \text{ V}$.

- 10-19 The circuit in Figure P10-19 is in the zero state. Find the s -domain relationship between the input $I_1(s)$ and the output $I_R(s)$.

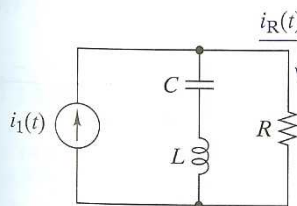
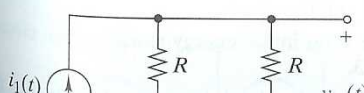


FIGURE P10-19

- 10-20 The circuit in Figure P10-20 is in the zero state. Find the s -domain relationship between the input $I_1(s)$ and the output $V_O(s)$.



- 10-21 The initial conditions in Figure P10-21 are $v_C(0) = V_0$ and $i_L(0) = 0$. Transform the circuit into the s domain and use superposition and voltage division to find the zero-state and zero-input components of $V_C(s)$.

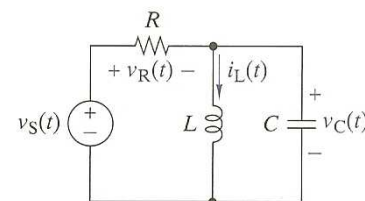


FIGURE P10-21

- 10-22 The initial conditions in Figure P10-21 are $v_C(0) = 0$ and $i_L(0) = I_0$. Transform the circuit into the s domain and use superposition and voltage division to find the zero-state and zero-input components of $V_R(s)$.

- 10-23 There is no energy stored in the capacitor in Figure P10-23 at $t = 0$. Transform the circuit into the s domain and use current division to find $v_O(t)$ when the input is $i_S(t) = 2.5 e^{-100t} u(t) \text{ mA}$. Identify the forced and natural poles in $V_O(s)$.

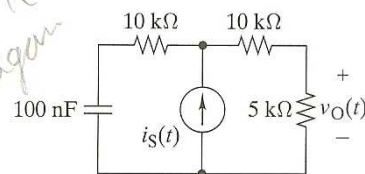
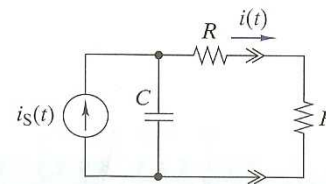


FIGURE P10-23

- 10-24 Repeat Problem 10-23 when $i_S(t) = 2.5 e^{-1000t} u(t) \text{ mA}$.

- 10-25 The circuit in Figure P10-25 is in the zero state. Use a Thévenin equivalent to find the s -domain relationship between the input $I_S(s)$ and the interface current $I(s)$.



Given a linear circuit:

- Determine the initial conditions (if not given) and transform the circuit into the s domain.
- Solve for zero-state and/or zero-input response transforms and waveforms using node-voltage or mesh-current methods.
- Identify the forced and natural poles or select circuit parameters to place the natural poles at specified locations. See Examples 10-10, 10-11, 10-13, 10-14, 10-15, 10-16, 10-17

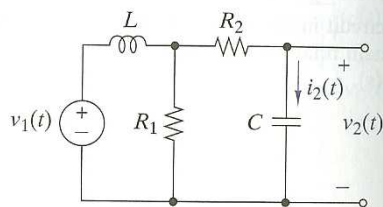


FIGURE P10-31

10-32 There is no initial energy stored in the circuit in Figure P10-31.

- Transform the circuit into the s domain and formulate node-voltage equations.
- Solve these equations for $V_2(s)$ in symbolic form.
- Find $v_2(t)$ for $v_1(t) = 10u(t)$ V, $R_1 = R_2 = 500 \Omega$, $L = 0.5$ H, and $C = 2 \mu\text{F}$.

10-33 There is no initial energy stored in the circuit in Figure P10-33.

- Transform the circuit into the s domain and formulate node-voltage equations.
- Solve these equations for $V_2(s)$ in symbolic form.
- Find $v_2(t)$ for $v_1(t) = 30u(t)$ V, $R_1 = 10 \text{ k}\Omega$, $R_2 = 20 \text{ k}\Omega$, $C_1 = 1 \mu\text{F}$, and $C_2 = 0.5 \mu\text{F}$.

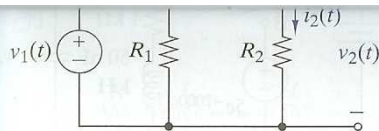


FIGURE P10-33

10-34 There is no initial energy stored in the circuit in Figure P10-33.

- Transform the circuit into the s domain and formulate mesh-current equations.
- Solve these equations for $I_2(s)$ in symbolic form.
- Find $i_2(t)$ for $v_1(t) = 4000r(t)$ V, $R_1 = 5 \text{ k}\Omega$, $R_2 = 20 \text{ k}\Omega$, $C_1 = 5 \mu\text{F}$, and $C_2 = C_1/3$.

10-35 There is no initial energy stored in the bridged-T circuit in Figure P10-35.

- Transform the circuit into the s domain and formulate mesh-current equations.
- Use the mesh-current equations to find the s -domain relationship between the input $V_1(s)$ and the output $V_2(s)$.

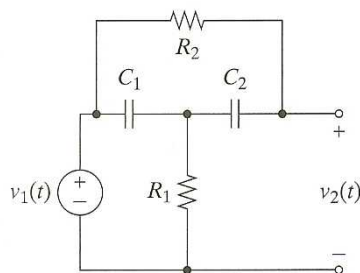


FIGURE P10-35

10-36 There is no initial energy stored in the bridged-T circuit in Figure P10-35.

- Transform the circuit into the s domain and formulate node-voltage equations.
- Use the node-voltage equations to find the s -domain relationship between the input $V_1(s)$ and the output $V_2(s)$.

10-37 There is no initial energy stored in the circuit shown in Figure P10-37. Find the zero-state mesh currents $i_A(t)$ and $i_B(t)$ when $v_1(t) = 100u(t)$ V.

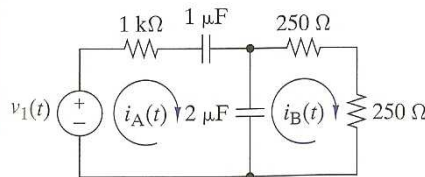


FIGURE P10-37

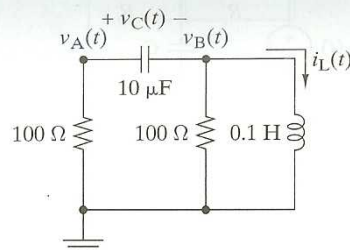


FIGURE P10-38

10-39 **D** The circuit in Figure P10-39 is in the zero state. Use node-voltage equations to find the circuit determinant. Select values of R , C , and μ so that the circuit has $\omega_0 = 5 \text{ krad/s}$ and $\zeta = 1/\sqrt{2}$.

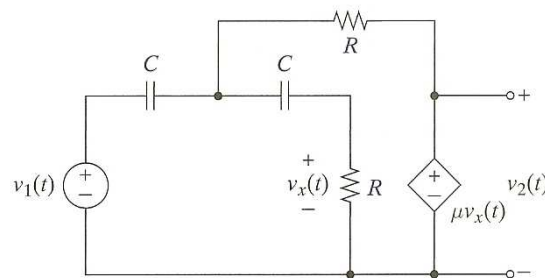


FIGURE P10-39

10-40 **D** The circuit in Figure P10-40 is in the zero state. Use mesh-current equations to find the circuit determinant. Select values of R , L , and C so that the circuit has $\omega_0 = 5 \text{ krad/s}$ and $\zeta = 1/\sqrt{2}$.

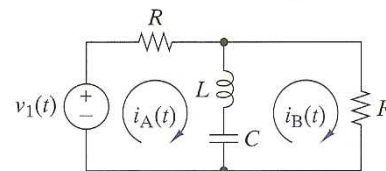


FIGURE P10-40

10-41 **D** The circuit in Figure P10-41 is in the zero state. Use node-voltage equations to find the circuit determinant. Select values of R , L , and C so that the circuit has $\omega_0 = 5 \text{ krad/s}$ and $\zeta = 1/\sqrt{2}$.

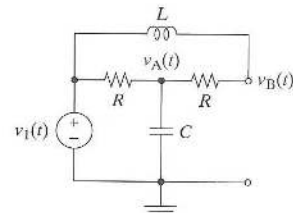


FIGURE P10-41

- 10-42 Three node voltages are shown in Figure P10-42.
- Explain why only one of the node voltages is independent.
 - Write a node-voltage equation in the independent node voltage.

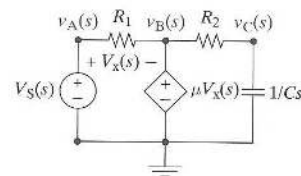


FIGURE P10-42

- 10-43 Three mesh currents are shown in Figure P10-43.
- Explain why only two of these mesh currents are independent.
 - Write s -domain mesh-current equations in two independent mesh currents.

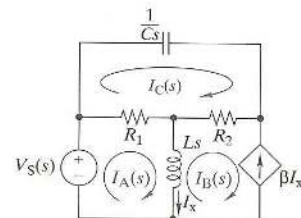


FIGURE P10-43

- 10-44 The switch in Figure P10-44 has been in position A for a long time and is moved to position B at $t = 0$. Solve for $V_C(s)$ and $v_C(t)$.

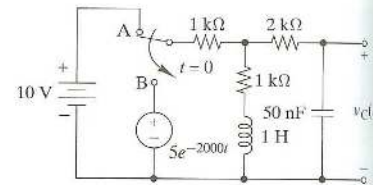


FIGURE P10-44

- 10-45 There is no energy stored in the circuit in Figure P10-45 at $t = 0$. Transform the circuit into the s domain and solve for $V_O(s)$ and $v_O(t)$.

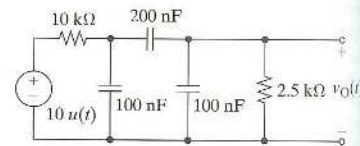


FIGURE P10-45

- 10-46 The switch in Figure P10-46 has been open for a long time and is closed at $t = 0$. Transform the circuit into the s domain and solve for $I_O(s)$ and $i_O(t)$.

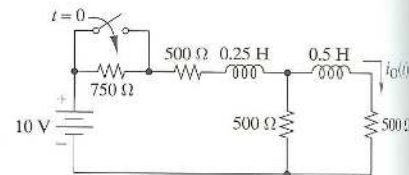


FIGURE P10-46

- 10-47 There is no initial energy stored in the circuit in Figure P10-47.

- Transform the circuit into the s domain and solve for $V_O(s)$ in symbolic form.
- Find $v_O(t)$ when $v_S(t) = 10^4 u(t)$ V, $R_1 = R_2 = 2 \text{ k}\Omega$, $C = 0.5 \text{ }\mu\text{F}$, and $L = 2 \text{ H}$.

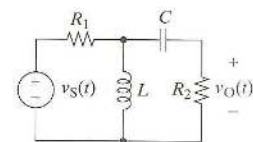


FIGURE P10-47

- 10-48 Repeat Problem 10-49. Show poles at $L = R^2 C$.

$v_S(t)$

F I

- 10-50 Find Figure P10-50.

V

f

INTEGRATING

- 10-51 A black switch and terminals are shown in Figure P10-51.

When a terminal is altered to the other terminal, the circuit is altered.

- What is the load in order to the load?

- When the switch is moved to the other terminal, the load is altered.
 - How does the load affect the circuit?
- Hint: Examine the circuit in Figure P10-51.