MAE140 Win12 HW7 Solutions

T R & T, 10.4, 10.8, 10.10, 10.12, 10.14, 10.20, 10.22, 10.24, 10.32, 10.35

10-4

$$Z_{EQ}(s) = 2R + \frac{1}{\frac{1}{R} + \frac{1}{Ls}} = R\left[\frac{3Ls + R}{Ls + R}\right] = 3R\left[\frac{s + \frac{R}{3L}}{s + \frac{R}{L}}\right]$$

Pole at s=-R/L

Zero at s=-R/(3L)

10-8

$$Z_{EQ}(s) = \frac{1}{2Cs} + \left(2R + \frac{1}{Cs}\right)||2R = \frac{8R^2C^2s^2 + 8RCs + 1}{8RC^2s^2 + 2Cs}$$

Poles at s=0 and s=-1/(4RC)

Zeros at $s = \frac{-2\pm\sqrt{2}}{4RC}$

10-10

$$Z_{EQ}(s) = 2Ls + \frac{1}{\frac{1}{R} + \frac{1}{Ls}} + 2R = \frac{(2Ls + R)(Ls + 2R)}{Ls + R}$$

Pole at s=-R/L. Select R=20kΩ, L=10H

10-12

The initial condition of the inductor is $i_L(0) = 0$.

Changing the circuit into s-domain and using KVL at t>0, we can get:

$$\frac{V_A}{s} - L[sI_L(s) - i_L(0)] - I_L(s)R = 0$$

Solving for $I_L(s)$, we get

$$I_L(s) = \frac{V_A}{L}\left[\frac{1}{s\left(s + \frac{R}{L}\right)}\right] = \frac{V_A}{R}\left[\frac{1}{s} - \frac{1}{s + \frac{R}{L}}\right]$$

$$i_L(t) = \mathcal{L}^{-1}\{I_L(s)\} = \frac{V_A}{R}\left(1 - e^{-\left(\frac{R}{L}\right)t}\right)u(t)$$

10-14

The initial conditions of the capacitor are $i_C(0) = 0$, $v_C(0) = -\frac{V_A}{2}$.

Changing the circuit into s-domain and using KCL at t>0, we can get:

$$\frac{\frac{V_A}{S} - V_C(s)}{R} - \frac{V_C(s)}{R} - I_C(s) = 0$$

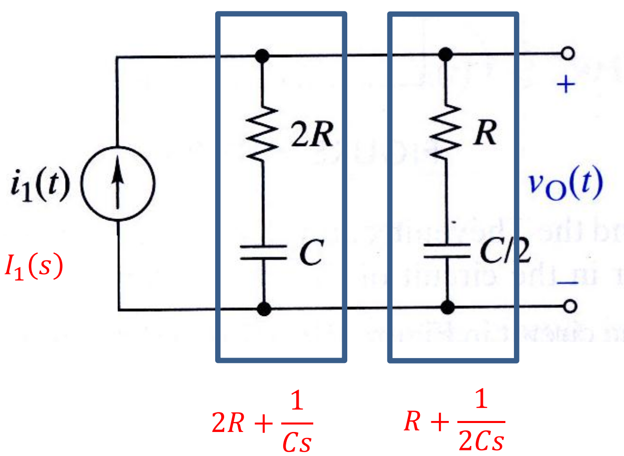
Where $I_C(s) = C\big(V_C(s)s - v_c(0)\big)$

Solving for $V_C(s)$, we can get

$$V_C(s) = V_A\left(\frac{1}{2s} - \frac{1}{s + \frac{2}{RC}}\right)$$

$$v_C(t) = \mathcal{L}^{-1}\{V_C(s)\} = \frac{V_A}{2} - V_A e^{-\frac{2}{RC}t}$$

10-20



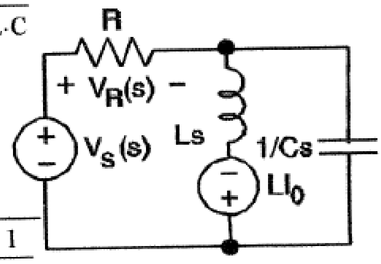
Change the circuit into s-domain and replace the elements by an equivalent impedance.

$$Z_{EQ} = \left(2R + \frac{1}{Cs}\right) || \left(R + \frac{2}{Cs}\right) = \frac{(2RCs + 1)(RCs + 2)}{3Cs(RCs + 1)}$$

$$V_O(s) = Z_{EQ} I_1(s) = \frac{(2RCs + 1)(RCs + 2)}{3Cs(RCs + 1)} I_1(s)$$

10-22 By voltage division

$$V_{Rzs}(s) = \frac{R\cdot V_S(s)}{\dfrac{1}{C\cdot s+\dfrac{1}{L\cdot s}}+R} = \frac{R\cdot V_S(s)\cdot\left(L\cdot C\cdot s^2+1\right)}{R\cdot L\cdot C\cdot s^2+L\cdot s+R} = \frac{\left(s^2+\dfrac{1}{L\cdot C}\right)\cdot V_S(s)}{s^2+\dfrac{s}{R\cdot C}+\dfrac{1}{L\cdot C}}$$



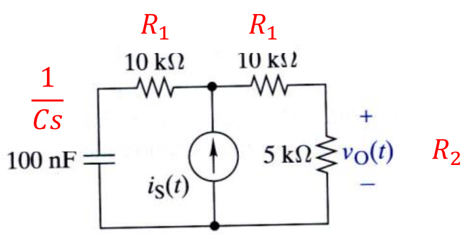
$$V_{Rzi}(s) = \frac{\left(\dfrac{1}{C\cdot s+\dfrac{1}{R}}\right)\cdot L\cdot I_0}{L\cdot s+\left(\dfrac{1}{C\cdot s+\dfrac{1}{R}}\right)} = \frac{R\cdot L\cdot I_0}{R\cdot L\cdot C\cdot s^2+L\cdot s+R} = \frac{\dfrac{I_0}{C}}{s^2+\dfrac{s}{R\cdot C}+\dfrac{1}{L\cdot C}}$$

(b)

$$V_R = V_{RZS} + V_{Rzi} = \frac{\left(s^2+\frac{1}{LC}\right)V_S(s)+\frac{I_0}{C}}{s^2+\frac{s}{RC}+\frac{1}{LC}} = \frac{\frac{15*s^2+\ 30000000}{s}+\ 10000}{s^2+\frac{2000*s}{3}+\ 2000000}$$

$$v_R(t) = \mathcal{L}^{-1}\{V_R(s)\} = 15u(t)$$

10-24



Transforming the circuit into the s-domain and using current division, we can get

$$I_O(s) = \frac{\frac{1}{R_1+R_2}}{\frac{1}{R_1+R_2}+\frac{1}{R_1+\frac{1}{Cs}}}I_S(s) = \frac{0.4(s+1000)}{s+400}I_S(s)$$

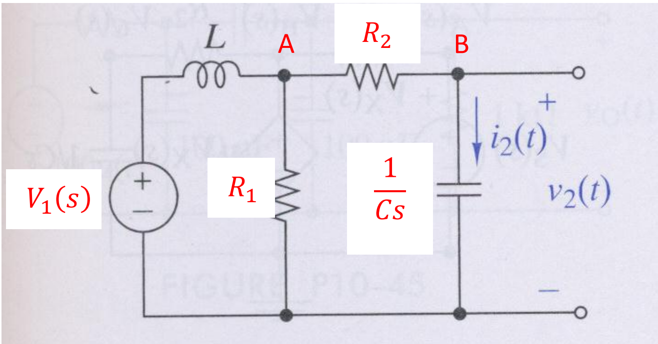
Where $I_S(s) = \mathcal{L}\{i_S(t)\} = 2.5\frac{1}{s+400}$.

$$I_O(s) = \frac{s+1000}{(s+400)^2} = \frac{1}{s+400} + \frac{600}{(s+400)^2}$$

$$i_O(t) = \mathcal{L}^{-1}\{I_O(s)\} = (e^{-400t} + 600te^{-400t})u(t)$$

The forced pole at s=-400rad/s, the natural pole at s=-400rad/s

10-32



(a) Node-Voltage
Node A:

$$\left(\frac{1}{R_1}+\frac{1}{Ls}+\frac{1}{R_2}\right)V_A-\frac{1}{R_2}V_B=\frac{V_1}{Ls}$$

Node B:

$$-\frac{1}{R_2}V_A+\left(\frac{1}{R_2}+Cs\right)V_B=0$$

$$\Delta(s)=\begin{vmatrix}\frac{1}{R_1}+\frac{1}{Ls}+\frac{1}{R_2} & -\frac{1}{R_2}\\ -\frac{1}{R_2} & \frac{1}{R_2}+Cs\end{vmatrix}=\frac{LC(R_1+R_2)s^2+(L+CR_1R_2)s+R_1}{LR_1R_2s}$$

$$\Delta_B(s)=\begin{vmatrix}\frac{1}{R_1}+\frac{1}{Ls}+\frac{1}{R_2} & \frac{V_1}{Ls}\\ -\frac{1}{R_2} & 0\end{vmatrix}=\frac{V_1}{LR_2s}$$

(b) Solve for $V_B(s)$ by Cramer's rule,

$$V_2(s)=V_B=\frac{\Delta_B(s)}{\Delta(s)}=\frac{R_1V_1}{LC(R_1+R_2)s^2+(L+CR_1R_2)s+R_1}$$

(c) The zeros of $\Delta(s)$ are natural poles:

$$s=\left(-\frac{1}{2}\right)\frac{L+CR_1R_2\pm\sqrt{(L+CR_1R_2)^2-4LCR_1(R_1+R_2)}}{LC(R_1+R_2)}$$

The pole of $\Delta_B(s)$ is the forced pole s=0.

(d) Substitute the values into the equation, we can get

$$V_2(s)=\frac{7500}{s\left(\frac{s^2}{1000}+s+500\right)}$$

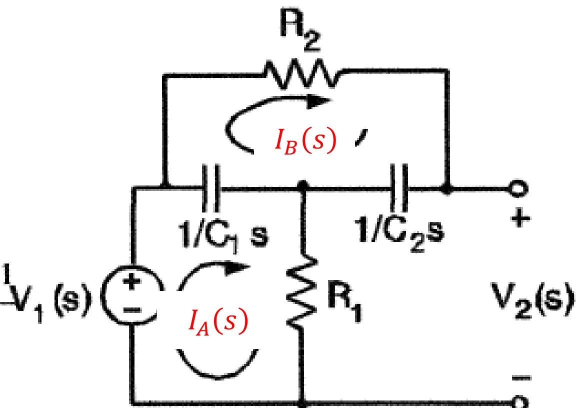
$$v_2(t)=\mathcal{L}^{-1}\{V_2(s)\}=\left(15\ -15\left(cos(500t)+\ sin(500t)\right)e^{-500t}\right)u(t)$$

10-35 (a) Mesh-current eqs.

$$\begin{pmatrix} R_1 + \dfrac{1}{C_1 \cdot s} & \dfrac{-1}{C_1 \cdot s} \\ \dfrac{-1}{C_1 \cdot s} & R_2 + \dfrac{1}{C_1 \cdot s} + \dfrac{1}{C_2 \cdot s} \end{pmatrix} \cdot \begin{pmatrix} I_A(s) \\ I_B(s) \end{pmatrix} = \begin{pmatrix} V_1(s) \\ 0 \end{pmatrix}$$

(b)

$$\Delta(s) = \frac{R_1 \cdot R_2 \cdot C_1 \cdot C_2 \cdot s^2 + (R_1 \cdot C_1 + R_2 \cdot C_2 + R_1 \cdot C_2) \cdot s + 1}{C_1 \cdot C_2 \cdot s^2}$$



$$\Delta_B = \begin{vmatrix} \dfrac{1}{R_1} + \dfrac{1}{C_1 s} & V_1 \\ -\dfrac{1}{C_1 s} & R_2 + \dfrac{1}{C_1 s} + \dfrac{1}{C_2 s} \end{vmatrix}$$

$$I_B(s) = \frac{\Delta_B(s)}{\Delta(s)} = \frac{C_2 s}{C_1 C_2 R_1 R_2 s^2 + (C_1 R_1 + C_2 R_1 + C_2 R_2)s + 1} V_1(s)$$

$$V_2(s) = V_1 - I_B R_2 = \frac{C_1 C_2 R_1 R_2 s^2 + (C_1 R_1 + C_2 R_1)s + 1}{C_1 C_2 R_1 R_2 s^2 + (C_1 R_1 + C_2 R_1 + C_2 R_2)s + 1} V_1(s)$$