MAE140 Win12 HW3 Solutions

T R & T, 3.1, 3.3, 3.6, 3.7, 3.9, 3.10, 3.12, 3.15 b) and d), 3.71, 3.74

3-1

(a) Writing node-voltage equations by inspection, we can get:

Node A:

$$\left(\frac{1}{40}+\frac{1}{30}\right)v_A-\frac{1}{30}v_B=2$$

Node B:

$$-\frac{1}{30}v_A+\left(\frac{1}{30}+\frac{1}{20}+\frac{1}{80}\right)v_B=0$$

Written in matrix form $\mathbf{Ax}=\mathbf{b}$

$$\begin{bmatrix}\left(\frac{1}{40}+\frac{1}{30}\right) & -\frac{1}{30}\\ -\frac{1}{30} & \left(\frac{1}{30}+\frac{1}{20}+\frac{1}{80}\right)\end{bmatrix}\begin{bmatrix}v_A\\ v_B\end{bmatrix}=\begin{bmatrix}2\\ 0\end{bmatrix}$$

(b) Solve the equations or matrix form, we can get

$$v_A=42.79\ V, v_B=14.88V$$

(c)

$$v_x=v_A-v_B=27.91V$$
$$i_x=\frac{v_B}{80}=0.1860\ A$$

3-3

(a) Writing node-voltage equations by inspection, we can get:

Node A:

$$\left(\frac{1}{4k\Omega}+\frac{1}{2k\Omega}\right)v_A-\frac{1}{2k\Omega}v_B=20mA-20mA=0$$

Node B:

$$-\frac{1}{2k\Omega}v_A+\left(\frac{1}{2k\Omega}+\frac{1}{4k\Omega}+\frac{1}{2k\Omega}\right)v_B=-20mA$$

Written in matrix form $\mathbf{Ax}=\mathbf{b}$

$$\begin{bmatrix}\left(\frac{1}{4k\Omega}+\frac{1}{2k\Omega}\right) & -\frac{1}{2k\Omega}\\ -\frac{1}{2k\Omega} & \left(\frac{1}{2k\Omega}+\frac{1}{4k\Omega}+\frac{1}{2k\Omega}\right)\end{bmatrix}\begin{bmatrix}v_A\\ v_B\end{bmatrix}=\begin{bmatrix}0\\ -20\end{bmatrix}$$

(b) Solve the equations or matrix form, we can get

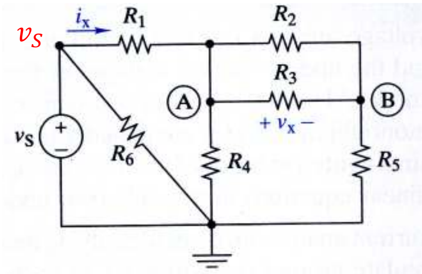
$$v_A=-14.545\ V$$
$$v_B=-21.818\ V$$

(c)

$$v_x=\frac{v_B}{2}=-10.909\ V$$

$$i_x=\frac{v_A-v_B}{2k\Omega}=3.6364\ mA$$

3-6



(a) Writing node-voltage equations by method 2, we can get:
Node A:

$$\left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}+\frac{1}{R_4}\right)v_A-\left(\frac{1}{R_2}+\frac{1}{R_3}\right)v_B=\frac{v_S}{R_1}$$

Node B:

$$-\left(\frac{1}{R_2}+\frac{1}{R_3}\right)v_A+\left(\frac{1}{R_2}+\frac{1}{R_3}+\frac{1}{R_5}\right)v_B=0$$

Written in matrix form $\mathbf{Ax}=\mathbf{b}$

$$\begin{bmatrix}\left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}+\frac{1}{R_4}\right) & -\left(\frac{1}{R_2}+\frac{1}{R_3}\right)\\ -\left(\frac{1}{R_2}+\frac{1}{R_3}\right) & \left(\frac{1}{R_2}+\frac{1}{R_3}+\frac{1}{R_5}\right)\end{bmatrix}\begin{bmatrix}v_A\\ v_B\end{bmatrix}=\begin{bmatrix}\frac{v_S}{R_1}\\ 0\end{bmatrix}$$

(b) By solving the two equations or the matrix form, we can get

$$v_A=9\,V,\, v_B=6V$$

$$i_x=\frac{v_S-v_A}{R_1}=1.5\,mA$$

$$v_x=v_A-v_B=3V$$

3-7

(a) Writing node-voltage equations by inspection, we can get:
Node A:

$$\left(\frac{1}{R_1}+\frac{1}{R_4}\right)v_A-\frac{1}{R_1}v_B-\frac{1}{R_4}v_C=i_{s1}$$

Node B:

$$-\frac{1}{R_1}v_A+\left(\frac{1}{R_1}+\frac{1}{R_2}\right)v_B+0\cdot v_C=i_{s2}$$

Node C:

$$-\frac{1}{R_4}v_A-0\cdot v_B+\left(\frac{1}{R_3}+\frac{1}{R_4}\right)v_c=-i_{s2}$$

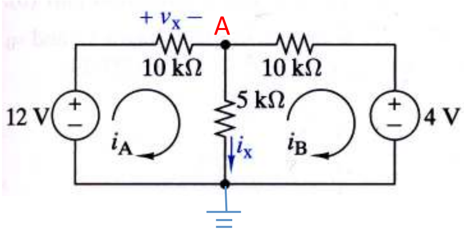
Written in matrix form $\mathbf{Ax}=\mathbf{b}$

$$\begin{bmatrix}\left(\frac{1}{R_1}+\frac{1}{R_4}\right) & -\frac{1}{R_1} & -\frac{1}{R_4}\\ -\frac{1}{R_1} & \left(\frac{1}{R_1}+\frac{1}{R_2}\right) & 0\\ -\frac{1}{R_4} & 0 & \left(\frac{1}{R_3}+\frac{1}{R_4}\right)\end{bmatrix}\begin{bmatrix}v_A\\ v_B\\ v_C\end{bmatrix}=\begin{bmatrix}i_{s1}\\ i_{s2}\\ -i_{s2}\end{bmatrix}$$

(c) By solving the two equations or the matrix form, we can get

$$v_A=5.98\,V,\, v_B=5.34\,V,\, v_C=-1.41\,V$$

3-9



(a) Writing node-voltage equations by inspection, we can get:

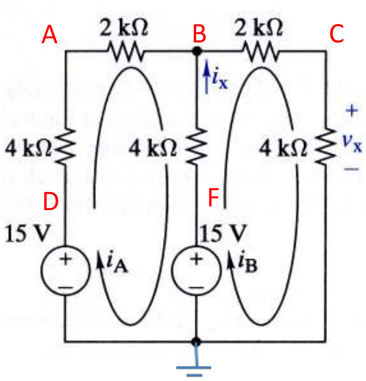
Node A: $\left(\frac{1}{10k\Omega}+\frac{1}{5k\Omega}+\frac{1}{10k\Omega}\right)v_A-\frac{1}{10k\Omega}\cdot 12V-\frac{1}{10k\Omega}\cdot 4V=0$

(b) Solving the equation, we can get $v_A=4V$.

So $i_A=\frac{12-v_A}{10k\Omega}=0.8\ mA$, $i_B=\frac{v_A-4}{10k\Omega}=0A$

(c) $v_x=12-v_A=8V$, $i_x=\frac{v_A}{5k\Omega}=0.8\ mA$

3-10



(a) Using method 2, writing node-voltage equations by inspection, we can get:

Node A: $\left(\frac{1}{2k\Omega}+\frac{1}{4k\Omega}\right)v_A-\frac{1}{2k\Omega}v_B-\frac{1}{4k\Omega}v_D=0$

Node B: $-\frac{1}{2k\Omega}v_A+\left(\frac{1}{2k\Omega}+\frac{1}{4k\Omega}+\frac{1}{2k\Omega}\right)v_B-\frac{1}{2k\Omega}v_C-\frac{1}{4k\Omega}v_F=0$

Node C: $-\frac{1}{2k\Omega}v_B+\left(\frac{1}{4k\Omega}+\frac{1}{2k\Omega}\right)v_C=0$

Knowing $v_D=15V$, $v_F=15V$, we can get the matrix form:

$$\begin{bmatrix}\left(\frac{1}{2k\Omega}+\frac{1}{4k\Omega}\right) & -\frac{1}{2k\Omega} & 0\\ -\frac{1}{2k\Omega} & \left(\frac{1}{2k\Omega}+\frac{1}{4k\Omega}+\frac{1}{2k\Omega}\right) & -\frac{1}{2k\Omega}\\ 0 & -\frac{1}{2k\Omega} & \left(\frac{1}{4k\Omega}+\frac{1}{2k\Omega}\right)\end{bmatrix}\begin{bmatrix}v_A\\ v_B\\ v_C\end{bmatrix}=\begin{bmatrix}\frac{15}{4k\Omega}\\ \frac{15}{4k\Omega}\\ 0\end{bmatrix}$$

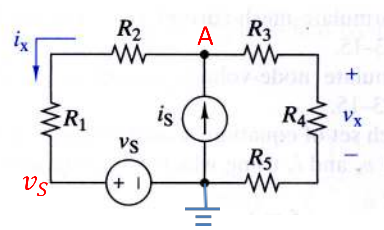
(b) Solving the matrix, we can get

$$v_A=12.14\ V, v_B=10.71V, v_C=7.14V$$

$$v_x=v_C=7.14V$$

$$i_x=\frac{15V-v_B}{4k\Omega}=1.07\ mA$$

3-12



(a) Use method 2 to write node-voltage equation.

Node A: $\left(\frac{1}{R_1+R_2}+\frac{1}{R_3+R_4+R_5}\right)v_A-\frac{1}{R_1+R_2}v_S-i_S=0 \;\Rightarrow\; v_A=37.5V$

(b) $v_x=\frac{R_4}{R_3+R_4+R_5}v_A=18.75V$, $i_x=\frac{v_A-v_S}{R_1+R_2}=0.025\ A$

(c) The total power $P=i_S v_A=3.75W$. Noting that the voltage source is absorbing power.

3-15

(b) Using method 2 to write node-voltage equation.

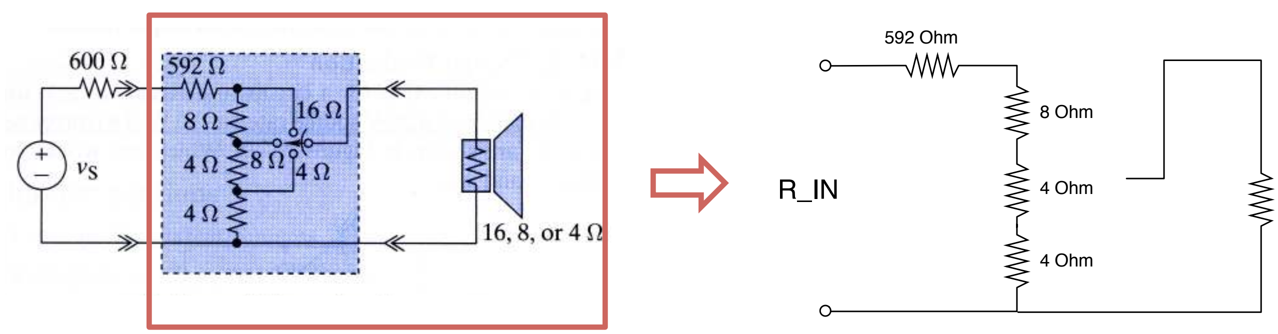
Node B: $-\frac{1}{2k\Omega}v_A+\left(\frac{1}{2k\Omega}+\frac{1}{4k\Omega}+\frac{1}{8k\Omega}\right)v_B-\frac{1}{8k\Omega}v_C=5mA$

Noting that $v_A=40V$, $v_C=-25V$, so we can get $v_B=25\ V$

(d) $v_x=v_B=25V$

$i_x=\frac{v_C-v_B}{8k\Omega}=-6.25mA.$

3-71



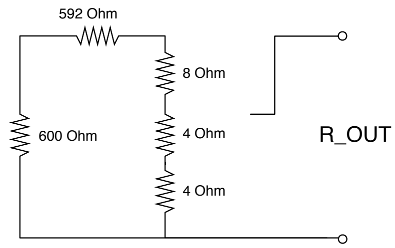
As shown in the figure, we can get R_{IN} when the connected speaker are 16, 8, 4Ω.

$$R_{IN}=592+(8+4+4)||16=600\ \Omega$$

$$R_{IN}=592+8+(4+4)||8=604\ \Omega$$

$$R_{IN}=592+8+4+4||4=608\ \Omega$$

The company claims that R_{IN} can vary in the range [588, 612]. So the claim for R_{IN} is true.



Now we calculate the value of R_{OUT}.

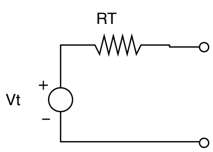
$R_{OUT} = (600 + 592)||(8 + 4 + 4) = 15.79\Omega$ It is in the range of $16 \pm 2\%$.

$R_{OUT} = (600 + 592 + 8)||(4 + 4) = 7.95\Omega$ It is in the range of $8 \pm 2\%$.

$R_{OUT} = (600 + 592 + 8 + 4)||4 = 3.98\Omega$ It is in the range of $4 \pm 2\%$.

The claim of the company is true.

3-74



To design a battery with large voltage, we need to combine batteries with small voltage. Since our goal voltage is 36V, there are only two ways to achieve it with given batteries: 4 first type or 9 second type.

If we choose first type, which is 4 batteries with $v_{OC} = 9V, R_t = 4\Omega$ in series, $R_T = 4\times4\Omega = 16\Omega > 10\Omega$.

If we choose second type, which is 9 batteries with $v_{OC} = 4V, R_t = 0.5\Omega$ in series, $R_T = 9\times0.5\Omega = 4.5\Omega < 10\Omega$.

It's clear that we have to choose second type. Also the weight of the combination is less for the second one.

The result is choosing 9 second type batteries in series.