MAE140 - Linear Circuits - Winter 2012
Midterm, February 7

Instructions

1. This exam is open book. You may use whatever written materials you choose, including your class notes and textbook. You may use a hand calculator with no communication capabilities
2. You have 70 minutes
3. Write your answers on your "Blue Book"
4. Do not forget to write your name, student number, and instructor

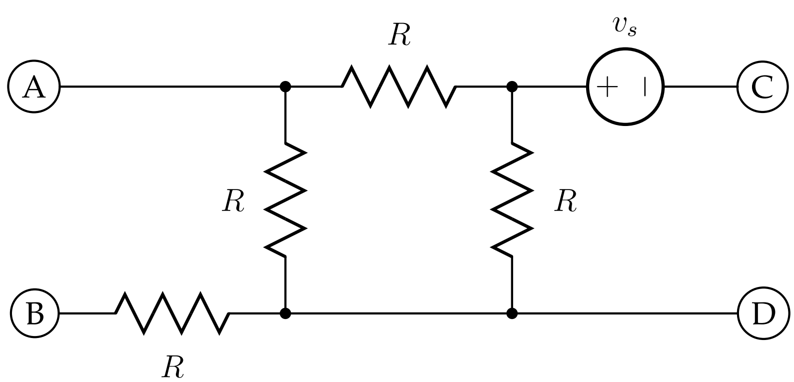
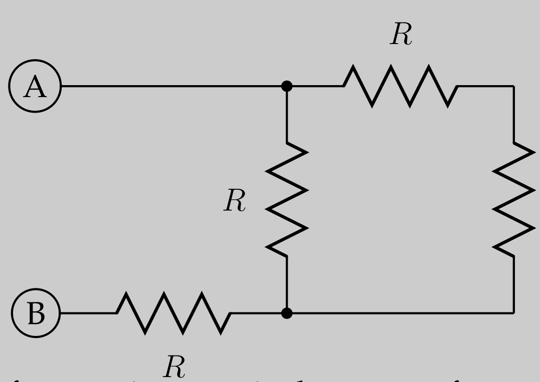


Figure 1: Circuit for Question 1.

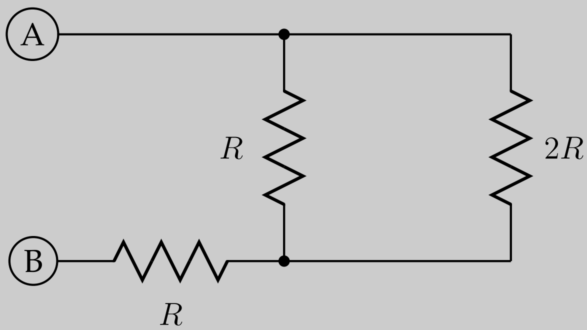
1. **Equivalent Circuits**
   Consider the circuit in Figure 1.
   (a) (2 points) Find the equivalent resistance as seen from terminals A and B.

Solution:
As the nodes C and D are not part of the analysis, the voltage source v_s can be ignored. The resulting circuit is



(+0.5 point)

After a resistor equivalence transformation we obtain



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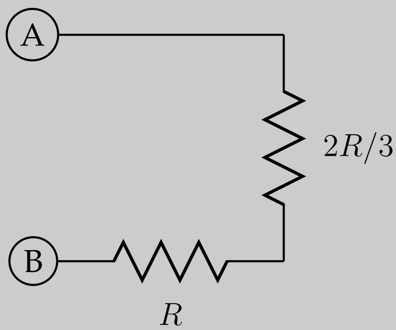

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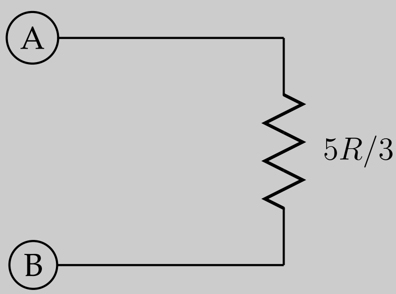
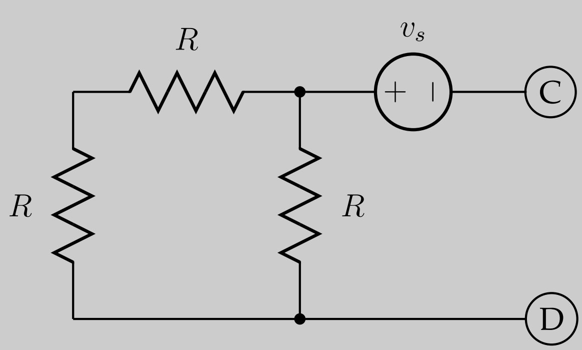


(+0.5 point)

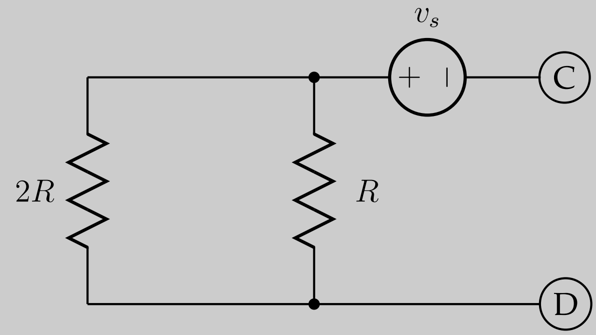
(b) (2 points) Find the Thévenin equivalent circuit as seen from terminals C and D.

Solution:
As the nodes A and B are not part of the analysis, the resistor R can be ignored. The resulting circuit is

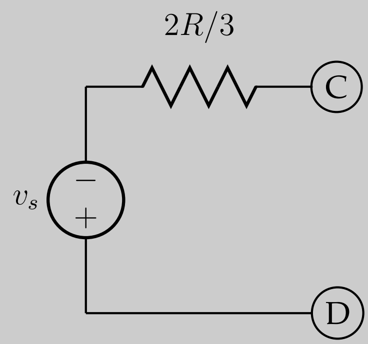


(+0.5 point)

After a series of resistor equivalence transformations we obtain



(+0.5 point)

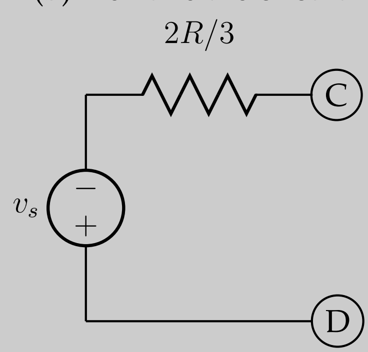


(+1 point)

(c) (2 points) Find the Norton equivalent circuit as seen from terminals C and D.

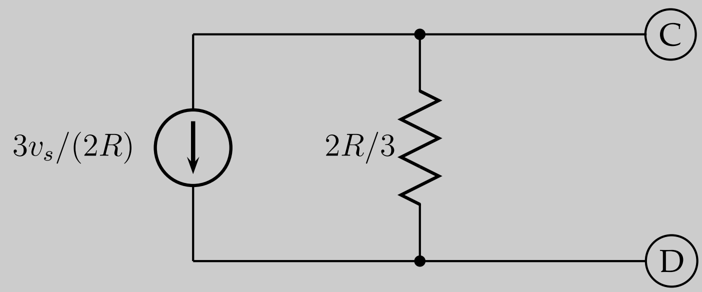
Solution:

Starting from the solution to item (b) we have the circuit



(+0.5 point)

A source transformation provides



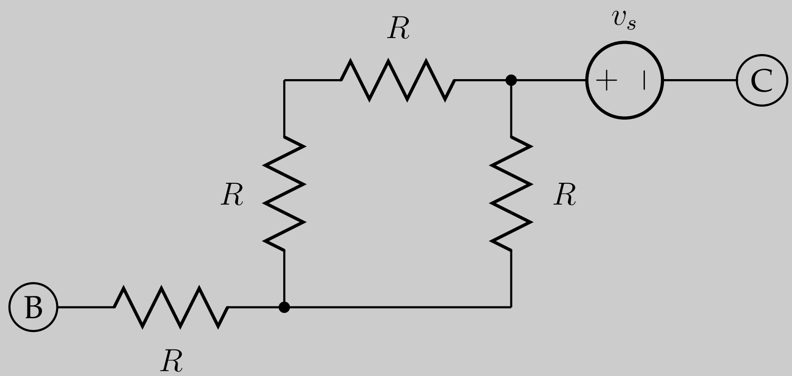
(+0.5 point)

(+1 point for having arrow in the right direction)

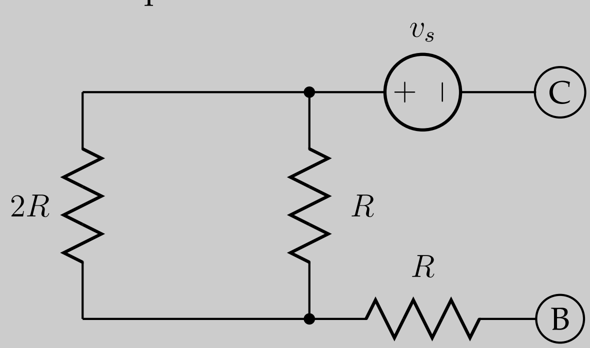
(d) (1 point (bonus)) Find the Norton equivalent circuit as seen from terminals C and B.

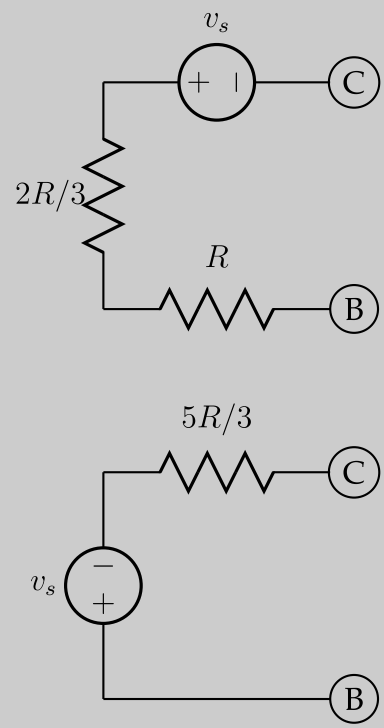
Solution:

From C and B the circuit looks like



After a series of resistor equivalence transformations we obtain





(+1 point)

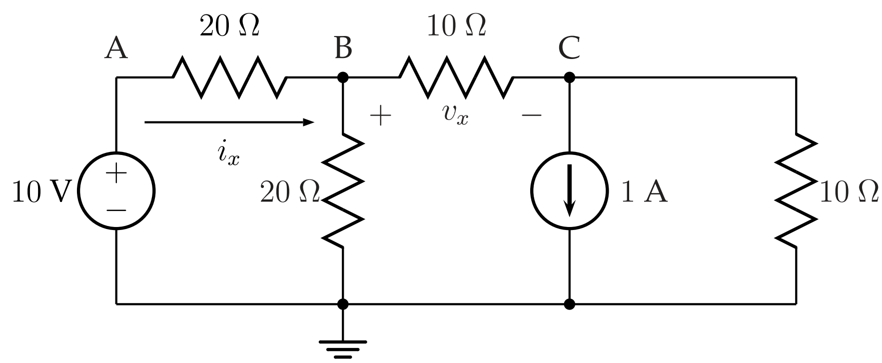


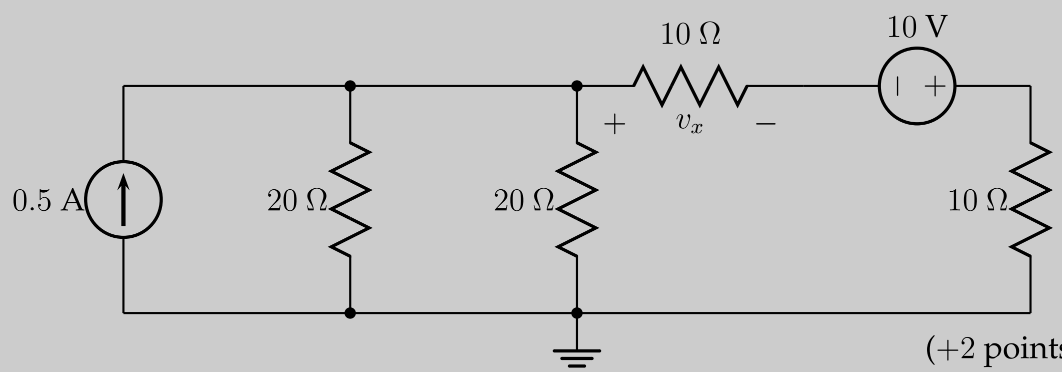
Figure 2: Circuit for Questions 2 and 3.

2. **Circuit Reduction and Analysis**
   Consider the circuit in Figure 2.

   (a) (4 points) Using resistor and source equivalence, transform the circuit in Figure 2 into a voltage divider circuit having a single voltage source from which the unknown voltage v_x can be directly calculated.

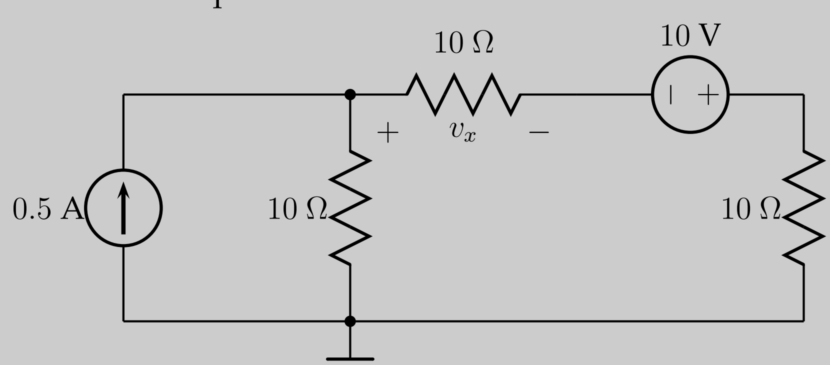
Solution:

Applying two source transformations we obtain the circuit



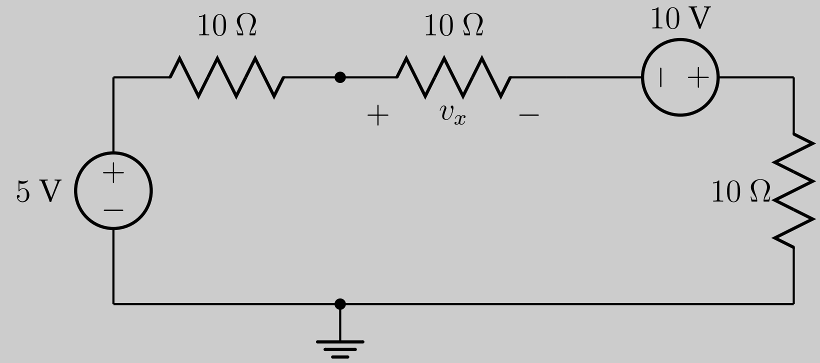
(+2 points)

After a resistor equivalence



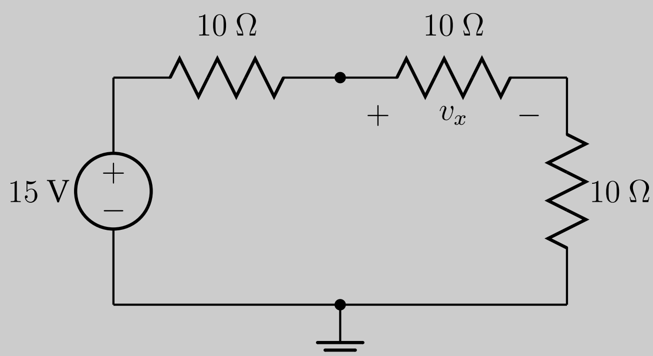
(+0.5 point)

and source transformation



(+1 point)

Combining the sources we obtain the voltage divider circuit



(+0.5 point)

(b) (2 points) Determine and compute the voltage v_x.

Solution:
From the voltage divider in part (a) we compute

$$v_x = \frac{10}{10 + 10 + 10} 15\text{V} = 5\text{V}. \quad \text{(+1 point)}$$

(+1 point for the correct numeric answer)

3. **Node Voltage Analysis**
Consider the circuit in Figure 2.

(a) (4 points) Formulate node-voltage equations for the circuit in Figure 2. Use the ground and node labels provided in the figure and clearly indicate the final equations, circuit unknowns and how you handled the presence of the voltage source.

Solution:
We will use Method # 2 to handle the voltage source (+0.5 point)
The voltage source determines the node voltage

$$v_A = 10\text{V}. \quad \text{(+1 point)}$$

Writing KCL by inspection for the remaining nodes:

$$\begin{bmatrix} -\frac{1}{20} & \frac{2}{20}+\frac{1}{10} & -\frac{1}{10} \\ 0 & -\frac{1}{10} & \frac{2}{10} \end{bmatrix} \begin{pmatrix} v_A \\ v_B \\ v_C \end{pmatrix} = \begin{pmatrix} 0 \\ -1 \end{pmatrix} \qquad \text{(+2 point)}$$

We should solve the above three equations on the unknowns v_A, v_B and v_C. (+0.5 point)

NOTE TO GRADERS: A solution by source transformation is also possible. In this case, assign 0.5 point for indicating the source transformation, 1 point for performing it correctly, 2 points for writing KCL correctly and 0.5 point for indicating which equations should be solved to determine the unknowns v_B and v_C.

(b) (2 points) Show how the voltage v_x and the current i_x are related to the node voltages.

Solution:

From the circuit diagram we obtain

$$v_x = v_B - v_C \qquad \text{(+1 point)}$$

and using Ohm's law

$$i_x = \frac{v_A - v_B}{20} \qquad \text{(+1 point)}$$

(c) (1 point (bonus)) Compute the voltage v_x.

Solution:

First we solve the equations obtained in part (a). Substituting for $v_A = 10$ V we obtain

$$\begin{bmatrix} \frac{1}{5} & -\frac{1}{10} \\ -\frac{1}{10} & \frac{1}{5} \end{bmatrix} \begin{pmatrix} v_B \\ v_C \end{pmatrix} = \begin{pmatrix} \frac{1}{2} \\ -1 \end{pmatrix}$$

from where

$$v_B = 0\text{V}, \qquad v_C = -5\text{V}$$

and

$$v_x = v_b - v_c = 0 - (-5) = 5\text{V}$$

(+1 point)

Do not modify the labels or the location of the ground. No need to solve any equations, except for the bonus point!