9.6) Find the Laplace transform of

$$f(t) = 30\left[2 - 5t - 2e^{-5t}\right]u(t)$$

$$F(s) = \mathcal{L}\{f(t)\} = 30\left(\frac{2}{s} - \frac{5}{s^2} - \frac{2}{(s+5)}\right)$$

$$= \frac{5s-25}{s^3(s+5)}$$

zero : 5

poles : 0, 0, 0, −4

9.8) a) Find the Laplace transform of

$$f_1(t) = 4\,\delta(t) + \left[20 e^{-20t} + 40 e^{-40t}\right] u(t)$$

$$\mathcal{L}\{f_1(t)\} = 4 + 20 \frac{1}{s+20} + 40 \frac{1}{s+40}$$

$$= \frac{4s^2 + 300s + 4800}{(s+20)(s+40)}$$

zeros: -51.86 , -23.14

poles: -20 , -40

b) $$f_2(t) = \left[15 + 15\cos(500t)\right] u(t)$$

$$\mathcal{L}\{f_2(t)\} = 15 \frac{1}{s} + 15 \frac{s}{s^2 + 500^2}$$

$$= \frac{15\,(2s^2 + 500^2)}{s^3 + 500^2 s}$$

zeros: $+353.55i$, $-353.55i$

poles: 0, $+5i$, $-5i$

9.16) Find the inverse Laplace of

a) $F_1(s) = \dfrac{s+100}{s(s+50)}$

Writing $F_1(s)$ as a partial fraction expansion

$$F_1(s) = \frac{k_1}{s} + \frac{k_2}{s+50} = \frac{s+100}{s(s+50)} = \frac{(k_1+k_2)s + k_1 50}{(s+50)s}$$

$$\Rightarrow \left.\begin{array}{l} k_1+k_2=1 \\ k_1 50 = 100 \end{array}\right\} \Rightarrow k_1 = 2, \; k_2 = -1$$

$$\mathcal{L}^{-1}\left\{ \frac{2}{s} - \frac{1}{s+50} \right\} = \left(2 - e^{-50t}\right) u(t)$$

b) $F_2(s) = \dfrac{s+10}{s(s+50)(s+100)}$

* Partial fraction expansion

$$F_2(s) = \frac{k_1}{s} + \frac{k_2}{s+50} + \frac{k_3}{s+100} = \frac{(k_1+k_2+k_3)s^2 + (150k_1+100k_2+50k_3)s + 5000k_1}{s(s+50)(s+100)}$$

$$= \frac{s+10}{s(s+50)(s+100)}$$

$$\left.\begin{array}{l} k_1+k_2+k_3=0 \\ 150k_1+100k_2+50k_3=1 \\ 5000k_1 = 10 \end{array}\right\} \quad \begin{array}{l} k_1 = \frac{1}{500} \\ k_2 = \frac{2}{125} \\ k_3 = \frac{-9}{500} \end{array}$$

9.16) cont.

$$\mathcal{L}^{-1}\left\{ \frac{1}{500}\,\frac{1}{s} + \frac{2}{125}\,\frac{1}{s+50} + \left(\frac{-9}{500}\right)\frac{1}{s+100} \right\}$$

$$= \left(\frac{1}{500} + \frac{2}{125} e^{-50t} - \frac{9}{500} e^{-100t} \right) u(t)$$

9.17) Find the inverse Laplace transforms

a) $F_1(s) = \dfrac{50(s+10)}{(s+5)(s+20)}$

* Partial fraction expansion

$$F_1(s) = \frac{k_1}{(s+5)} + \frac{k_2}{(s+20)} = \frac{(k_1+k_2)s + 20k_1 + 5k_2}{(s+5)(s+20)} = \frac{50s + 500}{(s+5)(s+20)}$$

$$\left.\begin{aligned} k_1 + k_2 &= 50 \\ 20k_1 + 5k_2 &= 500 \end{aligned}\right\} \Longrightarrow \begin{aligned} k_1 &= \frac{50}{3} \\ k_2 &= \frac{100}{3} \end{aligned}$$

9.17) cont.

$$\mathcal{L}^{-1}\left\{ \frac{50}{3}\frac{1}{s+5} + \frac{100}{3}\frac{1}{s+20} \right\} = \left(\frac{50}{3}e^{-5t} + \frac{100}{3}e^{-20t}\right)u(t)$$

b) $$F_2(s) = \frac{2s^2}{(s+200)(s+500)}$$

This is not strictly proper so converting to polynomial plus ~~st~~ strictly proper rational function

$$F_2(s) = 2 - \left(\frac{1400s + 200\,000}{(s+200)(s+500)}\right)$$

* Partial fraction expansion

$$\frac{1400s + 200\,000}{(s+200)(s+500)} = \frac{k_1}{(s+200)} + \frac{k_2}{(s+500)} = \frac{(k_1+k_2)s + 500k_1 + 200k_2}{(s+200)(s+500)}$$

$$\left.\begin{aligned} k_1 + k_2 &= 1400 \\ 500k_1 + 200k_2 &= 200\,000 \end{aligned}\right\} \Rightarrow \begin{aligned} k_1 &= \frac{-800}{3} \\ k_2 &= \frac{5000}{3} \end{aligned}$$

9.17) b) cont.

$$\mathcal{L}^{-1}\left\{2+\frac{800}{3}\frac{1}{s+200}-\frac{5000}{3}\frac{1}{s+500}\right\}$$

$$=2\delta(t)+\left(\frac{800}{3}e^{-200t}-\frac{5000}{3}e^{-500t}\right)u(t)$$

9.20) Find the inverse Laplace transforms

a) $F_1(s)=\dfrac{\alpha^2}{s^2(s+\alpha)}$

* Partial fraction expansion

$$\frac{\alpha^2}{s^2(s+\alpha)}=\frac{k_1 s+k_2}{s^2}+\frac{k_3}{(s+\alpha)}=\frac{(k_1+k_3)s^2+(k_1\alpha+k_2)s+k_2\alpha}{s^2(s+\alpha)}$$

$$\left.\begin{aligned} k_1+k_3&=0\\ k_1\alpha+k_2&=0\\ k_2\alpha&=\alpha\end{aligned}\right\}\quad \begin{aligned} k_1&=-1\\ k_2&=\alpha\\ k_3&=1\end{aligned}$$

9-20) Recall $\mathcal{L}\left\{\frac{df(t)}{dt}\right\} = sF(s) - f(0-)$

$$\Rightarrow \mathcal{L}^{-1}\left\{\frac{s}{(s+\alpha)^2}\right\} = \frac{d\left(te^{-\alpha t}\right)}{dt} = e^{-\alpha t} - \alpha t e^{-\alpha t}$$

$$\Rightarrow \mathcal{L}^{-1}\left\{\frac{1}{s} - \frac{s}{(s+\alpha)^2} - \frac{2\alpha}{s+\alpha}\right\}$$

$$= \left(1 - e^{-\alpha t} + \alpha t e^{-\alpha t} - 2\alpha t e^{-\alpha t}\right)u(t)$$

$$= \left(1 - (1+\alpha)e^{-\alpha t}\right)u(t)$$

9.20) cont.

$$\mathcal{L}^{-1}\left\{-\frac{1}{s}+\frac{\alpha}{s^2}+\frac{1}{s+\alpha}\right\}=\left(-1+t+e^{-\alpha t}\right)u(t)$$

b) $$F_2(s)=\frac{\alpha^2}{s(s+\alpha)^2}$$

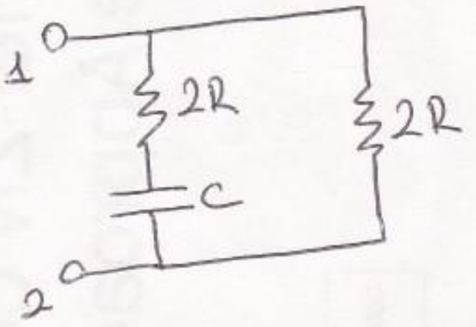
* Partial fraction expansion

$$\frac{\alpha^2}{s(s+\alpha)^2}=\frac{k_1}{s}+\frac{k_2 s+k_3}{(s+\alpha)^2}=\frac{(k_1+k_2)s^2+(2k_1\alpha+k_3)s+\alpha^2 k_1}{s(s+\alpha)^2}$$

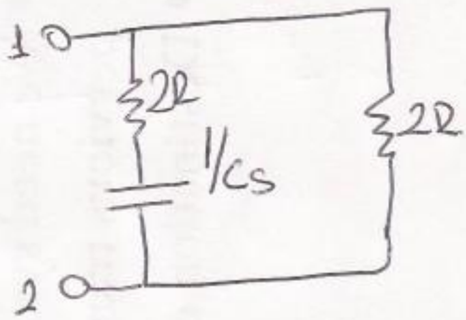
$$\left.\begin{aligned} k_1+k_2&=0\\ 2k_1\alpha+k_3&=0\\ \alpha^2 k_1&=\alpha^2 \end{aligned}\right\} \Rightarrow \begin{aligned} k_1&=1\\ k_2&=-1\\ k_3&=-2\alpha \end{aligned}$$

$$\mathcal{L}^{-1}\left\{\frac{1}{s}-\frac{s}{(s+\alpha)^2}-\frac{2\alpha}{(s+\alpha)}\right\}$$

10.1) Find $Z_{eq} = ?$



Laplace Domain $\Rightarrow$

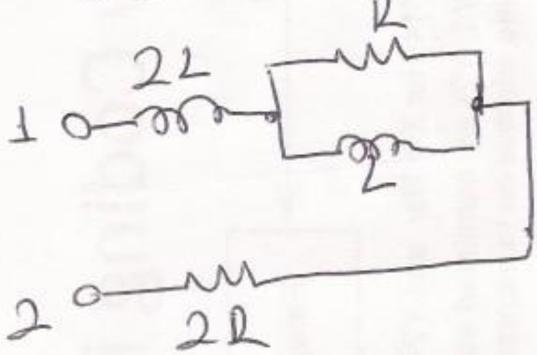


$$Z_{eq} = \left(\frac{1}{\left(2R + \frac{1}{Cs}\right)} + \frac{1}{2R} \right)^{-1} = \frac{Rs + \frac{1}{2C}}{s + \frac{1}{4RC}}$$

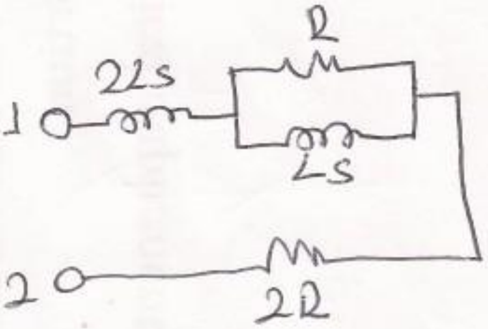
zero : $-\frac{1}{2RC}$

pole : $-\frac{1}{4RC}$

10.10) Find $Z_{eq}=?$. Locate pole at $s=-2000$ rad/s, find zeros.



Laplace domain $\Longrightarrow$



$$Z_{eq}(s) = 2Ls + \frac{RLs}{(R+Ls)} + 2R$$

$$= \frac{(2Ls+2R)(R+Ls)+RLs}{(Ls+R)}$$

$$= \frac{(Ls+2R)(2Ls+R)}{(Ls+R)}$$

Pole: $-\frac{R}{L}$

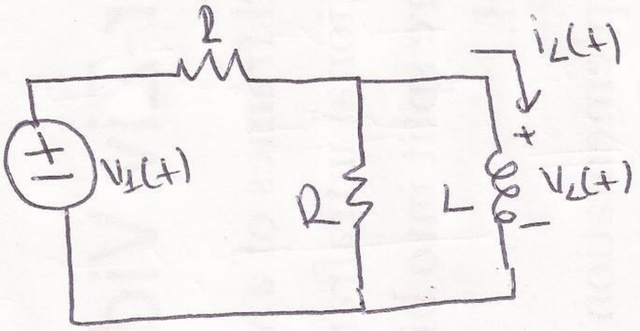
zeros: $-\frac{2R}{L}, -\frac{R}{2L}$

you can choose $L = 20\,H$, $R = 40\,k\Omega$

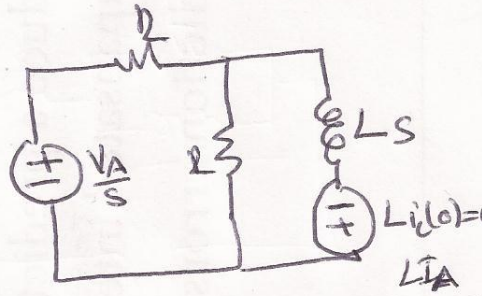
then zeros -4000 rad/sec and -1000 rad/sec

10.15) Find $I_L(s)$, $i_L(t)$, $V_L(s)$ and $V_L(t)$ for

$V_1(t) = V_A u(t)$ and $i_L(0) = I_A$

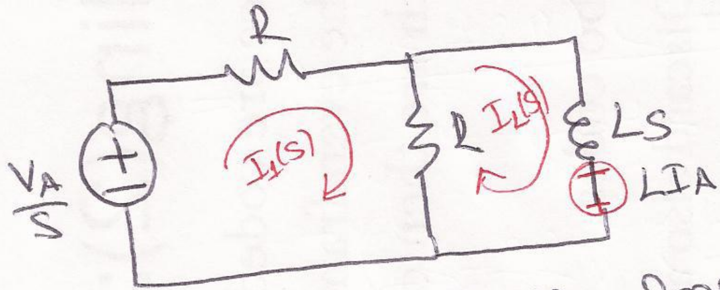


Laplace Domain $\Longrightarrow$



since $u(t) = \begin{cases} 0, & \text{for } t < 0 \\ 1, & \text{for } t \geq 0 \end{cases}$, $i_L(0) = I_A$ (initial energy)

$\Longrightarrow$



Use mesh analysis for $I_1(s)$ and $I_L(s)$

$$-\frac{V_A}{s} + 2R I_1(s) - R I_L(s) = 0 \quad (1)$$

$$(R + Ls) I_L(s) - R I_1(s) = +LI_A \quad (2)$$

10.15) cont. Solving eqn. (1) and (2)

$$I_2(s) = \frac{V_A + 2LI_A s}{s(R+2Ls)} = \frac{V_A/R}{s} - \frac{V_A/R}{s+R/2L} + \frac{I_A}{s+R/2L}$$

$$V_L(s) = Ls\,I_2(s) - LI_A = \frac{V_A/2 - RI_A}{s + R/2L}$$

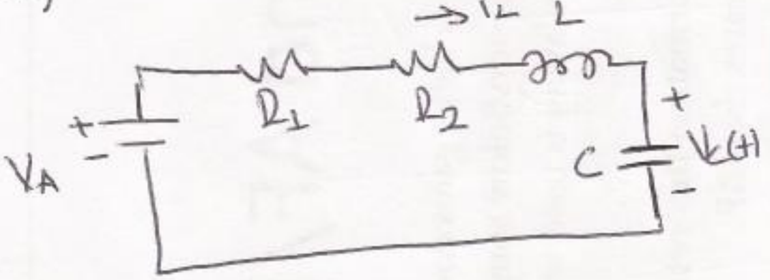
Inverse Laplace

$$i_2(t) = \left(\frac{V_A}{R}\left(1 - e^{-(R/2L)t}\right) + I_A e^{-(R/2L)t}\right)u(t)$$

$$v_L(t) = \left(\frac{V_A}{2} - RI_A\right)e^{-(R/2L)t}\,u(t)$$

10.17)

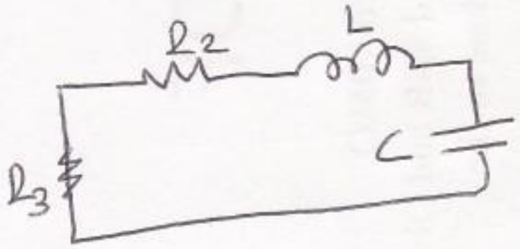
a) $t < 0$



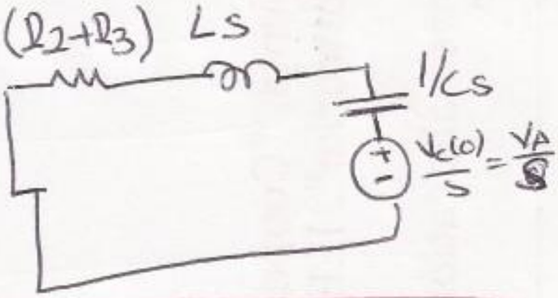
After a long time $V_C(t) = V_A$

and $i_L(0) = 0$

$t \geq 0$



Laplace domain $\Longrightarrow$



$$I_L(s) = \underset{\text{direction}}{\circlearrowleft} \frac{V_A/s}{\left(R_2+R_3+Ls+\frac{1}{Cs}\right)} = -\frac{V_A C}{LCs^2+(R_2+R_3)Cs+1}$$

10.17) cont.)

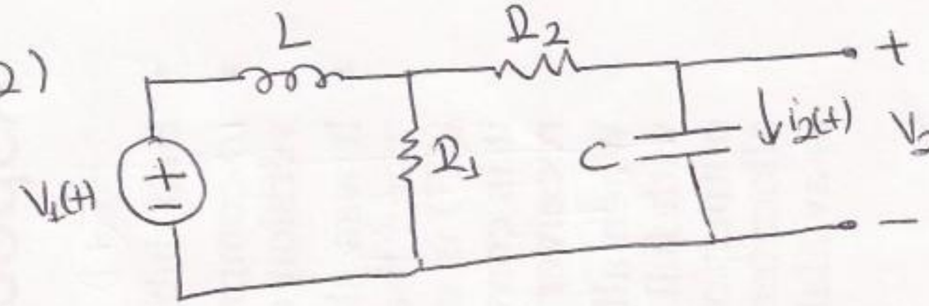
b) $$i_L(s) = -\frac{V_A/L}{\left(s^2 + \frac{(R_2+R_3)}{L}s + \frac{1}{LC}\right)}$$

for given values

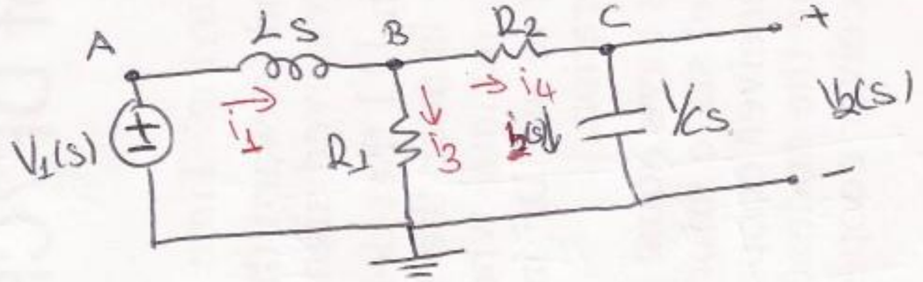
$$i_L(s) = -\frac{60}{\left(s^2 + 6000s + 1000\,000\right)}$$

$$i_L(t) = -\frac{3\sqrt{2}\,\sinh(2000\sqrt{2}\,t)}{200\,e^{(3000t)}}\,u(t)$$

10.32)



a)



Because of the voltage source and place of the ground, we can easily write $\boxed{V_A(s) = V_1(s)}$

and also $V_C(s) = V_2(s)$

@ node (B) $\quad i_2 - i_3 - i_4 = 0$

@ node (C) $\quad i_4 - i_2 = 0 \Rightarrow i_4 = i_2$

KCL @ node B

$$\frac{V_1 - V_B}{Ls} - \frac{V_B}{R_1} - \frac{V_B - V_2(s)}{R_2} = 0$$

10.32) cont.

$$\Rightarrow \quad \boxed{V_B\left(\frac{1}{Ls}+\frac{1}{R_1}+\frac{1}{R_2}\right)-\frac{V_2}{R_2}=\frac{V_1}{Ls} \quad (1)}$$

KCL @ node C

$$\Rightarrow \quad \frac{V_B - V_2(s)}{R_2} - V_2(s)Cs = 0$$

$$\Rightarrow \quad \boxed{V_B = V_2\left(1 + R_2Cs\right) \quad (2)}$$

b) By solving eqn(1) and eqn(2), we get

$$V_2(s) = \frac{V_1(s)\, R_1}{\left((R_1+R_2)LC\right)s^2 + \left(R_1R_2C+L\right)s + R_1}$$

10.32) cont.

c) There is no forced poles. ($f_p = V_1^{-1}(0)$, if exists)

The natural poles are:

$$^{n}p_{1,2}: \frac{-(R_1R_2C+L) \mp \sqrt{(R_1R_2C+L)^2 - 4(R_1+R_2)LCR_1}}{2(R_1+R_2)LC}$$

d) $$V_2(s) = \frac{7500}{10^{-3}s^3 + s^2 + 500s}$$

$$V_2(t) = 15\left(1 - \exp(-500t)\left(\cos(500t) + \sin(500t)\right)\right)u(t)$$