

3.71) Firstly, find our ranges

$$R_{in}: [588, 612]$$

$$R_{out}: [3.92, 4.08]$$

$$[7.84, 8.16]$$

$$[15.68, 16.32]$$

Let's check R_{in} and R_{out} for '16." Note voltage source does not affect resistance so we ignore it for the analysis

R_{out}



600 and 592 are series

8, 4, 4 are series

They are parallel to each other

$$\text{so } R_{out} = \left(\frac{1}{600 + 592} + \frac{1}{8 + 4 + 4} \right)^{-1}$$

$$= 15.7881 \quad \checkmark \text{ (in the range)}$$

3.71) cont.)

~~R_{in}~~

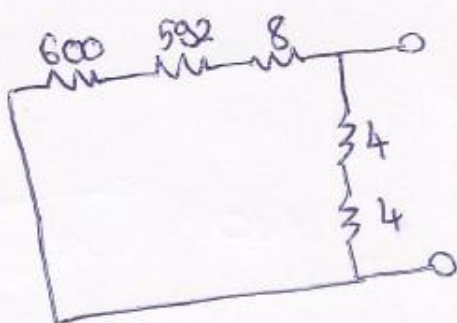


$$R_{in} = 592 + \left(\frac{1}{8+4+8} + \frac{1}{16} \right)^{-1}$$

$$= 600 \Omega \checkmark$$

Now for '8'

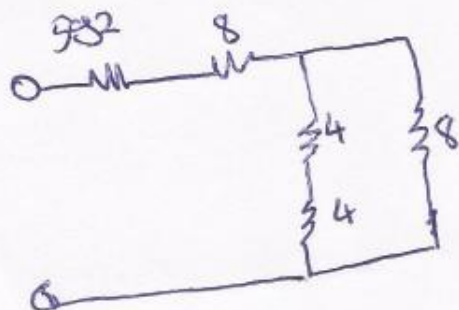
~~R_{in}~~



$$R_{in} = \left(\frac{1}{600+592+8} + \frac{1}{4+4} \right)^{-1}$$

$$= 7.9470 \checkmark$$

~~R_{in}~~



$$R_{in} = 592 + 8 + \left(\frac{1}{4+4} + \frac{1}{8} \right)^{-1}$$

$$= 604 \Omega \checkmark$$

3.71) cont.
for '4'

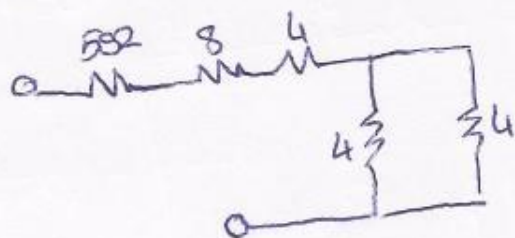
R_{out}



$$R_{out} = \left(\frac{1}{600 + 592 + 8 + 4} + \frac{1}{4} \right)^{-1}$$

$$= 3.9868 \Omega$$

R_{in}

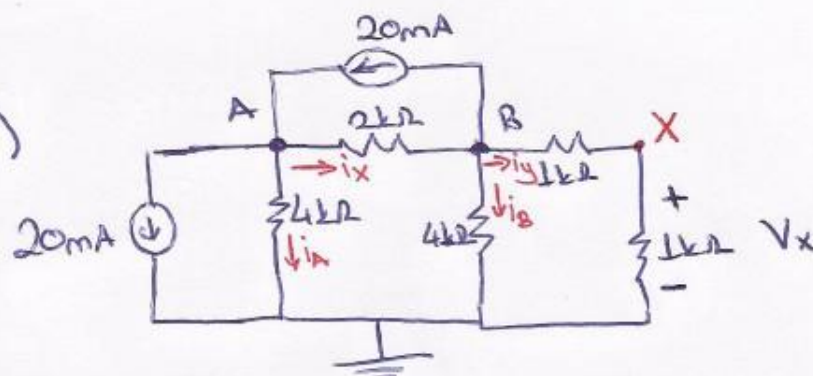


$$R_{in} = 592 + 8 + 4 + \left(\frac{1}{4} + \frac{1}{4} \right)^{-1}$$

$$= 606 \Omega$$

Yes! Their claim is correct.

3.3)



a) Node A, KCL

$$20\text{mA} - 20\text{mA} - i_X - i_A = 0$$

Node B, KCL

$$i_X - i_Y - i_B = 20\text{mA} = 0$$

$$i_A = \frac{V_A}{4\text{k}\Omega} \quad i_B = \frac{V_B}{4\text{k}\Omega} \quad , \quad i_Y = \frac{V_B}{(2\text{k}\Omega)}$$

$$i_X = \frac{V_A - V_B}{2\text{k}\Omega}$$

↳ to get rid of V_X (we will find it at part c), total ^{of} series resistance is used ($1\text{k}\Omega + 1\text{k}\Omega$)

Let's substitute currents into the equation we found by KCL

$$3V_A = 2V_B \rightarrow \text{from node A}$$

$$2V_A - 5V_B = 80 \rightarrow \text{from node B}$$

3.3) cont.

b) Substituting $V_A = \frac{2}{3} V_B$ into second equation, we get

$$\cancel{V_A} - \frac{1}{3} V_B = 80 \Rightarrow \boxed{V_B = -21.81 \text{ V}}$$

$$V_A = \frac{2}{3} (-21.81)$$

$$\boxed{V_A = -14.54 \text{ V}}$$

c) To find i_x , we can use the equation we have already had at part a

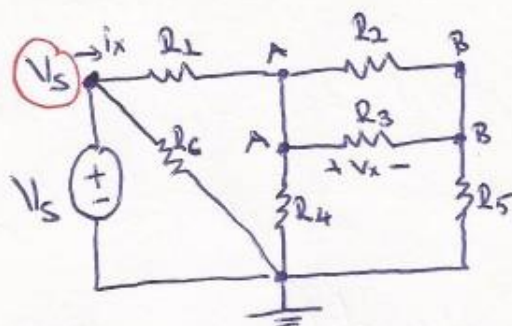
$$\boxed{i_x = \frac{V_A - V_B}{2 \text{ k}\Omega} = \frac{-14.54 + 21.81}{2 \text{ k}\Omega} = 3.635 \text{ mA}}$$

Using $V = IR$ formula, we get

$V_x = i_y \cdot (1 \text{ k}\Omega)$, substituting i_y

$$\boxed{V_x = \frac{V_B}{2 \text{ k}\Omega} 1 \text{ k}\Omega = -10.9 \text{ V}}$$

3.6)



a) KCL @ node A

$$\frac{V_s - V_A}{R_1} - \frac{V_A}{R_4} - \frac{V_A - V_B}{R_2} - \frac{V_A - V_B}{R_3} = 0$$

$$\Rightarrow V_A \left(-\frac{1}{R_1} - \frac{1}{R_2} - \frac{1}{R_3} - \frac{1}{R_4} \right) - V_B \left(-\frac{1}{R_2} - \frac{1}{R_3} \right) = -\frac{V_s}{R_1}$$

KCL @ node B

$$\frac{V_A - V_B}{R_2} + \frac{V_A - V_B}{R_3} - \frac{V_B}{R_5} = 0$$

$$\Rightarrow V_A \left(\frac{1}{R_2} + \frac{1}{R_3} \right) - V_B \left(\frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_5} \right) = 0$$

b)

$$i_x = \frac{V_s - V_A}{R_1}$$

$$V_x = V_A - V_B$$

3.6) b) cont.

$$R_1 = R_2 = R_3 = R_4 = R_5 = R_6 = 10\text{ k}\Omega$$

$$V_S = 24\text{ V}$$

Substituting values into the equations for nodes A and B,

$$-\frac{2}{5}V_A + \frac{1}{5}V_B = -\frac{24}{10}$$

$$\frac{1}{5}V_A - \frac{3}{10}V_B = 0$$

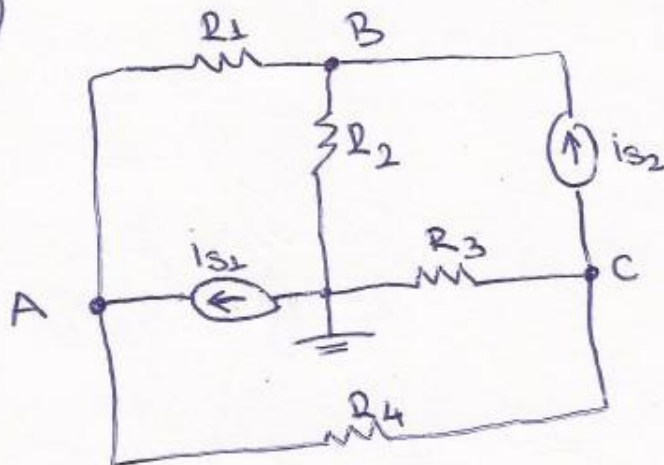
$$V_A = 9\text{ V}$$

$$V_B = 6\text{ V}$$

$$i_x = \frac{24 - 9}{10\text{ k}\Omega} = 1.5\text{ mA}$$

$$V_x = 9 - 6 = 3\text{ V}$$

3.7)



a) KCL @ node A

$$i_{s1} + \frac{V_B - V_A}{R_1} + \frac{V_C - V_A}{R_4} = 0$$

$$V_B \left(\frac{1}{R_1} \right) + V_C \left(\frac{1}{R_4} \right) - V_A \left(\frac{1}{R_1} + \frac{1}{R_4} \right) = -i_{s1}$$

KCL @ node B

$$i_{s2} - \frac{V_B}{R_2} - \frac{V_B - V_A}{R_1} = 0$$

$$V_A \left(\frac{1}{R_1} \right) - V_B \left(\frac{1}{R_1} + \frac{1}{R_2} \right) = -i_{s2}$$

3.7) a) cont.

KCL @ node C

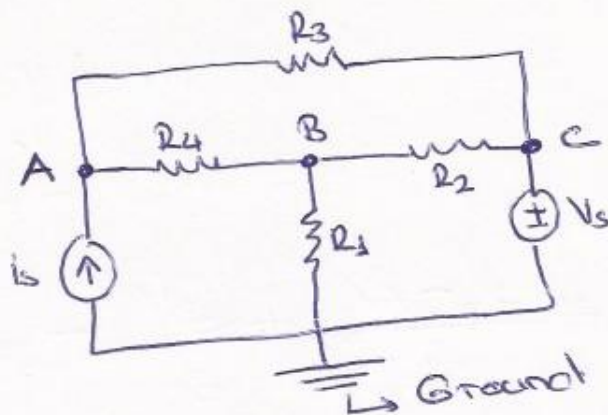
$$-i_{s2} - \frac{V_C}{R_3} + \frac{V_A - V_C}{R_4} = 0$$

$$V_A \left(\frac{1}{R_4} \right) - V_C \left(\frac{1}{R_3} + \frac{1}{R_4} \right) = i_{s2}$$

c) Solving three equation for three unknowns with given values, we get

$$V_A = 5.98, \quad V_B = 5.34, \quad V_C = -1.41$$

3.5)



a) Nodal Analysis

KCL @ node A

$$i_s - \frac{V_A - V_B}{R_4} - \frac{V_A - V_C}{R_3} = 0$$

$$V_A \left(-\frac{1}{R_4} - \frac{1}{R_3} \right) + V_B \left(\frac{1}{R_4} \right) + V_C \left(\frac{1}{R_3} \right) = -i_s$$

KCL @ node B

$$\frac{V_A - V_B}{R_4} - V_B \left(\frac{1}{R_1} \right) - \frac{V_B - V_C}{R_2} = 0$$

$$V_A \left(\frac{1}{R_4} \right) - V_B \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_4} \right) + V_C \left(\frac{1}{R_2} \right) = 0$$

3.5) a) cont.

Since we chose reference as shown on the figure, V_c can be easily found as V_s .

$$\boxed{V_c = V_s}$$

Then we have

two equations and two unknowns. Substituting the values which are given at part b, we get



$$-2V_A + V_B = -21$$

$$V_A - 3V_B = -20$$

$$V_B = \frac{61}{5}$$

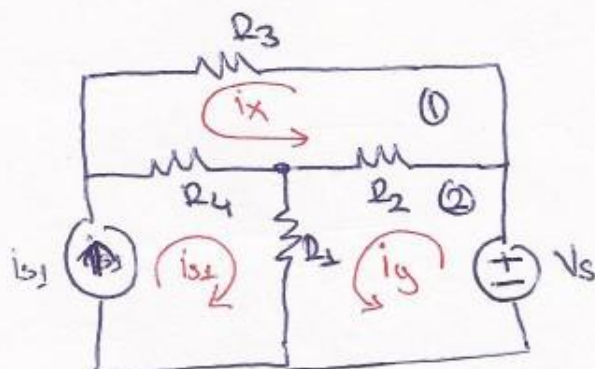
$$V_A = \frac{83}{5}$$

$$V_x = V_B - V_s = \frac{61}{5} - 20 = -\frac{39}{5} \text{ V}$$

$$i_x = \frac{V_s - V_A}{R_3} = \left(20 - \frac{83}{5} \right) \text{ mA} = \frac{17}{5} \text{ mA}$$

3.5) Mesh Analysis

b)



KVL @ mesh 1

$$R_3 i_x + R_4 (i_x + i_{s1}) + R_2 (i_x - i_y) = 0$$

$$\Rightarrow i_x (R_3 + R_4 + R_2) - i_y (R_2) = -i_{s1} (R_4)$$

KVL @ mesh 2

$$R_2 (i_y - i_x) + R_1 (i_y + i_{s1}) - V_s = 0$$

$$\Rightarrow -i_x (R_2) + i_y (R_1 + R_2) = V_s - R_1 i_{s1}$$

3.5) b) cont. Substituting the given values

$$3i_x - i_y = -1$$

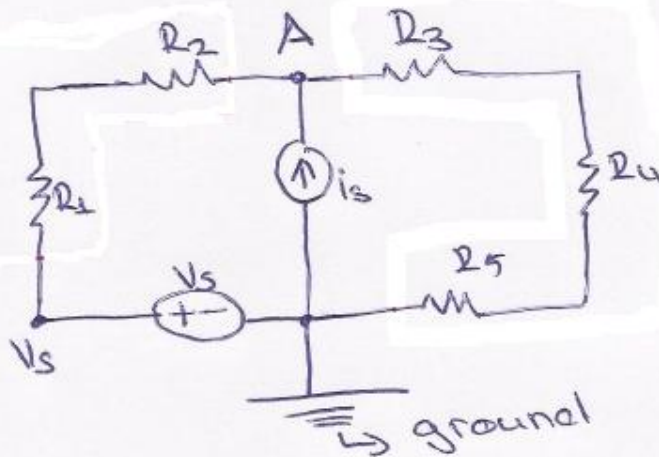
$$-i_x + 2i_y = 18$$

$$i_x = \frac{17}{5} \text{ mA}$$

$$i_y = \frac{112}{10} \text{ mA}$$

$$V_x = (i_x - i_y) R_2 = \left(\frac{34 - 112}{10} \right) 1 = -\frac{39}{5} \text{ V}$$

3.12)



a-b)

Nodal Analysis

KCL @ node A

$$i_s + \frac{(V_s - V_A)}{(R_1 + R_2)} - \frac{V_A}{(R_3 + R_4 + R_5)} = 0$$

$$V_A \left(\frac{1}{R_1 + R_2} + \frac{1}{R_3 + R_4 + R_5} \right) = i_s + \frac{V_s}{R_1 + R_2}$$

Solving for V_A with given values

$$V_A = \frac{75}{2} \text{ V}$$

3.12) cont.

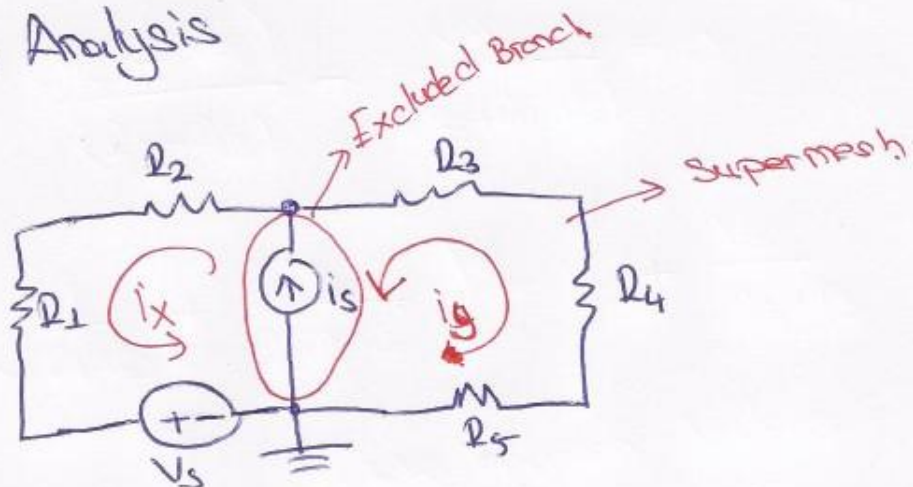
$$i_x = \frac{V_A - V_S}{R_1 + R_2} = 25 \text{ mA}$$

$$V_x = R_4 \frac{V_A}{(R_3 + R_4 + R_5)} = \frac{75}{4} \text{ V}$$

Current division

Mesh Analysis

a-b)



$$R_2 i_x + R_1 i_x + V_S + R_3 i_y + R_4 i_y + R_5 i_y = 0$$

$$i_x = i_y = i_s$$

$$\Rightarrow i_x (R_1 + R_2 + R_3 + R_4 + R_5) = (R_3 + R_4 + R_5) i_s - V_S$$

3.12) Mesh Analysis cont.

Using the given values, we get

$$\boxed{i_x = 25 \text{ mA}}$$

$$i_y = i_x - i_s = 25 - 100 = -75 \text{ mA}$$

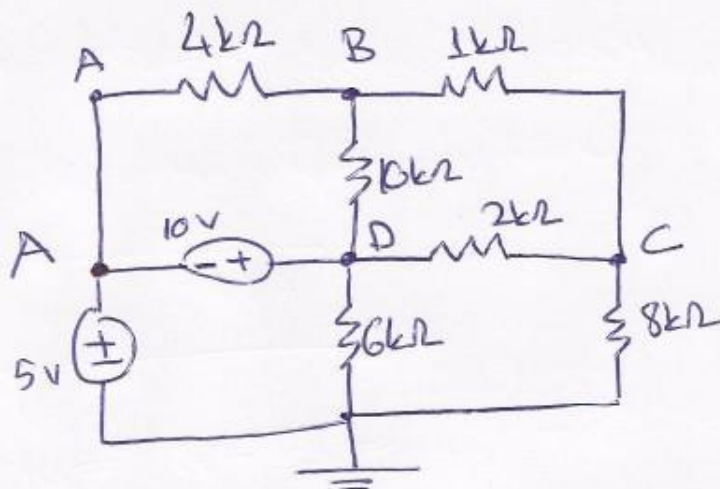
$$V_x = \ominus i_y \cdot R_4 = \frac{75}{4} \text{ V}$$

Because of
the direction.

c) Power can be found by using $P = VI$
and $P = I^2 \cdot R$ formulas for voltage source
and resistors respectively

$$\begin{aligned} P &= i_x^2 (R_1 + R_2) + i_y^2 (R_3 + R_4 + R_5) + V_s \cdot i_x \\ &= 3.75 \text{ W} \end{aligned}$$

3.20)



Nodal Analysis

We can easily find $V_A = 5V$ from the figure
and
 $V_D = 15V$

KCL @ node B

$$\frac{(5V - V_B)}{4k\Omega} - \frac{V_B - 15}{10k\Omega} - \frac{V_B - V_C}{1} = 0$$

$$V_B \left(-\frac{1}{4} - \frac{1}{10} \right) + V_C = -\frac{5}{4} - \frac{15}{10}$$

$$\boxed{-\frac{27}{20} V_B + V_C = -\frac{55}{20}}$$

3.20) cont

KCL @ node C

$$-\frac{V_C}{8} + \frac{V_B - V_C}{1} - \frac{V_C - 15}{2} = 0$$

$$V_B - V_C \left(\frac{1}{8} + 1 + \frac{1}{2} \right) = -\frac{15}{2}$$

$$\boxed{V_B - V_C \frac{13}{8} = -\frac{15}{2}}$$

Solving equations for V_A and V_B , we get

$$V_B = 10.03$$

$$V_C = 10.79$$

$$i_A = \frac{V_A - V_B}{4k\Omega} = \frac{5 - 10.03}{4} \approx -1.25 \text{ mA}$$

$$i_B = \frac{V_B - V_C}{1} = \frac{10.03 - 10.79}{1} = -0.76 \text{ mA}$$

$$i_C = \frac{V_C}{8} = \frac{10.79}{8} = 1.37 \text{ mA}$$

$$i_D = \frac{V_D}{6} + i_C = \frac{15}{6} + 1.37 = 3.87 \text{ mA}$$

3.20) (cont) Mesh Analysis

KVL @ mesh A

$$4i_A + 10(i_A - i_B) + 10 = 0$$

$$\Rightarrow i_A(4+10) - i_B(10) = -10$$

KVL @ mesh B

$$10(i_B - i_A) + 1i_B + 2(i_B - i_C) = 0$$

$$\Rightarrow i_A(-10) + i_B(10+1+2) + i_C(-2) = 0$$

KVL @ mesh C

$$2(i_C - i_B) + 8i_C + 6(i_C - i_D) = 0$$

$$\Rightarrow i_B(-2) + i_C(2+8+6) + i_D(-6) = 0$$

KVL @ mesh D

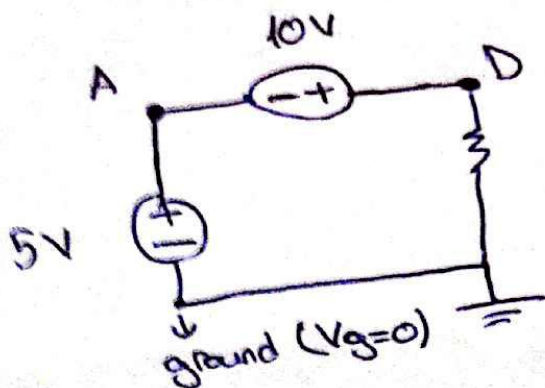
$$-5 - 10 + 6(i_D - i_C) = 0$$

$$\Rightarrow i_C(-6) + i_D(6) = 15$$

3.20) Solving 4 equations for 4 unknowns, we obtain

$$\begin{aligned} i_A &\approx -1.25 \text{ mA} \\ i_B &\approx -0.76 \text{ mA} \\ i_C &\approx 1.35 \text{ mA} \\ i_D &\approx 3.85 \text{ mA} \end{aligned}$$

P.S. Explanation about node D: We do not need to use KCL @ node to find V_D since we can justify that by following the potential differences between nodes



$$V_A - V_G = 5 \text{ and } V_G = 0$$

$$\Rightarrow \boxed{V_A = 5}$$

Same way

$$V_D - V_A = 10 \text{ and } V_A = 5$$

$$\Rightarrow \boxed{V_D = 15}$$