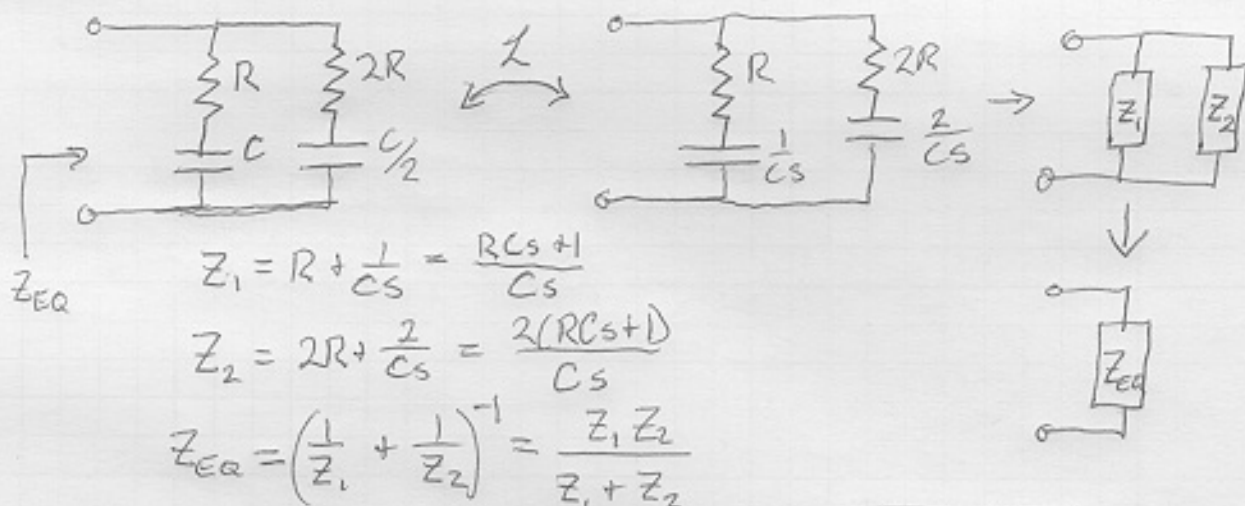


Homework VII

Ch.10: 2, 5, 10, 17, 20, 21, 25, 28, 31, 32

P.10.2 find Z_{EQ} , expressed as a rational function & locate poles & zeros



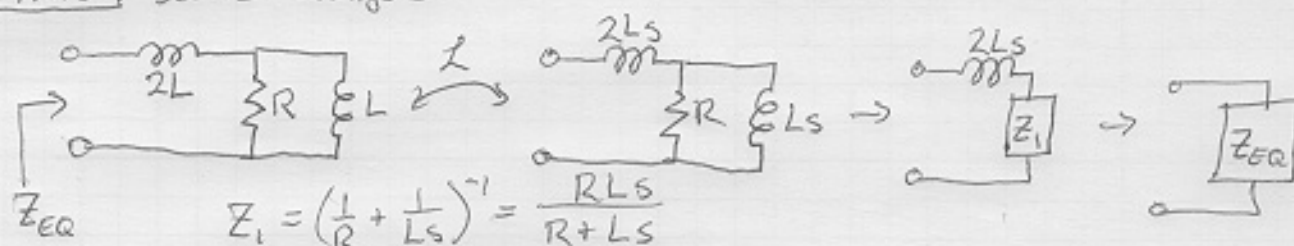
$$Z_1 = R + \frac{1}{Cs} = \frac{RCs + 1}{Cs}$$

$$Z_2 = 2R + \frac{2}{Cs} = \frac{2(RCs + 1)}{Cs}$$

$$Z_{EQ} = \left(\frac{1}{Z_1} + \frac{1}{Z_2} \right)^{-1} = \frac{Z_1 Z_2}{Z_1 + Z_2}$$

$$Z_{EQ} = \frac{2}{3} \frac{(RCs + 1)}{Cs} \quad \text{poles: } 0 \quad \text{zeros: } -\frac{1}{RC}$$

P.10.5 same analysis

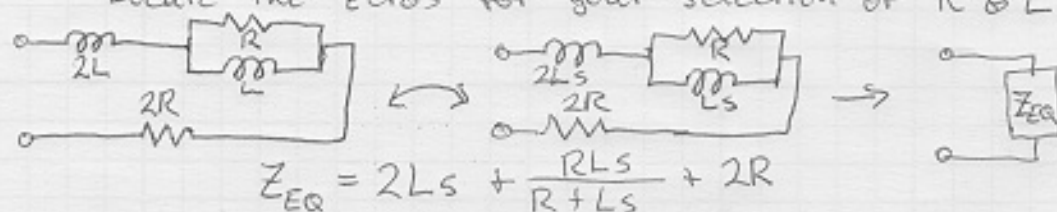


$$Z_1 = \left(\frac{1}{R} + \frac{1}{Ls} \right)^{-1} = \frac{RLs}{R + Ls}$$

$$Z_{EQ} = Z_1 + 2Ls = \frac{RLs}{R + Ls} + 2Ls$$

$$Z_{EQ} = \frac{2L^2 s^2 + 3RLs}{R + Ls} \quad \text{poles: } -\frac{R}{L} \quad \text{zeros: } 0, -\frac{3}{2} \frac{R}{L}$$

P.10.10 find Z_{EQ} , select R & L so that there is a pole @ $s = -1000 \text{ rad/s}$. Locate the zeros for your selection of R & L .



$$Z_{EQ} = 2Ls + \frac{RLs}{R + Ls} + 2R$$

$$Z_{EQ} = \frac{(2Ls + R)(Ls + 2R)}{R + Ls}$$

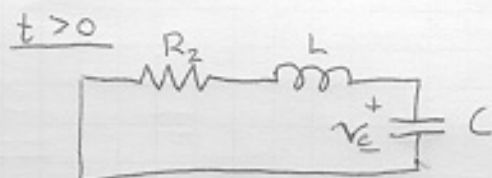
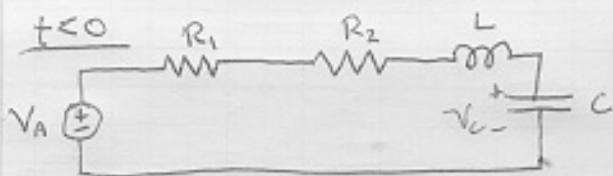
pole: $-\frac{R}{L}$

zeros: $-\frac{R}{2L}, -\frac{2R}{L}$

ex. $R = 10 \text{ k}\Omega$
 $L = 10 \text{ H}$

zeros = $-500 \text{ rad/s}, -2000 \text{ rad/s}$

P.10.17 ~~For circuit below~~ For circuit below



- a) Solve for $I_L(s)$, $t > 0$
initial conditions $v_C(0) = V_A$
 $i_L(0) = 0$

$$I_L(s) = \frac{-(V_A/s)}{R_2 + Ls + \frac{1}{Cs}}$$

$$I_L(s) = \frac{-V_A C}{LCs^2 + R_2Cs + 1}$$

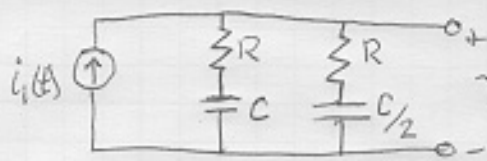


- b) find $i_L(t)$ for $R_1 = R_2 = 500 \Omega$, $L = 250 \text{ mH}$, $C = 4 \mu\text{F}$, $V_A = 15 \text{ V}$

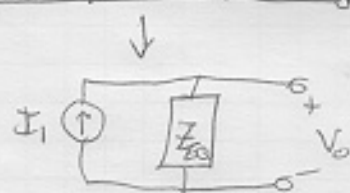
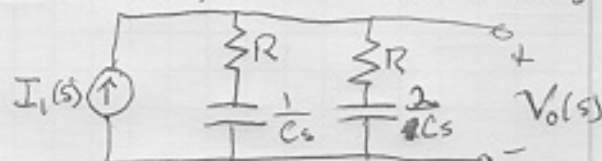
$$I_L(s) = \frac{60}{s^2 + 2000s + 1000000} = \frac{60}{(s + 1000)^2}$$

$$i_L(t) = 60t e^{-1000t} u(t)$$

P.10.20 find the zero state s-domain relationship between $I_1(s)$ & $V_0(s)$



$\xrightarrow{\mathcal{L}}$

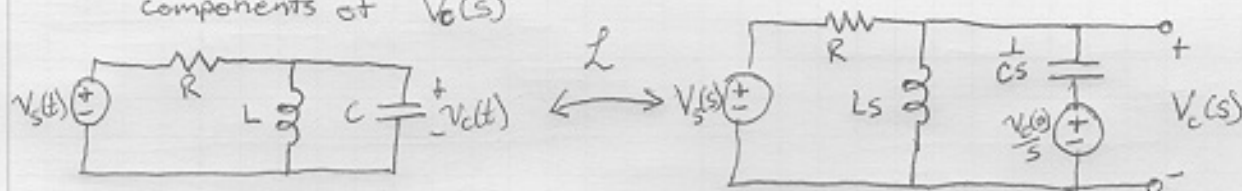


$$Z_{EQ} = \frac{(R + \frac{1}{Cs})(R + \frac{2}{Cs})}{R + \frac{1}{Cs} + R + \frac{2}{Cs}} = \frac{(RCs + 1)(RCs + 2)}{Cs(2RCs + 3)}$$

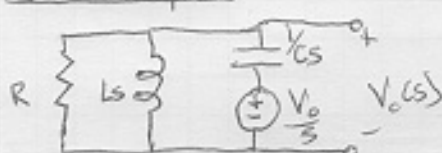
$$V_0(s) = Z_{EQ} I_1(s)$$

$$V_0(s) = \frac{(RCs + 1)(RCs + 2)}{Cs(2RCs + 3)}$$

P.10.21 With $i_L(0) = V_0$, $i_C(0) = 0$. Transform the circuit into the s-domain & use superposition to find the zero state & zero input components of $V_C(s)$



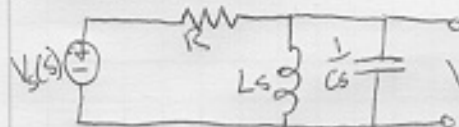
Zero-input



$$V_C(s) = \frac{\left(\frac{RLs}{R+Ls}\right) \frac{V_0}{s}}{\frac{1}{Cs} + \frac{RLs}{R+Ls}}$$

$$V_C(s) = \frac{V_0 RLCs}{RLCs^2 + Ls + R}$$

Zero-state

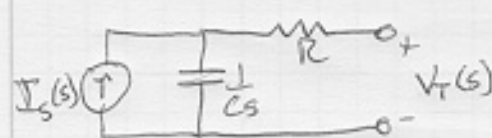
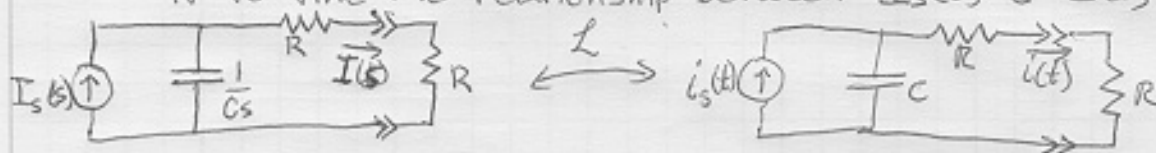


$$V_C(s) = \frac{\left(\frac{1}{Cs} \frac{1}{Ls + \frac{1}{Cs}}\right) V_s(s)}{R + \left(\frac{1}{Cs} \frac{1}{Ls + \frac{1}{Cs}}\right)}$$

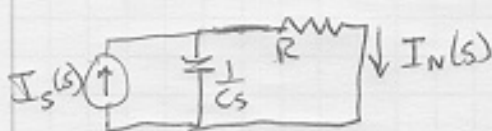
$$V_C(s) = \frac{Ls}{RLCs^2 + Ls + R} V_s(s)$$

P.10.25

find the zero state Thevenin equivalent & use it to find the relationship between $I_s(s)$ & $I(s)$

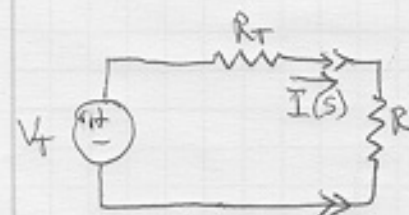


$$V_T = \frac{I_s}{Cs}$$



$$I_N(s) = \frac{(VR)}{\left(\frac{1}{Cs}\right) + Cs} I_s(s) = \frac{I_s(s)}{RCs + 1}$$

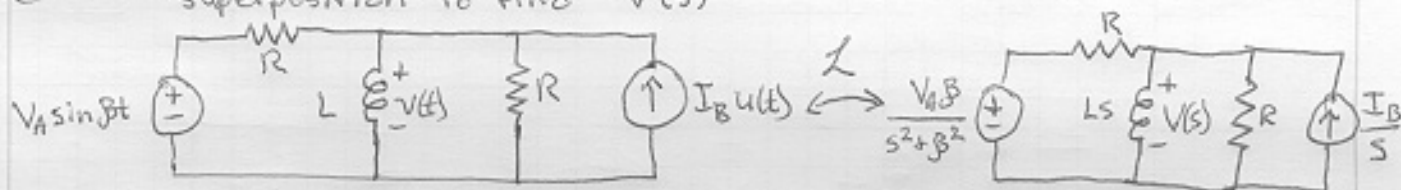
$$R_T = \frac{V_T}{I_N} = \frac{RCs + 1}{Cs}$$



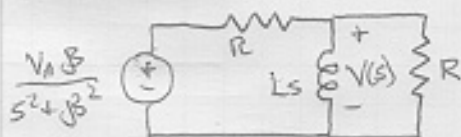
$$I(s) = \frac{V_T}{R_T + R} = \frac{I_s(s)}{Cs} \frac{1}{\frac{RCs + 1}{Cs} + R}$$

$$I(s) = \frac{I_s(s)}{2RCs + 1}$$

P.10.28 With zero initial energy stored in the circuit, use superposition to find $V(s)$

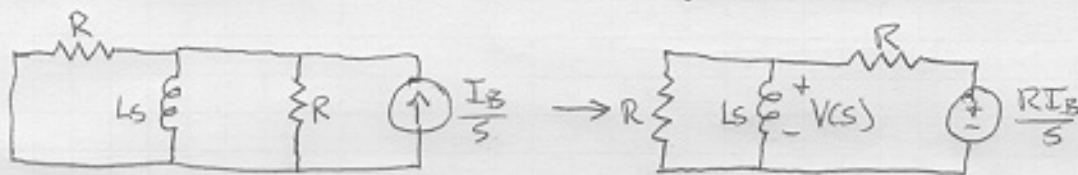


Superposition



$$V(s) = \frac{\left(\frac{RLs}{R+Ls} \right) \frac{V_A \beta}{s^2 + \beta^2}}{R + \left(\frac{RLs}{R+Ls} \right)}$$

$$V(s) = \frac{V_A \beta R L s}{(2RLs + R^2)(s^2 + \beta^2)} \quad \checkmark$$



$$V(s) = \frac{RLs}{R^2 + 2RLs} \frac{RI_B}{s} \quad \checkmark$$

total

$$V(s) = \frac{V_A \beta R L s}{(R^2 + 2RLs)(s^2 + \beta^2)} + \frac{I_B R^2 L s}{s(R^2 + 2RLs)}$$

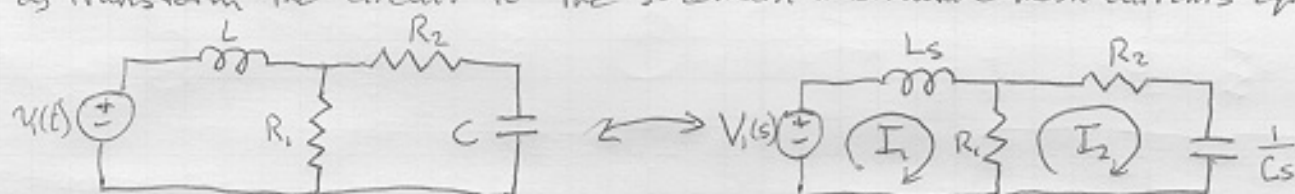
$$V(s) = \frac{L(V_A \beta s + I_B R(s^2 + \beta^2))}{(2Ls + R)(s^2 + \beta^2)}$$

Identify the forced & natural poles

poles :	$\pm j\beta$	forced
	$-\frac{R}{2L}$	natural

P.10.31 With no initial energy stored

a) Transform the circuit to the s-domain & formulate mesh-currents eqns.



$$\begin{array}{lcl} \text{mesh 1} & -V_1(s) + Ls I_1 + R_1(I_1 - I_2) & = 0 \\ \text{mesh 2} & R_1(I_2 - I_1) + (R_2 + \frac{1}{Cs}) I_2 & = 0 \end{array}$$

b) Solve these eqns for $I_2(s)$

$$\begin{aligned} I_1 &= \frac{R_1 + R_2 + \frac{1}{Cs}}{R_1} I_2 \\ (R_1 + Ls) \frac{R_1 + R_2 + \frac{1}{Cs}}{R_1} I_2 - R_1 I_2 &= V_1(s) \\ I_2 &= \frac{V_1(s) R_1 Cs}{(R_1 + Ls)(Cs(R_1 + R_2) + 1) - R_1^2 Cs} \\ \boxed{I_2 &= \frac{V_1(s) R_1 Cs}{(R_1 + R_2) L Cs^2 + (L + R_1 R_2 C) s + R_1}} \end{aligned}$$

c) Find $i_2(t)$ when $V_1(t) = 100 u(t)$ V, $R_1 = 1 k\Omega$, $R_2 = 2 k\Omega$, $L = 4$ H, & $C = 500$ nF

$$\begin{aligned} I_2(s) &= \frac{(25/3)}{s^2 + \frac{2500}{3}s + \frac{500000}{3}} \\ &= \frac{(25/3)}{(s+500)(s+1000/3)} \\ &= -\frac{0.05}{(s+500)} + \frac{0.05}{(s+1000/3)} \end{aligned}$$

$$\boxed{i_2(t) = 50 [e^{-\frac{1000t}{3}} - e^{-500t}] u(t) \text{ mA}}$$

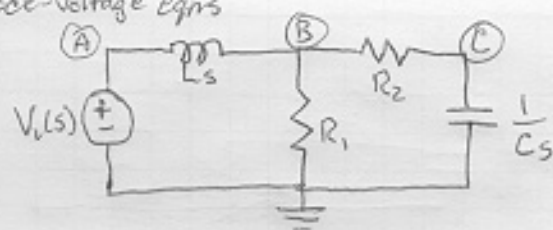
P.10.32 for the same circuit as in P.10.31

a) transform to the s-domain & develop node-voltage eqns

(A) $V_A = V_1(s)$

(B) $-\frac{1}{Ls} V_A + \left(\frac{1}{Ls} + \frac{1}{R_1} + \frac{1}{R_2}\right) V_B - \frac{1}{R_2} V_C = 0$

(C) $-\frac{1}{R_2} V_B + \left(\frac{1}{R_2} + Cs\right) V_C = 0$



b) Solve these eqns for $V_2 = V_C$

$$V_B = (R_2Cs + 1)V_C$$

$$\frac{(R_1R_2 + R_2Ls + R_1Ls)(R_2Cs + 1)}{R_1R_2Ls} V_C - \frac{1}{R_2} V_C = \frac{V_1(s)}{Ls}$$

$$V_C = \frac{V_1(s) R_1}{Cs(R_1R_2 + R_2Ls + R_1Ls) + R_1 + Ls}$$

$$= \frac{V_1(s) R_1}{((R_1 + R_2)LC)s^2 + (R_1R_2C + L)s + R_1}$$

c) find $v_2(t)$ when $v_1(t) = 10u(t)V$, $R_1 = R_2 = 500\Omega$, $L = 0.5H$, $C = 2\mu F$

$$V_2(s) = \frac{5000}{s(0.001s^2 + s + 500)}$$

$$= \frac{5000000}{s(s^2 + 1000s + 500000)} \quad \text{poles: } 0, -500 \pm j500$$

want form $V_2(s) = \frac{k_1}{s} + \frac{k_2}{s+500-j500} + \frac{k_2^*}{s+500+j500}$

$$k_1 = \lim_{s \rightarrow 0} \frac{5000000}{s^2 + 1000s + 500000} = 10$$

$$k_2 = \lim_{s \rightarrow -500+j500} \frac{5000000}{s(s+500+j500)} = -\frac{10}{1+j} = -5+j5$$

$$k_2^* = -5-j5$$

$$V_2(s) = \frac{10}{s} + \frac{(-5+j5)}{s+500-j500} + \frac{(-5-j5)}{s+500+j500}$$

$$v_2(t) = [10 + (-5+j5)e^{(-500+j500)t} + (-5-j5)e^{(-500-j500)t}]u(t)$$

$$v_2(t) = 10[1 + \sqrt{2}e^{-500t} \cos(500t + \frac{3\pi}{4})]u(t)$$

$$v_2(t) = 10[1 + e^{-500t}(\sin 500t - \cos 500t)]u(t)$$