

Homework VI

Ch. 9: 1, 3, 5, 8, 16, 19, 23, 31, 46, 48

P. 9.1 find $F(s)$, poles & zeros for

$$f(t) = A[e^{-\alpha t} - 2e^{-\gamma t}] u(t)$$

$$\mathcal{L}\{f(t)\} = A \left(\frac{1}{s+\alpha} - \frac{2}{s+\gamma} \right)$$

$$F(s) = -\frac{A(s+2\alpha-\gamma)}{(s+\alpha)(s+\gamma)} \quad \begin{array}{l} \text{poles: } -\alpha, -\gamma \\ \text{zeros: } \gamma - 2\alpha \end{array}$$

P. 9.3 find $F(s)$, poles & zeros for

$$f(t) = A[1 - 2\cos(\beta t)] u(t)$$

$$\mathcal{L}\{f(t)\} = A \left(\frac{1}{s} - \frac{2s}{s^2 + \beta^2} \right)$$

$$F(s) = \frac{A(\beta^2 - s^2)}{s(s^2 + \beta^2)} \quad \begin{array}{l} \text{poles: } 0, \pm j\beta \\ \text{zeros: } \pm \beta \end{array}$$

P. 9.5 find $F(s)$, poles & zeros for

$$f(t) = A\delta(t) - 2A\beta e^{-\beta t} \cos(\beta t) u(t)$$

$$\mathcal{L}\{f(t)\} = A - 2A\beta \frac{s+\beta}{(s+\beta)^2 + \beta^2}$$

$$F(s) = A \left(\frac{s^2}{(s+\beta)^2 + \beta^2} \right) \quad \begin{array}{l} \text{poles: } -\beta \pm j\beta \\ \text{zeros: } 0 \end{array}$$

P. 9.8 find $F(s)$ and plot the pole-zero diagrams for

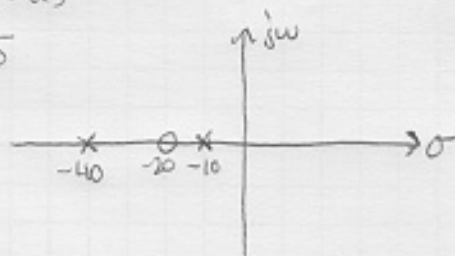
a) $f_1(t) = 3\delta(t) + [10e^{-10t} - 40e^{-40t}] u(t)$

$$\mathcal{L}\{f_1(t)\} = 3 + 10 \frac{1}{s+10} - 40 \frac{1}{s+40}$$

$$F_1(s) = \frac{3(s+20)^2}{(s+10)(s+40)}$$

poles: -10, -40

zeros: -20



b) $f_2(t) = [20 - 15\cos(500t)] u(t)$

$$\mathcal{L}\{f_2(t)\} = \frac{20}{s} - 15 \frac{s}{s^2 + 500^2}$$

$$F_2(s) = \frac{5(s^2 + 1000000)}{s(s^2 + 250000)}$$

poles: 0, $\pm j500$ zeros: $\pm j1000$ 

P.9.16 find $f(t)$ for

$$a) F_1(s) = \frac{s+20}{s(s+10)}$$

find form

$$F_1(s) = \frac{k_1}{s} + \frac{k_2}{s+10}$$

cover-up method

$$k_1 = \lim_{s \rightarrow 0} s F_1(s) = \lim_{s \rightarrow 0} \frac{s+20}{s+10} = 2$$

$$k_2 = \lim_{s \rightarrow -10} (s+10) F_1(s) = \lim_{s \rightarrow -10} \frac{s+20}{s} = -1$$

$$F_1(s) = \frac{2}{s} - \frac{1}{s+10}$$

$$\mathcal{L}^{-1}\{F_1(s)\} = 2u(t) - e^{-10t}u(t)$$

$$f_1(t) = [2 - e^{-10t}]u(t)$$

$$b) F_2(s) = \frac{s^2+10s+10}{s(s+10)} = 1 + \frac{10}{s(s+10)}$$
$$= 1 + \frac{1}{s} - \frac{1}{s+10}$$

$$\mathcal{L}^{-1}\{F_2(s)\} = \delta(t) + u(t) - e^{-10t}u(t)$$

$$f_2(t) = \delta(t) + [1 - e^{-10t}]u(t)$$

P.9.19 find $f(t)$ and sketch it for $\beta > 0$

$$a) F_1(s) = \frac{\beta(s+\beta)}{s(s^2+\beta^2)}$$

get to form

$$\frac{k_1}{s} + \frac{k_2}{s-j\beta} + \frac{*k_2}{s+j\beta}$$

cover up method

$$k_1 = \lim_{s \rightarrow 0} s F_1(s) = 1$$

$$k_2 = \lim_{s \rightarrow j\beta} (s-j\beta) F_1(s) = -\frac{1}{2} - j\frac{1}{2}$$

$$F_1(s) = \frac{1}{s} + \frac{-\frac{1}{2} - j\frac{1}{2}}{(s-j\beta)} + \frac{-\frac{1}{2} + j\frac{1}{2}}{(s+j\beta)}$$

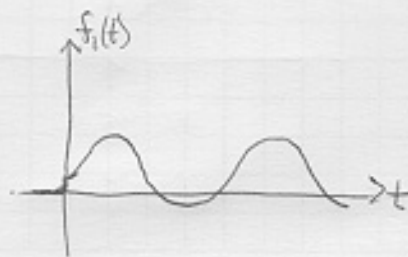
$$= \frac{1}{s} + \frac{1}{2} \frac{-s+\beta}{s^2+\beta^2} + \frac{1}{2} \frac{-s+\beta}{s^2+\beta^2}$$

$$= \frac{1}{s} + \frac{-s+\beta}{s^2+\beta^2}$$

$$F_1(s) = \frac{1}{s} - \frac{s}{s^2+\beta^2} + \frac{\beta}{s^2+\beta^2}$$

$$\mathcal{L}^{-1}\{F_1(s)\} = u(t) - \cos(\beta t)u(t) + \sin(\beta t)u(t)$$

$$f_1(t) = [1 - \cos \beta t + \sin \beta t]u(t)$$



P.9.19 continued

$$b) F_2(s) = \frac{s(s+\beta)}{s^2+\beta^2} = 1 + \frac{\beta(s-\beta)}{(s-j\beta)(s+j\beta)}$$

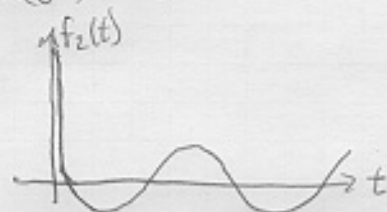
want form $1 + \frac{k}{(s-j\beta)} + \frac{*k}{(s+j\beta)}$

$$k = \lim_{s \rightarrow j\beta} (s-j\beta) F_2(s) = \frac{\beta}{2} (1+j)$$

$$\begin{aligned} F_2(s) &= 1 + \frac{\beta}{2} \frac{(1+j)}{(s-j\beta)} \frac{(s+j\beta)}{(s+j\beta)} + \frac{\beta}{2} \frac{(1-j)}{(s+j\beta)} \frac{(s-j\beta)}{(s-j\beta)} \\ &= 1 + \frac{\beta}{2} \left(\frac{s-\beta+j(s+\beta)}{s^2+\beta^2} + \frac{s-\beta-j(s+\beta)}{s^2+\beta^2} \right) \\ &= 1 + \frac{\beta(s-\beta)}{s^2+\beta^2} = 1 + \frac{\beta s}{s^2+\beta^2} - \frac{\beta^2}{s^2+\beta^2} \end{aligned}$$

$$\mathcal{L}^{-1}\{F_2(s)\} = \delta(t) + \beta \cos(\beta t) u(t) - \beta \sin(\beta t) u(t)$$

$$f_2(t) = \delta(t) + \beta [\cos \beta t - \sin \beta t] u(t)$$



P.9.23 find $f(t)$ of the following

$$a) F_1(s) = \frac{(s+4)(s+8)}{s(s+2)(s+6)} = \frac{k_1}{s} + \frac{k_2}{s+2} + \frac{k_3}{s+6}$$

cover-up method

$$k_1 = \lim_{s \rightarrow 0} s F_1(s) = \frac{8}{3}$$

$$k_2 = \lim_{s \rightarrow -2} (s+2) F_1(s) = \frac{1}{2} - \frac{3}{2}$$

$$k_3 = \lim_{s \rightarrow -6} (s+6) F_1(s) = \frac{1}{6} - \frac{1}{6}$$

$$F_1(s) = \frac{8}{3} \frac{1}{s} - \frac{1}{2} \frac{1}{s+2} - \frac{1}{6} \frac{1}{s+6}$$

$$\mathcal{L}^{-1}\{F_1(s)\} = \frac{8}{3} u(t) - \frac{1}{2} e^{-2t} u(t) - \frac{1}{6} e^{-6t} u(t)$$

$$f_1(t) = \left[\frac{8}{3} - \frac{1}{2} e^{-2t} - \frac{1}{6} e^{-6t} \right] u(t)$$

$$b) F_2(s) = \frac{3s^4+10s^2+4}{s(s^2+1)(s^2+4)} = \frac{3s^4+10s^2+4}{s(s-j)(s+j)(s-j2)(s+j2)}$$

$$= \frac{k_1}{s} + \frac{k_2}{s-j} + \frac{*k_2}{s+j} + \frac{k_3}{s-j2} + \frac{*k_3}{s+j2}$$

$$k_1 = \lim_{s \rightarrow 0} s F_2(s) = 1$$

$$k_2 = \lim_{s \rightarrow j} (s-j) F_2(s) = \frac{1}{2}$$

$$k_3 = \lim_{s \rightarrow 2j} (s-2j) F_2(s) = \frac{1}{2}$$

P.9.23.b continued

$$F_2(s) = \frac{1}{s} + \frac{1}{2} \frac{1}{s-j} \frac{(s+j)}{(s+j)} + \frac{1}{2} \frac{1}{s+j} \frac{(s-j)}{(s-j)} + \frac{1}{2} \frac{1}{s-j^2} \frac{(s+j^2)}{(s+j^2)} + \frac{1}{2} \frac{1}{s+j^2} \frac{(s-j^2)}{(s-j^2)}$$
$$= \frac{1}{s} + \frac{s}{s^2+1} + \frac{s}{s^2+4}$$

$$\mathcal{L}^{-1}\{F_2(s)\} = u(t) + \cos(t) u(t) + \cos(2t) u(t)$$

$$f_2(t) = [1 + \cos t + \cos 2t] u(t)$$

P.9.3] Use Laplace transformation to find $y(t)$ that satisfies the following

a) $50 \frac{dy}{dt} + 250y = 0$ with $y(0^-) = 10$

$$\mathcal{L}\{50 \frac{dy}{dt} + 250y = 0\} = 50(sY(s) - y(0^-)) + 250Y(s) = 0$$

$$\Downarrow$$
$$50sY(s) - 500 + 250Y(s) = 0$$
$$(50s + 250)Y(s) = 500$$

$$Y(s) = \frac{500}{50s + 250}$$
$$= \frac{10}{s + 5}$$

$$y(t) = \mathcal{L}^{-1}\{Y(s)\}$$

$$y(t) = 10 e^{-5t} u(t)$$

b) $\frac{dy}{dt} + 20y = 40 u(t)$

$$\mathcal{L}\{\frac{dy}{dt} + 20y = 40 u(t)\} = sY(s) - y(0^-) + 20Y(s) = \frac{40}{s}$$

$$\Downarrow$$
$$(s + 20)Y(s) = \frac{40}{s} - 10$$

$$Y(s) = \frac{(\frac{40}{s} - 10)}{s + 20}$$

$$= \frac{10(4-s)}{s(s+20)} = \frac{k_1}{s} + \frac{k_2}{s+20}$$

$$k_1 = \lim_{s \rightarrow 0} sY(s) = 2$$

$$k_2 = \lim_{s \rightarrow -20} (s+20)Y(s) = -12$$

$$y(t) = \mathcal{L}^{-1}\left\{\frac{2}{s} - \frac{12}{s+20}\right\}$$

$$y(t) = [2 - 12e^{-20t}] u(t)$$

P.9.46 Use initial & final value properties to find initial & final values of $f(t)$ s from P.9.23

a) $F_1(s) = \frac{(s+4)(s+8)}{s(s+2)(s+6)}$

initial $\rightarrow \lim_{t \rightarrow 0^+} f(t) = \lim_{s \rightarrow \infty} s F_1(s) = 1$

final $\rightarrow \lim_{t \rightarrow \infty} f(t) = \lim_{s \rightarrow 0} s F_1(s) = \frac{8}{3}$

b) $F_2(s) = \frac{3s^4 + 10s^2 + 41}{s(s^2+1)(s^2+4)}$

initial $\rightarrow \lim_{t \rightarrow 0^+} f(t) = \lim_{s \rightarrow \infty} s F_2(s) = \infty$

final $\rightarrow$ not applicable because $F_2(s)$ has poles on the j -axis.

P.9.48 Do the same for the following

a) $F_1(s) = \frac{s(s+5)}{s^2+6s+9}$

initial $\rightarrow$ not applicable because $F_1(s)$ is not a proper rational function.

final $\rightarrow \lim_{t \rightarrow \infty} f(t) = \lim_{s \rightarrow 0} s F_1(s) = 0$

b) $F_2(s) = \frac{10(s^2+10s-20)}{s(s^2+100)}$

initial $\rightarrow \lim_{t \rightarrow 0^+} f(t) = \lim_{s \rightarrow \infty} s F_2(s) = 1$

final $\rightarrow$ not applicable because $F_2(s)$ has poles on the j -axis.