MAE140 – Linear Circuits – Summer Session II – 2011
Midterm, August 18

Instructions

(i) This exam is open book. You may use whatever written materials you choose, including your class notes and textbook. You may use a hand calculator with no communication capabilities.

(ii) You have 60 minutes.

(iii) Do not forget to write your name and student number.

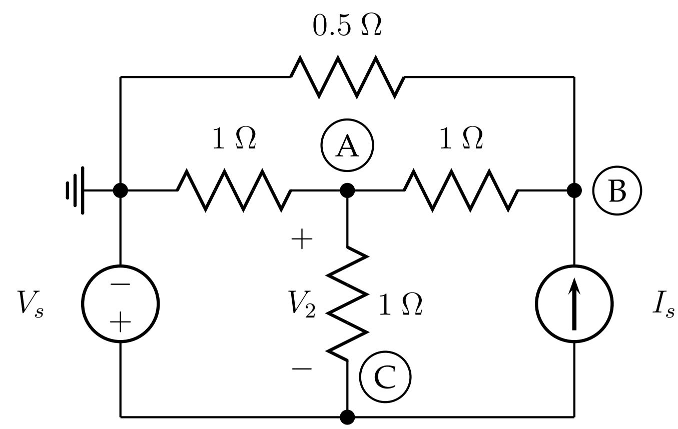


Figure 1: Circuit for Question 1.

1. **Node Voltage Analysis**

   (a) (6 points) Formulate node-voltage equations for the circuit in Figure 1. Use the ground and node labels provided in the figure and clearly indicate the final equations, circuit unknowns and how you handle the presence of the voltage source. You do not have to solve any equations.

Solution:

By inspection, and using Method #2 not to write KCL @ node C due to the presence of the voltage source we obtain two KCL equations (+1 point)

$$\begin{bmatrix} 1+1+1 & -1 & -1 \\ -1 & 1+2 & 0 \end{bmatrix} \begin{pmatrix} V_A \\ V_B \\ V_C \end{pmatrix} = \begin{pmatrix} 0 \\ I_s \end{pmatrix} \qquad \text{(+3 points)}$$

and one equation from the voltage source

$$V_C = V_s \qquad \text{(+1 point)}$$

Those three equations should be simultaneously solved for the node voltages (unkwowns) V_A, V_B and V_C. (+1 point)

(b) (2 points (bonus)) Using the results of part (a), show how the voltage V_2 is related to the sources V_s and I_s through

$$V_2 = \frac{1}{8}(I_s - 5V_s).$$

Hint: Use the fact that $\begin{bmatrix} 3 & -1 \\ -1 & 3 \end{bmatrix}^{-1} = \frac{1}{8}\begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix}$.

Solution:
Use $V_C = V_s$ to reduce the KCL equations to

$$\begin{bmatrix} 3 & -1 \\ -1 & 3 \end{bmatrix}\begin{pmatrix} V_A \\ V_B \end{pmatrix} = \begin{pmatrix} V_s \\ I_s \end{pmatrix} \qquad \text{(+1 point)}$$

Using the hint we have

$$\begin{pmatrix} V_A \\ V_B \end{pmatrix} = \begin{bmatrix} 3 & -1 \\ -1 & 3 \end{bmatrix}^{-1}\begin{pmatrix} V_s \\ I_s \end{pmatrix} = \frac{1}{8}\begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix}\begin{pmatrix} V_s \\ I_s \end{pmatrix} = \frac{1}{8}\begin{pmatrix} 3V_s + I_s \\ V_s + 3I_s \end{pmatrix}$$

and hence

$$V_2 = V_A - V_C = \frac{1}{8}(3V_s + I_s) - V_s = \frac{1}{8}(I_s - 5V_s) \qquad \text{(+1 point)}$$

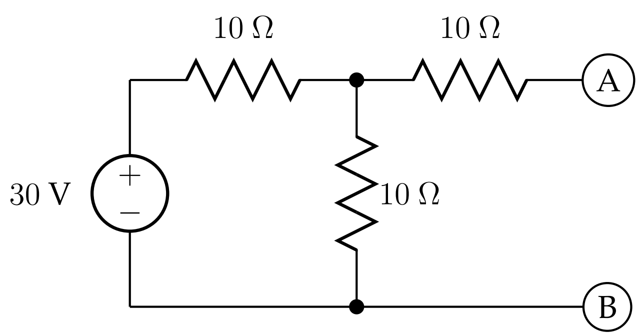
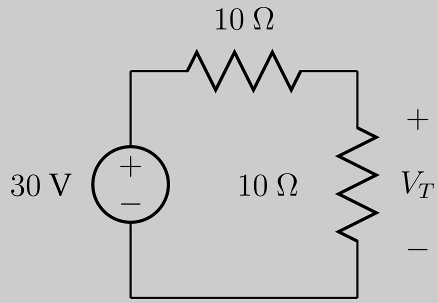


Figure 2: Circuit for Question 2.

2. **Circuit Equivalence**

(a) (3 points) Determine the Thevenin Voltage and the Norton Current for the circuit shown in Figure 2. Then use these quantities to compute the Thevenin equivalent for the circuit as seen from terminals A and B.

Solution:
The Thevenin Voltage V_T is determined when A-B is an open circuit, that is:

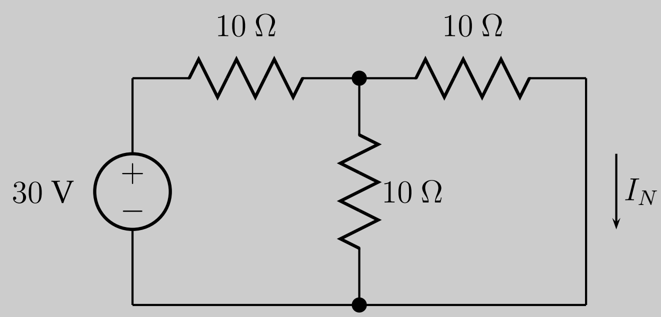


(+0.5 point)

where using voltage division we obtain

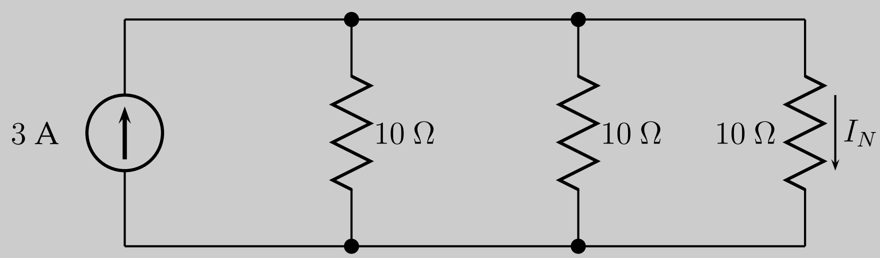
$$V_T = \frac{10}{10+10} 30\text{V} = 15\text{V} \qquad \text{(+0.5 point)}$$

The Norton Current I_N is determined when A-B is a short circuit, that is:



(+0.5 point)

After a souce transformation the circuit becomes

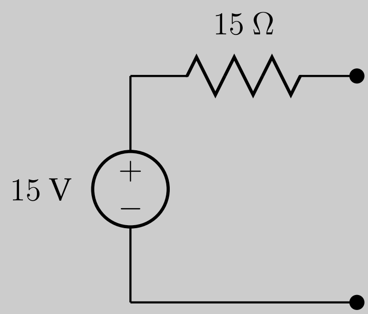


(+0.5 point)

where using current division we obtain

$$I_N = \frac{\frac{1}{10}}{\frac{1}{10}+\frac{1}{10}+\frac{1}{10}} 3\text{A} = \frac{1}{3} 3\text{A} = 1\text{A} \qquad \text{(+0.5 point)}$$

The Thevenin equivalent is then



(+0.5 point)

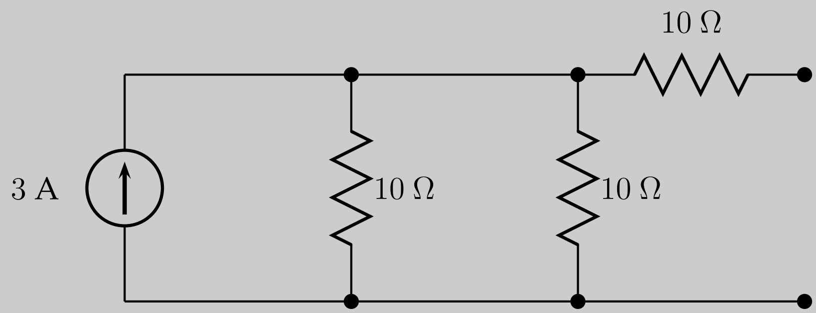
where R is computed as

$$R = \frac{V_T}{I_N} = 15\ \Omega$$

(b) (3 points) Using source transformations and circuit reduction techniques, find the Thevenin equivalent for the circuit shown in Figure 2 as seen from terminals A and B.

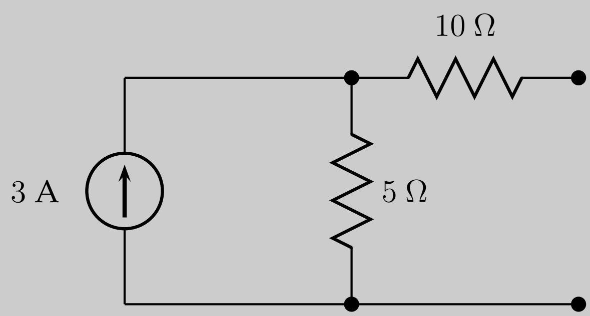
Solution:

After a souce transformation the circuit becomes



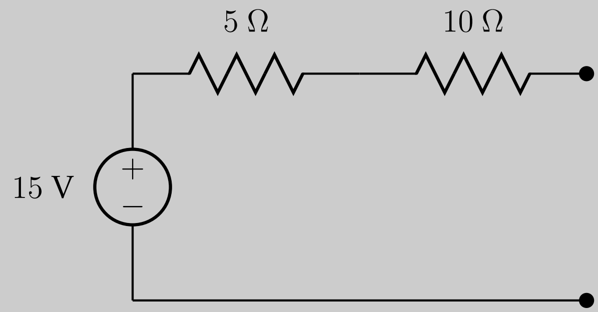
(+1 point)

Associating the resistors in parallel



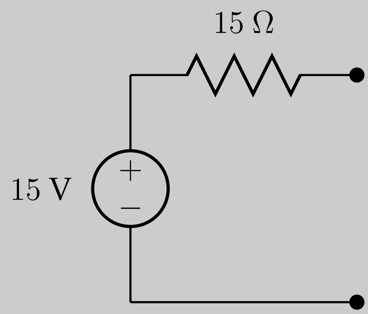
(+1 point)

Another source transformation leads to



(+0.5 point)

and finally



(+0.5 point)

after associating the resistors in series.

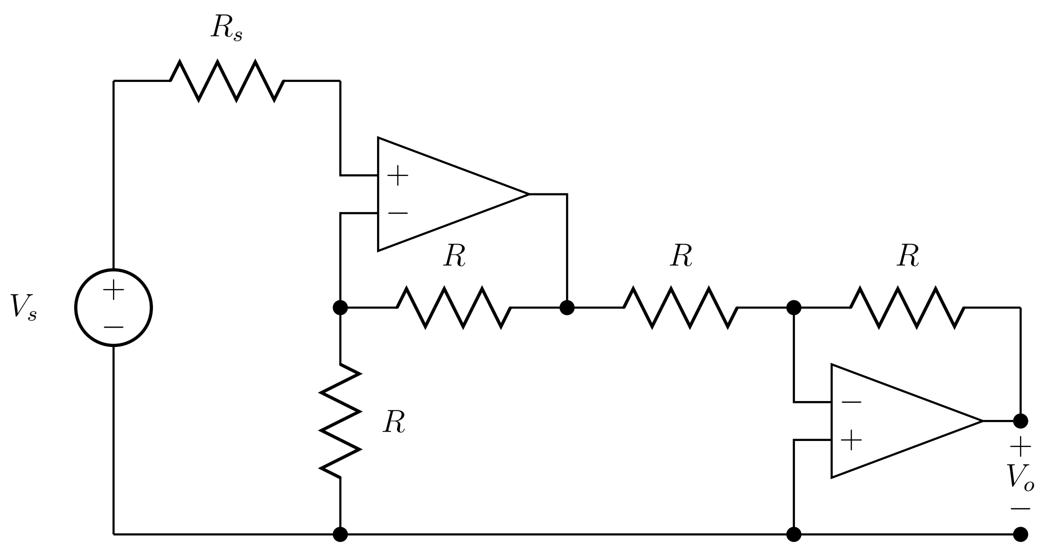


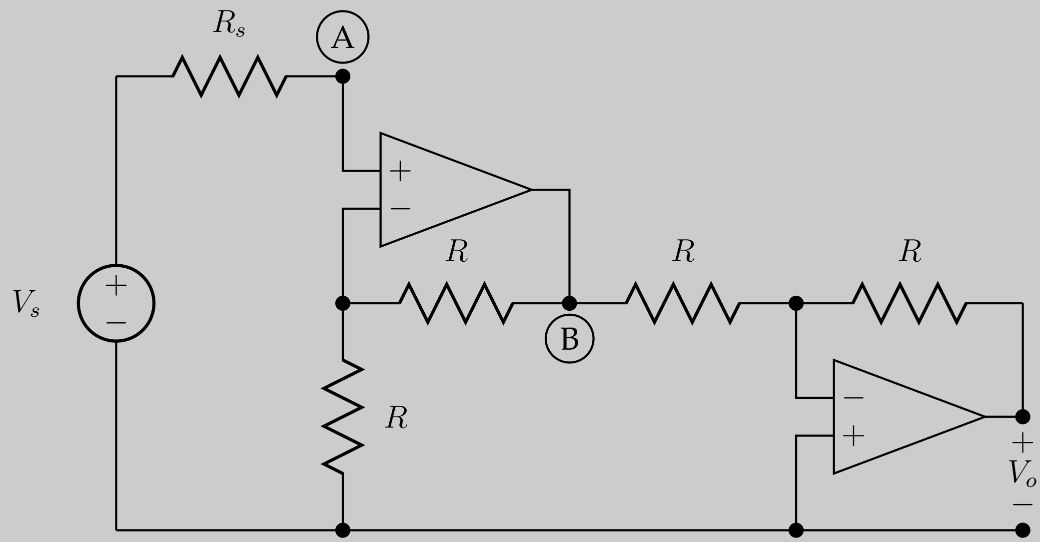
Figure 3: Circuit for Question 3.

3. **Op-Amp Analysis**

   (a) (6 points) Determine the gain V_o/V_s for the Op-Amp circuit in Figure 3.

Solution:

Let's first label some nodes:



Recognize that from A to B we have a non-inverting Op-Amp block. (+1 point)
Therefore

$$V_B = \frac{R+R}{R} V_A = 2V_A \qquad \text{(+1 point)}$$

Recognize that from B to V_o we have a inverting Op-Amp block. (+1 point)
Therefore

$$V_o = -\frac{R}{R} V_B = -V_B \qquad \text{(+1 point)}$$

Furthermore, because $i_p = 0$ we have that

$$V_A = V_s \qquad \text{(+1 point)}$$

Putting it all together

$$V_o = -V_B = -2V_A = -2V_s \quad \Longrightarrow \quad V_o/V_s = -2 \qquad \text{(+1 point)}$$