MAE140 Win12 HW6 Solutions

T R & T, 9.1, 9.3, 9.8, 9.10, 9.11, 9.16, 9.20, 9.25, 9.26, 9.27

9-1

$$F(s) = \mathcal{L}\{f(t)\} = \mathcal{L}\{10[e^{-50t} - 2e^{-100t}]u(t)\}$$
$$= \frac{10}{s+50} - \frac{20}{s+100}$$
$$= -\frac{10s}{(s+50)(s+100)}$$

Zeros: s=0

Poles: s=-50, s=-100

9-3

$$F(s) = \mathcal{L}\{f(t)\} = \mathcal{L}\{25[1 - 3\cos(500t)]u(t)\}$$
$$= \frac{25}{s} - \frac{75s}{s^2+500^2} = \frac{25(-2s^2+500^2)}{s(s^2+500^2)}$$

Zeros: $s = \pm 250\sqrt{2}$

Poles: $s = 0, s = \pm i500$

9-8

(a) $F_1(s) = \mathcal{L}\{f_1(t)\} = 4 + \frac{20}{s+20} + \frac{40}{s+40} = \frac{4s^2+300s+4800}{(s+20)(s+40)}$

Zeros: -51.86, -23.14; Poles: -20, -40

(b) $F_2(s) = \mathcal{L}\{f_2(t)\} = \frac{15}{s} + \frac{15s}{s^2+500^2} = \frac{15(2s^2+500^2)}{s^3+500^2 s}$

Zeros: $\pm i250\sqrt{2}$, Poles: $\pm i500$, 0

9-10

a) $3e^{-2s}$

b) $\frac{10e^{-s}}{s+50}$

c) $\frac{10e^{-2s}}{s+50}$

9-11

$$\mathcal{L}\{f(t)\} = \frac{10}{s} + \frac{2}{s+10} + \frac{5e^{-0.05s}s}{s^2+100^2}$$

9-16

a)

$$F_1(s) = \frac{s+100}{s(s+50)} = \frac{2}{s} - \frac{1}{s+50}$$
$$\mathcal{L}^{-1}\{F_1(s)\} = (2 - e^{-50t})u(t)$$

b)

$$F_2(s) = \frac{s+10}{s(s+50)(s+100)} = \frac{k_1}{s} + \frac{k_2}{s+50} + \frac{k_3}{s+100}$$

We can solve for k_1, k_2, k_3 and get

$$F_2(s) = \frac{1}{500}\frac{1}{s} + \frac{2}{125}\frac{1}{s+50} - \frac{9}{500}\frac{1}{s+100}$$
$$\mathcal{L}^{-1}\{F_2(s)\} = \left(\frac{1}{500} + \frac{2}{125}e^{-50t} - \frac{9}{500}e^{-100t}\right)u(t)$$

9-20

a)

$$F(s) = \frac{\alpha^2}{s^2 \cdot (s+\alpha)} = \frac{-1}{s} + \frac{\alpha}{s^2} + \frac{1}{s+\alpha} \qquad f_1(t) = \left(-1 + \alpha \cdot t + e^{-\alpha \cdot t}\right) \cdot u(t)$$

b)

$$F_2(s) = \frac{\alpha^2}{s(s+\alpha)^2} = \frac{1}{s} - \frac{1}{s+\alpha} - \frac{\alpha}{(s+\alpha)^2} \;\Rightarrow\; f_2(t) = (1 - e^{-\alpha t} - \alpha t e^{-\alpha t})u(t)$$

9-25

a)

$$F_1(s) = \frac{4(s^2+16)}{s(s+4+4i)(s+4-4i)} = \frac{k_1}{s} + \frac{k_2}{s+4+4i} + \frac{k_2^*}{s+4-4i}$$

We can solve for k_1, k_2, k_2^* and get

$$k_1 = 2, \qquad k_2 = 1-3i, \qquad k_2^* = 1+3i.$$

$$|k_2| = \sqrt{10}, \theta = \arctan(3)$$

$$F_1(s) = \frac{2}{s} + \frac{1-3i}{s+4+4i} + \frac{1+3i}{s+4-4i}$$

$$\mathcal{L}^{-1}\{F_1(s)\} = \left(2 + 2\sqrt{10}e^{-4t}\cos\left(4t + acrtan(3)\right)\right)u(t)$$

b)

$$F_2(s) = \frac{2(s^2+30s+800)}{s(s+40)(s+10)} = \frac{4}{s} + \frac{2}{s+40} - \frac{4}{s+10}$$

$$\mathcal{L}^{-1}\{F_2(s)\} = (4 + 2e^{-40t} - 4e^{-10t})u(t)$$

9-26

a)

$$F_1(s) = \frac{(s+50)^2}{(s+20)^2(s+100)} = \frac{0.3906}{s+100} + \frac{0.6094}{s+20} + \frac{11.25}{(s+20)^2}$$

$$\mathcal{L}^{-1}\{F_1(s)\} = 0.3906e^{-100t} + 0.6094e^{-20t} + 11.25te^{-20t}$$

b)

$$F_2(s) = \frac{(s+20)^2}{(s+50)^2(s+100)} = \frac{2.56}{s+100} - \frac{1.56}{s+50} + \frac{18}{(s+50)^2}$$

$$\mathcal{L}^{-1}\{F_2(s)\} = 2.56e^{-100t} - 1.56e^{-20t} + 18te^{-20t}$$

9-27

$$F(s) = \frac{s+r}{s+20}$$

a)

$$\mathcal{L}\{f(t)\} = 1 - \frac{5}{s+20} = \frac{s+15}{s+20} \;\Rightarrow r = 15$$

b)

$$\mathcal{L}\{f(t)\} = 1 = \frac{s+20}{s+20} \;\Rightarrow r = 20$$

c)

$$\mathcal{L}\{f(t)\} = 1 + \frac{5}{s+20} = \frac{s+25}{s+20} \;\Rightarrow r = 25$$