

MAE 140 – Linear Circuits – Winter 2009
Midterm

Instructions

- 1) This exam is open book. You may use whatever written materials you choose, including your class notes and textbook. You may use a hand calculator with no communication capabilities.
- 2) You have 70 minutes.
- 3) Write your name, student number and instructor.

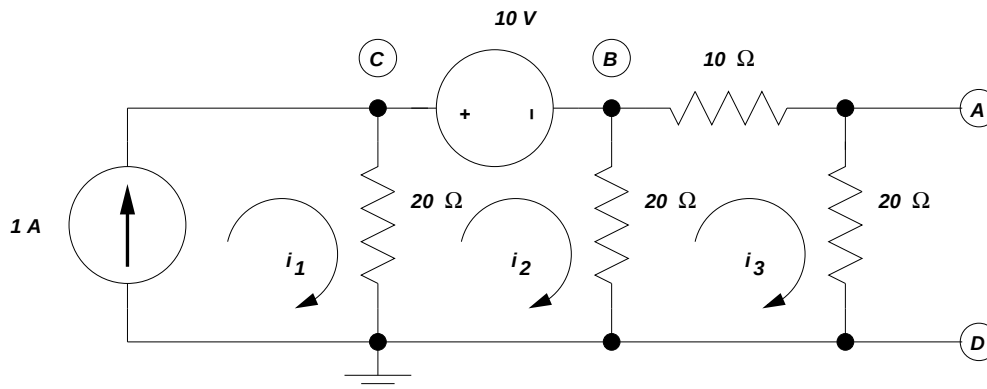


Figure 1: Circuit for questions 1-4.

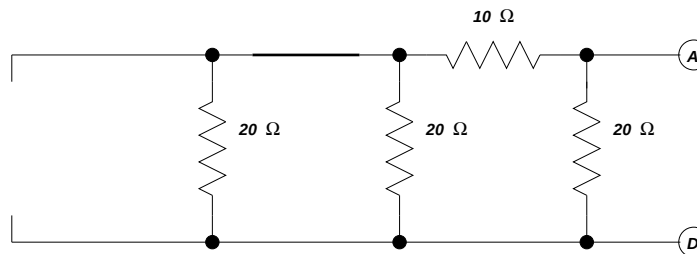
Questions

1. Equivalent circuits

- (a) (2 points) Turn off all the sources in the circuit of Figure 1 and find the equivalent resistance as seen from terminals A and D.

Solution: First determine the circuit with all sources off:

(1 point)



The equivalent resistance is then

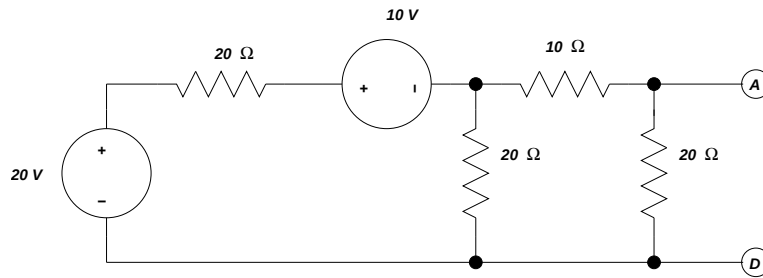
$$[(20 // 20) + 10] // 20 = [10 + 10] // 20 = 20 // 20 = 10 \Omega$$

(1 point)

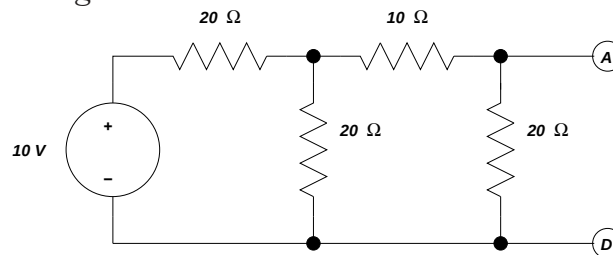
- (b) (3 points) Find the Thévenin equivalent as seen from terminals A and D.

Solution: The simplest possible way may be to use a source transformation on the current source to obtain the following circuit:

(1 point)

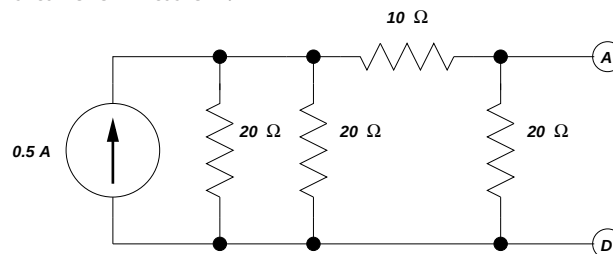


Then associate the voltage sources in series:

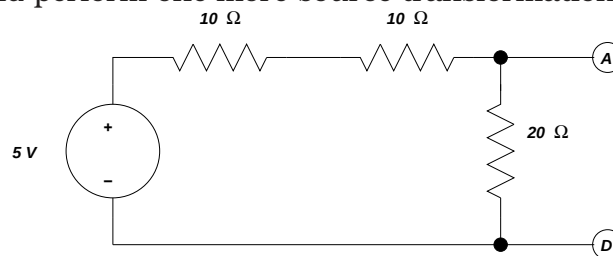


and apply a source transformation:

(1 point)



Associate 20//20 and perform one more source transformation:

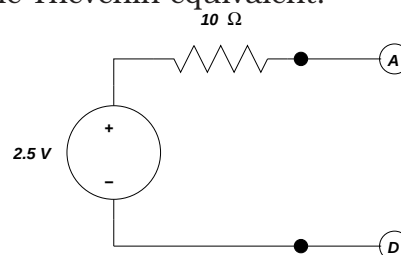


Calculate the open-circuit voltage V_T directly using voltage division

$$V_T = \frac{20}{10 + 10 + 20} 5 \text{ V} = 2.5 \text{ V}$$

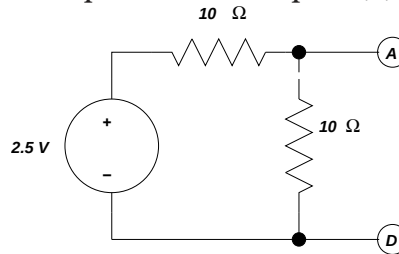
and use part (a) to draw the Thévenin equivalent:

(1 point)



(c) (1 point) Find the power absorbed by a 10Ω resistor if connected to terminals A and D.

Solution: Using the Thévenin equivalent from part (b):



Compute the voltage on the resistor using voltage division:

$$v = \frac{10}{10 + 10} 2.5 = 1.25V$$

then the power absorbed is

$$p = \frac{v^2}{10} = \frac{1.25^2}{10} \approx 156 \text{ mW} \quad (1 \text{ point})$$

2. Mesh current analysis

- (a) (6 points) Formulate mesh-current equations for the circuit in Figure 1. Use the mesh currents shown in the figure and clearly indicate how you handle the presence of a current source, the final equations and the unknowns they must be solved for. **Do not modify the circuit or the labels in any way. Do not use source transformation. Do not solve any equations!**

Solution: Because the current source appears in a single mesh we have

$$i_1 = 1 \text{ A}, \quad (\text{eq. 1})$$

which determines i_1 . (1 point)

The remaining two currents can be determined by the mesh-current equations

$$\begin{bmatrix} -20 & 20 + 20 & -20 \\ 0 & -20 & 20 + 10 + 20 \end{bmatrix} \begin{pmatrix} i_1 \\ i_2 \\ i_3 \end{pmatrix} = \begin{pmatrix} -10 \\ 0 \end{pmatrix} \quad (\text{eq. 2})$$

which were obtained by inspection of the circuit. (3 points)

One must solve (eq. 1) and (eq. 2) for currents i_1 , i_2 and i_3 . That is $i_1 = 1$ and solve

$$\begin{bmatrix} 40 & -20 \\ -20 & 50 \end{bmatrix} \begin{pmatrix} i_2 \\ i_3 \end{pmatrix} = \begin{pmatrix} 10 \\ 0 \end{pmatrix}$$

for i_2 and i_3 . (2 points)

3. Nodal voltage analysis

- (a) (6 points) Assuming that the node labeled D is the ground node (reference), formulate node-voltage equations for the circuit in Figure 1. Use the node labels provided in the figure and clearly indicate how you handle the presence of a voltage source, the final equations, and the unknowns they must be solved for. **Do not modify the circuit or the labels in any way. Do not use source transformation. Do not solve any**

equations!

Hint: Use a super-node.

Solution: We start by writing the node-voltage equations by inspection

$$\begin{bmatrix} \frac{1}{10} + \frac{1}{20} & -\frac{1}{10} & 0 \\ -\frac{1}{10} & \frac{1}{10} + \frac{1}{20} & 0 \\ 0 & 0 & \frac{1}{20} \end{bmatrix} \begin{pmatrix} v_A \\ v_B \\ v_C \end{pmatrix} = \begin{pmatrix} 0 \\ -i_v \\ 1 + i_v \end{pmatrix} \quad (\text{eq. 3})$$

where i_v is the unknown current on the voltage source.

(2 points)

Furthermore, we have the relationship

$$v_C - v_B = 10 \text{ V} \quad (\text{eq. 4})$$

because of the voltage source.

(1 point)

In order to determine the node-voltages we can form the *super-node* equation formed by adding the second and third rows of (eq. 4)

$$\begin{bmatrix} -\frac{1}{10} & \frac{1}{10} + \frac{1}{20} & \frac{1}{20} \end{bmatrix} \begin{pmatrix} v_A \\ v_B \\ v_C \end{pmatrix} = 1 \quad (\text{eq. 5})$$

which should be solved for v_A , v_B and v_C along with the first equation of (eq. 3) and (eq. 4).

(2 points)

The final equations are

$$\begin{bmatrix} \frac{1}{10} + \frac{1}{20} & -\frac{1}{10} & 0 \\ -\frac{1}{10} & \frac{1}{10} + \frac{1}{20} & \frac{1}{20} \\ 0 & -1 & 1 \end{bmatrix} \begin{pmatrix} v_A \\ v_B \\ v_C \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 10 \end{pmatrix}$$

to be solved for v_A , v_B and v_C .

(1 point)

4. Bonus Question

- (a) (1 point) If you were allowed to modify the circuit in Figure 1, describe what would you do in order to avoid having to use a super-node in Question 3? **Do not write or solve any equations!**

Solution: One could choose nodes B (or C) as references.

(1 point)

In this way v_C (or v_B) would be determined and one would simply not write KCL at node C. This is method # 2.

Alternatively, a more complicate solution would be to leave D as the reference node and apply a source transformation on the current source and then associate the voltage sources as done in part (b) of Question 1.